A

PIMS EDUCATIONAL ACADEMY Regd. No. : 130/10

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+2 MODEL EXAMINATION

PART III - MATHEMATICS

[English Version]

Time : 3 Hrs.] **[Max. Marks : 200**

SECTION - A

Note : (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is

1) 1 2) 2 3) 3 4) any real number

2. If I is the unit matrix of orders n, where $k \neq 0$ is a constant, then adj (kI) =

1) k^n (adj A) 2) k (adjI) 3) k^2 (adj (I)) 4) k^{n-1} (adj I)

3. In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has

1) unique solution 2) two solutions
3) infinitely many solutions 4) no solutions

4. $(A^T)^{-1}$ = ____

1) A^{-1} 2) A^T 3) A^1 4) $(A^{-1})^T$

5. If $\vec{a}$ and $\vec{b}$ include an angle 120 and their magnitude are 2 and $\sqrt{3}$ then $\vec{a} \cdot \vec{b}$ is equal to

1) $\sqrt{3}$ 2) $-\sqrt{3}$ 3) 2 4) $-\frac{\sqrt{3}}{2}$

6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then

1) $\vec{a}$ is parallel to $\vec{b}$ 2) $\vec{a}$ is perpendicular to $\vec{b}$
3) $|\vec{a}| = |\vec{b}|$ 4) $\vec{a}$ and $\vec{b}$ are unit vectors

7. If $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = 64$ then $[\vec{a}, \vec{b}, \vec{c}]$ is

1) 32 2) 8 3) 128 4) 0

8. If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{i} + a\vec{j} - \vec{k}$ acting through the point $\vec{i} + \vec{j}$ is $\sqrt{8}$ then the value of 'a' is

1) 1 2) 2 3) 3 4) 4

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9. The angle between the line $\vec{r} = \vec{a} + t\,\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = q$ is connected by the relation

1) $\cos\theta = \dfrac{\vec{a} \cdot \vec{n}}{q}$ 2) $\cos\theta = \dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{h}|}$ 3) $\sin\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{n}|}$ 4) $\sin\theta = \dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|}$

10. The centre and radius of sphere $|\vec{r} - (2\vec{i} - \vec{j} + 4\vec{k})| = 5$

1) (2, -1, 4) , 5 2) (2, 1, 4) , 5 3) (-2, 1, 4) , 6 4) (2, +1, -4) , 5

11. If $x^2 + y^2 = 1$ then the value of $\dfrac{1 + x + iy}{1 + x - iy}$ is

1) x - iy 2) 2x 3) - 2iy 4) x + iy

12. If the point represented by the complex number *iz* is rotated about the origin through the angle $\dfrac{\pi}{2}$ in the counter clockwise direction then the complex number representing the new position is

1) iz 2) - iz 3) - z 4) z

13. $z_1 = 4 + 5i,\ z_2 = -3 + 2i$ then $\dfrac{z_1}{z_2}$ is

1) $\dfrac{2}{13} - \dfrac{22}{13}i$ 2) $-\dfrac{2}{13} + \dfrac{22}{13}i$ 3) $\dfrac{-2}{13} - \dfrac{23}{13}i$ 4) $\dfrac{2}{13} + \dfrac{22}{13}i$

14. The value of $\left[\dfrac{-1 + i\sqrt{3}}{2}\right]^5 + \left[\dfrac{-1 - i\sqrt{3}}{2}\right]^5$

1) 2 2) 0 3) −1 4) 1

15. The length of the latus rectum of the parabola whose vertrex is (2, -3) and the directrix $x = 4$ is

1) 2 2) 4 3) 6 4) 8

16. The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is

1) 4 2) 3 3) 2 4) 6

17. The length of the latus rectum of the rectangular hyperbola $xy = 32$ is

1) $8\sqrt{2}$ 2) 32 3) 8 4) 16

18. Equation of Latus rectum of the parabola $x^2 = 20y$

1) x - 5 = 0 2) y - 5 = 0 3) y + 5 = 0 4) x + 5 = 0

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19. The slope of the normal to the curve $y = 3x^2$ at the point whose x coordinate is 2 is mathstimes.com

1) $\frac{1}{13}$ 2) $\frac{1}{14}$ 3) $\frac{-1}{12}$ 4) $\frac{1}{12}$

20. If the length of the diagonal of a square is increasing at the rate of 0.1 cm / sec. What is the rate of increase of its area when the side of $\frac{15}{\sqrt{2}}$ cm ?

