B

Register Number

PIMS® EDUCATIONAL ACADEMY

Regd. No. : 130/10

+2 MODEL EXAMINATION

PART III - MATHEMATICS

[English Version]

Time : 3 Hrs.] **[Max. Marks : 200**

SECTION - A

Note : (i) All questions are compulsory.

(ii) Each question carries one mark.

(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. If $\vec{p}$, $\vec{q}$ and $\vec{p} + \vec{q}$ are vectors of magniture λ then the magnitude of $|\vec{p} - \vec{q}|$ is

   1) 2λ 2) $\sqrt{3}\lambda$ 3) $\sqrt{2}\lambda$ 4) 1

2. If $u = \dfrac{1}{\sqrt{x^2+y^2}}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

   1) $\frac{1}{2}u$ 2) u 3) $\frac{3}{2}u$ 4) $-u$

3. If A is a square matrix of order n then $|\text{adj } A|$ is

   1) $|A|^2$ 2) $|A|^n$ 3) $|A|^{n-1}$ 4) $|A|$

4. If $f(x) = \dfrac{A}{\pi}\dfrac{1}{16+x^2}$ $-\infty < x < \infty$ is a p.d.f. of a continuous random variable X, then the value of A is

   1) 16 2) 8 3) 4 4) 1

5. The Rolle's constant for the function $y = x^2$ on $[-2, 2]$ is

   1) $\dfrac{2\sqrt{3}}{3}$ 2) 0 3) 2 4) -2

6. A monoid becomes a group if it satisfies the

   1) closure axiom 2) associative axiom 3) identity axiom 4) inverse axiom

7. The value of $[3] +_{11} ([5] +_{11} [6])$ is

   1) [0] 2) [1] 3) [2] 4) [3]

8. The surface area of the solid of revolution of the region bounded by $y = 2x$, $x = 0$ and $x = 2$ about x-axis is

   1) $8\sqrt{5}\pi$ 2) $2\sqrt{5}\pi$ 3) $\sqrt{5}\pi$ 4) $4\sqrt{5}\pi$

9. The co-ordinate of the vertices of the rectangular hyperbola $xy = 16$ are

   1) $(4, 4), (-4, -4)$ 2) $(2, 8), (-2, -8)$ 3) $(4, 0), (-4, 0)$ 4) $(8, 0), (-8, 0)$

10. In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system is

1) unique solution 2) two solutions 3) infinitely many solutions 4) no solutions

11. Chord AB is a diameter of the sphere $|\vec{r} - (2\vec{i} + \vec{j} - 6\vec{k})| = \sqrt{18}$ with co-ordinate A as (3, 2, -2). The co-ordinate of B is

1) (1, 0 , 10) 2) (– 1, 0 , – 10) 3) (–1, 0, 10) 4) (1, 0 , –10)

12. Which of the following is incorrect regarding n^{th} roots of unity ?

1) the number of distinct roots in n
2) the roots are in G.P. with common ratio is $\frac{2\pi}{n}$
3) the argument are in A.P. with common difference $\frac{2\pi}{n}$
4) product of the root is 0, the sum of the root is + 1

13. If p.d.f. of the standard normal variable z is $\phi(z)$

1) $\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}z^2}$ 2) $\frac{1}{\sqrt{2\pi}} e^{-z^2}$ 3) $\frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}z^2}$ 4) $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

14. If 2cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours without replacement, is

1) $\frac{1}{2}$ 2) $\frac{26}{51}$ 3) $\frac{25}{51}$ 4) $\frac{25}{102}$

15. The vector equation of the plane whose distance from the origin is p and perpendicular to a unit vector $\hat{n}$

1) $\vec{r} \cdot \vec{n} = p$ 2) $\vec{r} \cdot \hat{n} = q$ 3) $\vec{r} \times \vec{n} = p$ 4) $\vec{r} \cdot \hat{n} = p$

16. The stationary point of $f(x) = x^{3/5}(4 - x)$ occures at x =

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) 0 4) 4

17. If $Z_n = \cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}$ then $z_1 z_2 ..... z_6$ is

1) 1 2) – 1 3) i 4) -i

18. The amount present in a ratio active element disintegrates at a rate proportional to its amount. The differential equation corresponding to the above statment is (k is negative)

1) $\frac{dp}{dt} = \frac{k}{p}$ 2) $\frac{dp}{dt} = kt$ 3) $\frac{dp}{dt} = kp$ 4) $\frac{dp}{dt} = -kt$

