MODEL HIGHER SECONDARY EXAMINATION

KEY FOR MATHS

PART - I

Q. No.	Key	Answer	Q. No.	Key	Answer
1.	a	1	25.	b	4
2.	c	$k \neq -4$	26.	d	inverse axiom
3.	a	1	27.	d	[3]
4.	c	$\vec{u} = \vec{0}$	28.	c	$\frac{1}{9}$
5.	a	$\tan^{-1} 1/3$	29.	d	8
6.	d	abc	30.	a	1
7.	a	(0, 0, -4)	31.	c	$1 - P(x < a)$
8.	a	$(-\frac{1}{2}, -8)$	32.	d	Q - {0}
9.	a	purely imaginary	33.	d	2, 2
10.	a	the straight line x = 1/4	34.	b	$-\int_c^d x\,dy$
11.	c	8	35.	a	1
12.	c	$2 \tan^{-1}(\frac{3}{4})$	36.	b	8/3
13.	c	$x = -\frac{17}{4}$	37.	b	q
14.	c	$\frac{x}{b}$	38.	c	$\lvert\vec{r}\rvert = \lvert\vec{a}\rvert$
15.	c	$3\theta = 27t + 80$	39.	a	$\pi/4$
16.	a	$-\frac{\pi}{4}$	40.	c	consistant
17.	b	an asymptote parallel to y-axis			
18.	b	$\frac{1}{11}$			
19.	b	$\frac{1}{30}$			
20.	b	$\sqrt{2} - 1$			
21.	a	20π			
22.	d	-tan x			
23.	b	A cos x + B sin x			
24.	b	$\frac{d^2y}{dx^2} = 0$			

41. $|A| = 2$ **- 1 mark**

adj A = $\begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix}$ **- 1 mark**

A (adj A) = $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = |A| I$ **- 2 marks**

(adj A) A = $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = |A| I$ **- 2 marks**

A (adj A) = (adj A) A = $|A| I_2$

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45. A ~ $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ -2 & 1 & 3 & 4 \end{bmatrix}$ $R_1 \leftrightarrow R_3$ **- 1 mark**

~ $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 11 & 18 \end{bmatrix}$ R_1, R_2, $R_3 \to R_3 + 2R_1$ **- 2 marks**

~ $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 4 \end{bmatrix}$ R_1, R_2, $R_3 \to R_3 - 7R_2$ **- 2 marks**

$\rho(A) = 3$ **- 1 mark**

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43. (a) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = 8\vec{i} - 10\vec{j} + 4\vec{k}$ **- 1 mark**

Area = $|\vec{a} \times \vec{b}| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{180}$ **- 1 mark**

Area = $6\sqrt{5}$ sq units **- 1 mark**

(b) V = $[\vec{a}\ \vec{b}\ \vec{c}]$ **- 1 mark**

$$\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

$\lambda = -3$ **- 2 marks**

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44. $\vec{r}^{\,2} = x^2 + y^2 + z^2$ **- 1 mark**

$\Rightarrow x^2 + y^2 + z^2 - 8x + 6y - 10z - 50 = 0$ **- 3 marks**

Centre = (4, -3, 5) **- 1 mark**

Radius = 10 units **- 1 mark**

45. $| Z_1 + Z_2 | \leq | Z_1 | + | Z_2 |$ **- 2 marks**

$| Z_1 + Z_2 |^2 = (Z_1 + Z_2)\,(\overline{Z_1 + Z_2})$ **- 1 mark**

$= |Z_1|^2 + | Z_2|^2 + 2\text{Re}\,(Z_1\,\overline{Z}_2)$ **- 1 mark**

$\leq | Z_1 |^2 + | Z_2 |^2 + 2\,| Z_1\,\overline{Z}_2 |$ **- 1 mark**

$| Z_1 + Z_2 | \leq | Z_1 | + | Z_2 |$ **- 1 mark**

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46. $r = 2$ **- 1 mark**

$\theta = \pi/6$ **- 1 mark**

$(\sqrt{3} + i)^n = 2^n\,(\cos n\pi/6 + i \sin n\,\pi/6)$ **- 2 marks**

$(\sqrt{3} - i)^n = 2^n\,(\cos n\,\pi/6 - i \sin n\,\pi/6)$ **- 1 mark**

$(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos n\,\pi/6$ **- 1 mark**

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47. $a = 4\sqrt{2}$ **- 1 mark**