1) 1.5 cm^2/sec 2) 3 cm^2/sec 3) $3\sqrt{2}$ cm^2/sec 4) 0.15 cm^2/sec

21. If $f(x) = x^2 - 4x + 5$ on $[0, 3]$ then the absolute maximum value is

1) 2 2) 3 3) 4 4) 5

22. The statement : If f has a local extremum (maximum or minimum) at C and $f^1(c)$ exists then $f^1(c) = 0$ is

1) the extreme value theorem 2) Fermat's theorem

3) Law of mean 4) Rolle's theorem

23. The curve $y^2(x - 2) = x^2(1+x)$ has

1) an asymptote parallel to x-axis 2) an asymptote parallel to y-axis

3) asymptotes parallel to both axes 4) no asymptotes

24. If $u = y \sin x$, then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to

1) cos x 2) cos y 3) sin x 4) 0

25. The value of $\int_{-\pi/2}^{\pi/2} \left(\frac{\sin x}{2 + \cos x}\right) dx$ is

1) 0 2) 2 3) log 2 4) log 4

26. The volume generated when the region bounded by $y = x$, $y = 1$, $x = 0$ is rotated about y - axis is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$

27. The surface area of the solid of revolution of the region bounded by $y = 2x$, $x = 0$ and $x = 2$ about x-axis is

1) $8\sqrt{5}\pi$ 2) $2\sqrt{5}\pi$ 3) $\sqrt{5}\pi$ 4) $4\sqrt{5}\pi$

28. $\int_a^b f(x)\, dx$ = _______

1) $2\int_b^a f(x)\, dx$ 2) $\int_a^b f(a-x)dx$ 3) $\int_a^b f(b-x)\, dx$ 4) $\int_a^b f(a + b - x)\, dx$

29. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x^2}$ is

1) e^x 2) log x 3) $\frac{1}{x}$ 4) e^{-x}

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30. The differential equation satisfied by all the straight lines in xy plane is

1) $\frac{dy}{dx}$ = a constant 2) $\frac{d^2y}{dx^2} = 0$ 3) $y + \frac{dy}{dx} = 0$ 4) $\frac{d^2y}{dx^2} + y = 0$

31. The particular integral of the differential equation $f(D)\,y = e^{ax}$ where $f(D) = (D - a)g(D)$, $g(a) \neq 0$ is

1) me^{ax} 2) $\frac{e^{ax}}{g(a)}$ 3) $g(a)\,e^{ax}$ 4) $\frac{xe^{ax}}{g(a)}$

32. The order and degree of the differential equation $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$ are

1) 2, 1 2) 1, 2 3) 2, ½ 4) 2, 2

33. The conditional statement $p \rightarrow q$ is equivalent to

1) $p \vee q$ 2) $p \vee \sim q$ 3) $\sim p \vee q$ 4) $p \wedge q$

34. In the set of integers with operation * defined by $a * b = a + b - ab$, the value of 3 * (4 * 5) is

1) 25 2) 15 3) 10 4) 5

35. Which of the following is correct ?

1) An element of a group can have more thane one inverse
2) If every element of a group is its own inverse, then the group is abelian.
3) The set of all 2 x 2 real matrices forms a group under matrix multiplication.
4) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$

36. '_' is a binary operation on

1) N 2) Q - {0} 3) R - {0} 4) Z

37. A random variable X has the following probability mass function as follows :

X	-2	3	1
(P(X = x)	$\frac{l}{6}$	$\frac{l}{4}$	$\frac{l}{12}$

Then the value of λ is

1) 1 2) 2 3) 3 4) 4

38. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls without replacement, is

1) $\frac{1}{20}$ 2) $\frac{18}{125}$ 3) $\frac{4}{25}$ 4) $\frac{3}{10}$

39. In a Poisson distribution if $P(X = 2) = P(X = 3)$ then the value of its parameter λ is a

1) 6 2) 2 3) 3 4) 0

40. Mean and variance of standard normal distribution are

1) μ, σ^2 2) μ, σ 3) 0, 1 4) 1, 1

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Note : (i) Answer any *ten* questions.
(ii) Question No.55 in compulsory and choose any nine questions from the remaining.
(iii) Each question carries six marks. **10 x 6 = 60**