19. If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

1) 0 2) 32 3) -16 4) -32

20. If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{i} + a\vec{j} - \vec{k}$ acting through the point $\vec{i} + \vec{j}$ is $\sqrt{8}$ then the value of 'a' is

1) 1 2) 2 3) 3 4) 4

21. Order and degree of the differential equation $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$

1) 2,1 2) 1,2 3) 2, ½ 4) 2, 2

22. Cramer's rule is applicable only when

1) $\Delta \neq 0$ 2) $\Delta = 0$ 3) $\Delta = 0, \Delta x \neq 0$ 4) $\Delta x = \Delta y = \Delta z = 0$

23. The point of intersection of the lines $\vec{r} = (-\vec{i} + 2\vec{j} + 3\vec{k}) + t(-2\vec{i} + \vec{j} + \vec{k})$ and $\vec{r} = (2\vec{i} + 3\vec{j} + 5\vec{k}) + s(\vec{i} + 2\vec{j} + 3\vec{k})$ is

1) (2,1,1) 2) (1,2,1) 3) (1,1,2) 4) (1,1,1)

24. $\int_0^{\infty} x^6 e^{\frac{-x}{2}} dx$

1) $\frac{\angle 6}{2^7}$ 2) $\frac{\angle 6}{2^6}$ 3) $2^6 \angle 6$ 4) $2^7 \angle 6$

25. The P.I. of $(3D^2 + D - 14)y = 13e^{2x}$ is

1) $26x\, e^{2x}$ 2) $13x\, e^{2x}$ 3) $x\, e^{2x}$ 4) $x^2 / 2e^{2x}$

26. The chord of contact of tangents from any point on the directrix of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through its

1) vertex 2) focus 3) directrix 4) latus rectum

27. If $f(a) = 2$; $f'(a) = 1$; $g(a) = -1$; $g'(a) = 2$ then the value of

$$\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$$

1) 5 2) -5 3) 3 4) -3

28. $\div$ is not a binary operation on ______

1) N 2) R 3) Z 4) C - { 0 }

29. The value of $\int_0^{\pi/4} \cos^3 2x\, dx$ is

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) 0 4) $\frac{2\pi}{3}$

30. The angle between the curves $y = e^{mx}$ and $y = e^{-mx}$ for $m > 1$ is

1) $\tan^{-1}\left(\frac{2m}{m^2-1}\right)$ 2) $\tan^{-1}\left(\frac{2m}{1-m^2}\right)$ 3) $\tan^{-1}\left(\frac{-2m}{1+m^2}\right)$ 4) $\tan^{-1}\left(\frac{2m}{m^2+1}\right)$

31. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

1) $\frac{1}{k^2} I$ 2) $\frac{1}{k^3} I$ 3) $\frac{1}{k} I$ 4) $k\, I$

32. If $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 8$ then $[\vec{a}, \vec{b}, \vec{c}]$

1) 4 2) 16 3) 32 4) -4

33. Which of the following is a tautology ?

1) $p \vee q$ 2) $p \wedge q$ 3) $p \vee \sim q$ 4) $p \wedge \sim p$

34. For a Poisson distribution with parameter $\lambda = 0.25$ the value of the 2^{nd} moment about the origin is

1) 0.25 2) 0.3125 3) 0.0625 4) 0.025

35. If $a = \cos\alpha - i\sin\alpha$, $b = \cos\beta - i\sin\beta$, $c = \cos\gamma - i\sin\gamma$ then $(a^2 c^2 - b^2) / abc$ is

1) $\cos 2(\alpha - \beta + \gamma) + i \sin 2(\alpha - \beta + \gamma)$ 2) $-2\cos(\alpha - \beta + \gamma)$
3) $-2i\sin(\alpha - \beta + \gamma)$ 4) $2\cos(\alpha - \beta + \gamma)$

36. In which region the curve $y^2(a+x) = x^2(3a - x)$ does not lie ?

1) $x > 0$ 2) $0 < x < 3a$ 3) $x < -a$ and $x > 3a$ 4) $-a < x < 3a$

37. The angle between the two tangents drawn from the point $(-4, 4)$ to $y^2 = 16x$ is

1) 45^0 2) 30^0 3) 60^0 4) 90^0

38. The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxillary circle is

1) $\pi b(a - b)$ 2) $2\pi a(a - b)$ 3) $\pi a(a - b)$ 4) $2\pi b(a - b)$

39. The length of the semi-major and the length of semi minor axis of the ellipse $\frac{x^2}{144} + \frac{y^2}{169} = 1$ are