$c^2 = 16$ **- 1 mark**

The centre is the midpoint of AA' = (1, 3) **- 1 mark**

Equation of the R.H with centre (1, 3) is $(x - 1)(y - 3) = 16$ **- 1 mark**

The combined equation of the asymptotes is $(x - 1)(y - 3) = 0$ **- 1 mark**

Separate equations are $x - 1 = 0$ and $y - 3 = 0$ **- 1 mark**

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48. Let $f(x) = \log_e (1 + x)\ ;\ f(o) = 0$ **- 1 mark**

$f'(x) = \dfrac{1}{1 + x}\ ;\ f'(o) = 1$ **- 1 mark**

$f''(x) = -\dfrac{1}{(1 + x)^2}\ ;\ f''(o) = -1$ **- 1 mark**

$f'''(x) = \dfrac{2}{(1 + x)^3}\ ;\ f'''(0) = 2$ **- 1 mark**

$f(x) = f(0) + \dfrac{x}{1!}\ f'(0) + \dfrac{x^2}{2!}\ f''(0) + \dfrac{x^3}{3!}\ f'''(0) + ...$ **- 1 mark**

$\log (1 + x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + ...$ **- 1 mark**

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49. Diagram **- 1 mark**

$A = \int_a^b y\, dx$

$A = 4\int_0^a \dfrac{b}{a} \sqrt{a^2 - x^2}\, dx$ **- 3 marks**

$A = \pi ab$ sq. unit **- 2 marks**

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50. $(x - a)^2 + y^2 = 1$ **- 2 marks**

$(x - a) + yy' = 0 = (x - a) = -yy'$ **- 2 marks**

$y^2 [(y')^2 + 1] = 1$ **- 2 marks**

51.

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee (\sim r)$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

4^{th} Column **- 2 marks**

5^{th} Column **- 2 marks**

6^{th} Column **- 2 marks**

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52. Truth table $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth table for $((\sim p) \vee q) \wedge ((\sim q) \vee p)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$(\sim q) \vee p$	$((\sim p) \vee q) \wedge (\sim q) \vee p)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

3^{rd} Column **- 1 mark**

4^{th} Column **- 1 mark**

5^{th} Column **- 1 mark**

6^{th} Column **- 1 mark**

7^{th} Column **- 1 mark**

$p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$ **- 1 mark**

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53. $P(X = x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$ **- 1 mark**

$$\frac{e^{-\lambda} \lambda^2}{2!} = \frac{e^{-\lambda} \lambda^3}{3!}$$

$\lambda = 3$ **- 2 marks**

$P(x = 5) = \dfrac{e^{-3} (3)^5}{5!} = 0.101$ **- 3 marks**

54. $P = \frac{4}{5}$ **- 1 mark**

$q = \frac{1}{5}$ **- 1 mark**

$P[\text{Atleast 5 pass}] = P(x \geq 5)$ **- 1 mark**

$= P(x = 5) + P(x = 6)$ **- 1 mark**

$P(X \geq 5) = \frac{2048}{5^5}$ **- 2 marks**

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55. (a)

$x + y = 100$ **- 1 mark**

$P = xy = x(100 - X) = 100x - x^2$ **- 1 mark**

$P'(x) = 100 - 2x$ **- 1 mark**

$P'' = -2 < 0$ **- 1 mark**

P is maximum, when $x = 50$ **- 1 mark**

$\therefore \quad y = 50$ **- 1 mark**

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55. (b) $\frac{dw}{dt} = \frac{\delta w}{\delta x} \cdot \frac{dx}{dt} + \frac{\delta w}{\delta y} \cdot \frac{dy}{dt} + \frac{\delta w}{\delta z} \cdot \frac{dz}{dt}$ **- 2 marks**