41. Solve by matrix inversion method :
$7x + 3y = -1,\ 2x + y = 0$

42. Examine the consistency of the equations. If it is consistent then solve the same.
$x - 4y + 7z = 14 \qquad 3x + 8y - 2z = 13 \qquad 7x - 8y + 26z = 5$

43. a) Show that the vectors $2\vec{i} - \vec{j} + \vec{k},\ \vec{i} - 3\vec{j} - 5\vec{k},\ -3\vec{i} + 4\vec{j} + 4\vec{k}$ form the sides of a right angled triangle

b) The work done by the force $\vec{F} = a\vec{i} + \vec{j} + \vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight line is given to be 5 units. Find the value of a

44. Show that diameter of a sphere subtends a right angle at a point on the surface.

45. If $\arg(z - 1) = \frac{\pi}{6}$ and $\arg(z + 1) = 2\frac{\pi}{3}$ then prove that $|z| = 1$

46. Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}$

47. Find the equation of the rectangular hyperbola which has its centre at (2, 1), one of its asymptotes $3x - y - 5 = 0$ and which passes through the point (1, -1).

48. A cylindrical hole 4mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12 mm. Estimate the amount of metal removed.

49. Find two positive numbers whose product is 100 and whose sum is minimum.

50. If $w = x + 2y + z^2$ and $x = \sin t$; $y = \cos t$; $z = t$. Find $\frac{dw}{dt}$

51. Find the volume of the solid that results when the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b > 0$) is revolved about the minor axis

52. Solve : $(2D^2 + 5D + 2)y = e^{-\frac{1}{2}x}$

53. Find the order of each element in the group $(z_5 - \{[0]\}, .5)$

54. i) For the probability density function
$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x < 0 \end{cases}, \text{ find } F(2)$$
ii) Prove that the total probability is one.

55. (a) Find the Mean and Variance for probability density function : $f(x) = \begin{cases} \frac{1}{24}, & -12 < x < 12 \\ 0, & \text{otherwise} \end{cases}$

(OR)

(b) Construct the truth tabe for $(p \wedge q) \vee (\sim r)$

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SECTION - C

Note : (i) Answer any *ten* questions.

(ii) Question No.70 is compulsory and choose any nine questions from the remaining.

(iii) Each question carries ten marks. 10 x 10 = 100

56. Verify whether the given system of equations is consistent. If it is consistent, solve them.

$2x + 5y + 7z = 52,\ \ x + y + z = 9,\ \ 2x + y - z = 0$

57. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find their point of intersection.

58. Derive the equation of the plane in the intercept form.

59. Solve : $x^4 + x^3 + x^2 + x + 1 = 0$

60. A kho-kho player in a practice session while running realises that the sum of the distances from the two kho-kho poles from his is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

61. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, (2,4) is the centre of the hyperbola and it passes through (2, 0)

62. Evaluate : $\lim_{x \to 0} (\cot x)^{\sin x}$

63. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ when $u = \sin 3x \cos 4y$

64. Find the area bounded by x - axis and an arch of the cycloid $x = a\ (2t - \sin 2t)$, $y = a\ (1 - \cos 2t)$

65. Find the surface area of the solid generated by revolving the arc of the parabola $y^2 = 4ax$, bounded by its latus rectum about x - axis.

66. Solve : $dx + xdy = e^{-y} \sec^2 y\ dy$

67. A drug is excreted in a patients urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of durg at time $t = 0$, which is excreted at a Rate of $-3t^{1/2}$ mg/h. (i) What is the general equation for the amount of drug in the patient at time $t > 0$? (ii) When will the patient be drug free ?

68. Show that the set G of all rational numbers except -1 forms an abelian group with respect to the operation

*given by $a * b = a + b + ab$ for all $a, b \in G$.

69. The mean weight of 500 male students in a certain college in 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh (i) between 120 and 155 pounds (ii) more than 185 pounds.

Z	2.067	0.2667	2.2667
Area	0.4803	0.1026	0.4881

70. a) Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectum of the parabola and hence draw the graph : $y^2 - 8x + 6y + 9 = 0$

(OR)

b) Find the points of inflection and determine the intervals of convexity and concavity of the Gaussion curve $y = e^{-x^2}$

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