1) 26, 12 2) 13, 24 3) 12, 26 4) 13, 12

40. The differential equation obtained by eliminating 'a' and 'b' from $y = ae^{3x} + be^{-3x}$ is

1) $\frac{d^2y}{dx^2} + ay = 0$ 2) $\frac{d^2y}{dx^2} - 9y = 0$ 3) $\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0$ 4) $\frac{d^2y}{dx^2} + 9x = 0$

SECTION - B

Note : (i) Answer any *ten* questions.
(ii) Question No.55 in compulsory and choose any nine questions from the remaining.
(iii) Each question carries six marks. 10 x 6 = 60

41. Solve the following non-homogeneous equations of three unknowns

$$x + y + 2z = 4$$
$$2x + 2y + 4z = 8$$
$$3x + 3y + 6z = 10$$

42. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

43. a) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ then

Prove that $(\vec{c} \times \vec{a}) \times \vec{b} = \vec{0}$

b) Find the distance between the parallel planes $\vec{r} \cdot (-\vec{i} - \vec{j} + \vec{k}) = 3$ and $\vec{r} \cdot (\vec{i} + \vec{j} - \vec{k}) = 5$

44. Show that the points A (1 , 2, 3) B (3, -1, 2), C (-2 , 3 , 1) and D (6, -4 , 2) are lying on the same plane.

45. Prove that the points representing the complex numbers $2i$, $1 + i$, $4 + 4i$ and $3 + 5i$ on the Argand plane are the vertices of a rectangle.

46. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

47. Obtain the Maclaurin's Series for : arc tan x or $\tan^{-1} x$

48. Find the intervals in which $f(x) = 2x^3 + x^2 - 20x$ is increasing and decreasing.

49. The time of swing T of a pendulum is given by $T = k\sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of the pendulum l changes from 32.1 cm to 32.0 cm.

50. Evaluate : $\int_0^1 x\, e^{-4x}\, dx$

51. a) Solve : $(D^2 + 6D + 9)\, y = 0$

b) Form the differential equation from the following : $y = e^{2x}(A + Bx)$

52. Find the Mean and Variance for the probability density functions : $f(x) = \begin{cases} xe^{-x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$

53. State and prove cancellation laws on group.

54. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

55. (a) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

(OR)

(b) Find the square root of $(-8 - 6i)$

SECTION - C

Note : (i) Answer any *ten* questions.

(ii) Question No.70 is compulsory and choose any nine questions from the remaining.

(iii) Each question carries ten marks. 10 x 10 = 100

56. Investigate for what values of λ, μ the simultaneus equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

57. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find their point of intersection.

58. Find the vector and cartesian equation of the plane containing the line.

$$\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3} \text{ and parallel to the line } \frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$$

59. *P* represents the variable complex number *z,* find the locus of *P* if

$$\mathrm{Re}\left(\frac{z+1}{z+i}\right) = 1$$

60. The arch of a bridge is in the shape of a semi-ellipse having a horizontal span of 40 ft and 16ft high at the centre. How high is the arch, 9ft from the right or left of the centre.

61. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points (6, 0) and (-3, 0).

62. A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find (i) the initial velocity of the missile, (ii) the time when the height of the missile is a maximum (iii) the maximum height reached and (iv) the velocity with which the missile strikes the ground.

63. A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area ?

64. Using Euler's theorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ if

$$u = \sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$$

65. Compute the area between the curve $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

66. Prove that the curved surface area of a sphere of radius *r* intercepted between two parallel planes at a distance *a* and *b* from the centre of the sphere is $2\pi r\,(b - a)$ and hence deduct the surface area of the sphere. $(b > a)$

67. Solve $(1 - x^3)\frac{dy}{dx} - 3x^2 y = \sec^2 x$.

68. Let G be the set of all rational numbers except 1 and * be defined on G by $a * b = a + b - ab$ for all $a, b \in G$. Show that (G, *) is an infinite abelian group.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year [$e^{-3} = 0.0498$]

70. a) In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and the end of 4 hours 21 grams. How many grams of the substance was there initially ?

(OR)

b) Find the eccentricity, centre, foci and vertices of the hyperbola :

$$x^2 - 3y^2 + 6x + 6y + 18 = 0$$