$\frac{\delta w}{\delta x} = 1; \quad \frac{dx}{dt} = -\sin t$ **- 1 mark**

$\frac{\delta w}{\delta y} = 2; \quad \frac{dt}{dt} = \cos t$ **- 1 mark**

$\frac{\delta w}{\delta y} = 2z; \quad \frac{dz}{dt} = 1$ **- 1 mark**

$\frac{dw}{dt} = 1(-\sin t) + 2\cos t + 2z$

$= -\sin t + 2\cos t + 2t$ **- 1 mark**

SECTION - C

56. The system of equations can be written as $Ax = B$

$[A, B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$ **- 1 mark**

$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & -1 & \mu-12 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix}$ $R_2 \rightarrow R_2 - 4R_1$, $R_3 \rightarrow R_3 - 2R_1$ **- 2 marks**

$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu-12 & 0 \\ 0 & 0 & 8-\mu & 0 \end{bmatrix}$ $R_2 \rightarrow R_3 + R_2$ **- 1 mark**

Case (i) If $\mu \neq 8$, $\rho[A] = \rho[A, B] = 3$

The system has the trivial solution **- 3 marks**

Case (ii) If $\mu = 8$, $\therefore$ $\rho(a) = \rho[A, B] = 2$ < number of unknowns

The given system is equivalant to $x + y + 3z = 0$; $y + 4z = 0$

Taking $z = k$, we get $x = k$, $y = -4k$, $z = k$ [$k \in R - \{0\}$]

which are non-trivial solutions **- 3 marks**

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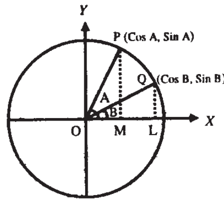
57. Diagram **- 2 marks**



$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$ **- 2 marks**

$\vec{OQ} = \cos B\,\vec{i} + \sin B\,\vec{j}$ **- 2 marks**

$\vec{OQ} \times \vec{OP} = \text{Sin}(A - B)\,\vec{k}$ **- 1 mark**

$\vec{OQ} \times \vec{OP} = \vec{k}\,[\sin A \cos B - \cos A \sin B]$ **- 2 marks**

$\sin(A - B) = \sin A \cos B - \cos A \sin B$ **- 1 mark**

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58. $\vec{a} = 2\vec{i} + 2\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 4\vec{j} + 2\vec{k}$, $\vec{c} = 7\vec{i} + 6\vec{k}$ **- 2 marks**

$\vec{r} = (1 - s - t)(2\vec{i} + 2\vec{j} - \vec{k}) + s(3\vec{i} + 4\vec{j} + 2\vec{k}) + t(7\vec{i} + 6\vec{k})$ **- 3 marks**

Cartesian equation of the plane is $\begin{vmatrix} x - 2 & y - 2 & z + 1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ **- 3 marks**

$5x + 2y - 3z = 17$ **- 2 marks**

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59. $\arg\left(\dfrac{z - 1}{z + 3}\right) = \pi/2 \Rightarrow \arg(z - 1) - \arg(z + 3) = \pi/2$ **- 1 mark**

$\tan^{-1}\left(\dfrac{y}{x - 1}\right) - \tan^{-1}\left(\dfrac{y}{x + 3}\right) = \pi/2$ **- 2 marks**

$\tan^{-1}\left(\dfrac{\dfrac{y}{x - 1} - \dfrac{y}{x + 3}}{1 + \dfrac{y}{x - 1} \cdot \dfrac{y}{x + 3}}\right) = \pi/2$ **- 2 marks**

$\Rightarrow 1 + \dfrac{y}{x - 1} \cdot \dfrac{y}{x + 3} = 0$ **- 2 marks**

$\therefore$ locus of P is $x^2 + y^2 + 2x - 3 = 0$ **- 2 marks**

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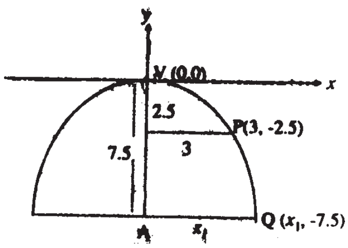
60. Diagram **- 2 marks**



$x^2 = -4ay$ **- 1 mark**

$a = 9/10$ **- 2 marks**

$x^2 = -4 \times 9/10\ y$ **- 1 mark**

The point $(x_1, -7.5)$ lies on the parabola **-2 marks**

$x_1 = 3\sqrt{3}$ **- 2 marks**

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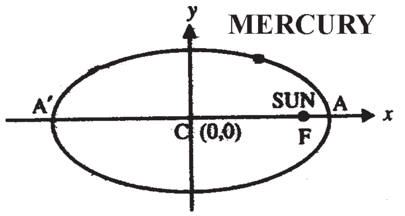
61. Diagram **- 2 marks**



Closest distance F_1A $= CF - CF_1$

$= a - ae$ **- 2 marks**

$= 36\,(1 - 0.206)$

$= 36 \times 0.794$

$= 28.584$ million miles **- 2 marks**

The forthest position F_1A^1 $= F_1C + CA^1$

$= ae + a$ **- 2 marks**

$= a\,(e + 1)$

$= 36\,(1 + 0.206)$

$= 36 \times 1.206$

$= 43.416$ million miles **- 2 marks**

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62. $\lim_{x \to 0} x^{\sin x}$ is of the form 0^0 **- 1 mark**

$y = x^{\sin x} \Rightarrow \log y = \sin x \log x$ **- 1 mark**

$\log y = \dfrac{\log x}{\operatorname{cosec} x}$ **- 1 mark**

$\lim_{x \to 0+} \log y = \lim_{x \to 0+} \dfrac{\log x}{\operatorname{cosec} x}$ which is of the type $\dfrac{-\infty}{\infty}$ **- 1 mark**

$= \lim_{x \to 0} \dfrac{-\sin^2 x}{x \cos x}$ (of the type $\dfrac{0}{0}$) **- 2 marks**

$\lim_{x \to 0+} \log y = 0$ **- 2 marks**

By composite function theorem, we have

$\log \lim_{x \to 0+} y = 0 \Rightarrow \lim_{x \to 0+} y = e^0 = 1$ **- 2 marks**

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63. **(i) Domain, extent, Intercept and origin :**

When $x > 0$, y is well defined, As $x \to \infty$, $y \to \pm\infty$

The curve exists in first and fourth quadrant only

The intercepts with the axes are given by **- 2 marks**

$x = 0, y = 0$ and when $y = 0, x = 0$

clearly the curve passes through origin

(ii) Symmetry : The curve is symmetric about x -axis only **- 1 mark**

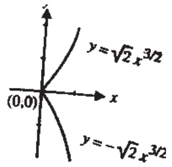
(iii) Asymptotes : The curve does not admit asymptotes - **1 mark**

(iv) Monotonicity : For the branch $y = \sqrt{2}x^{1/2}$ of the curve is increasing - **1 mark**

For the branch $y = -\sqrt{2}\, x^{3/2}$ of the curve is decreasing - **1 mark**

(v) Special points : (0, 0) is not a point of inflexion - **1 mark**

Diagram : - **3 marks**



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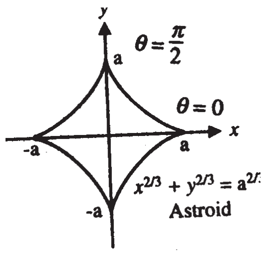
64. Diagram - **1 mark**



$\frac{dx}{dt} = -3a\cos^2 t \sin t$ - **1 mark**

$\frac{dy}{dt} = 3a\sin^2 t \cos t$ - **1 mark**

$\sqrt{+\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 3a \sin t \cos t$ - **2 marks**

$\therefore$ Length of the entire curve $= 4\int_0^{\pi/2} 3a \sin t \cos t\, dt$ - **3 marks**

$= 6a$ - **2 marks**

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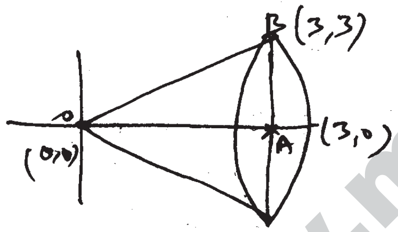
65. Diagram - **3 marks**



Equation of OB is $y = 3/3\, x \Rightarrow y = x$ - **2 marks**

$V = \pi\int_0^4 y^2\, dx$ - **1 mark**

$= \pi\int_0^4 x^2\, dx$ - **1 mark**

$= 9\pi$ - **3 marks**

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66. Put $x + y = z$ - **1 mark**

$\Rightarrow 1 + dy/dx = dz/dx$ - **1 mark**

$\Rightarrow \frac{z^2}{1+z^2}\, dz = dx$ - **3 marks**

$\Rightarrow \int\left(1 - \frac{1}{1+z^2}\right) dz = \int dx + c$ - **2 marks**

$\Rightarrow z - \tan^{-1} z = x + c$ - **1 mark**

$\Rightarrow y - \tan^{-1}(x + y) = c$ - **2 marks**

67. $\frac{dA}{dt} = KA$ **- 1 mark**

$A = ce^{kt}$ **-1 mark**

when t = 0, A = 130000 $\Rightarrow$ C = 130000 **- 1 mark**

$A = 130000\ e^{kt}$ **- 1 mark**

when t = 30, A = 160000, $e^{30k} = 16/13$ **- 2 marks**

when t = 60, A = 130000 x e^{60k} **- 2 marks**

The approximate population is 2020 is 197000 **- 2 marks**

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68. (i) closure axion :

$A = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \in G,\ B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in G$

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in G$ (therefore $x \neq 0,\ y \neq 0 \Rightarrow 2xy \neq 0$) **- 2 marks**

(ii) Matrix multiplication is always associative **- 1 mark**

(iii) $E = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \in G$ **- 3 marks**

(iv) $A^{-1} = \begin{pmatrix} 1/4x & 1/4x \\ 1/4x & 1/4x \end{pmatrix} \in G$ **- 3 marks**

$\therefore$ G is a group under matrix multiplication **- 1 mark**

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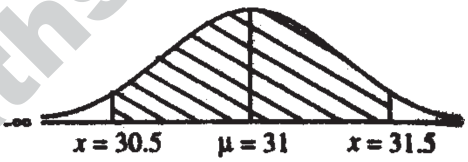
69. Given $\mu = 31$ and $\sigma = 0.2$

(i) (a) when x = 30.5 , Z = – 2.5 **- 1 mark**

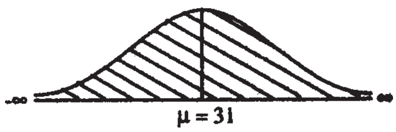
when x = 31.5, Z = 2.5 **- 1 mark**



Required probability p (30.5 < x < 31.5) = 0.9876 **- 1 mark**

(b) when x = 30, z = −5 **- 1 mark**

when x = 32, z = 5 **- 1 mark**



p (30 < x < 32) = p (−5 < z < 5)

= 1 (app) **- 1 mark**

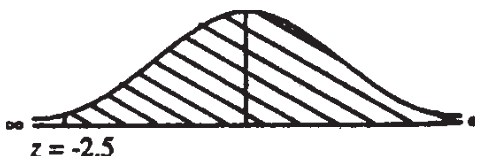
(ii) when x = 30.5, z = – 2.5 **- 1 mark**

p (x > 30.5) = p (z > – 2.5)

= 0.5 + p (0 < z < 2.5) **- 1 mark**

= 0.5 + 0.4938

= 0.9938 **- 2 marks**



70. (a) Point of intersection as $(K^{2/3}, K^{1/3})$ **- 2 marks**

$M_1 = \frac{1}{2K^{1/3}}$ **- 2 marks**

$M_2 = -\frac{1}{K^{1/3}}$ **- 2 marks**

But they are orthogonal and therefore $M_1 \times M_2 = -1$

$$\Rightarrow \frac{1}{2K^{1/3}} \left(-\frac{1}{K^{1/3}}\right) = -1$$ **- 2 marks**

$$8k^2 = 1$$ **- 2 marks**

(b) The condition for $y = mx + c$ to be a tangent to a

hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $c^2 = a^2 m^2 - b^2$ **- 2 marks**

$m = -\frac{5}{12}$ **- 1 mark**

$c = 3/4$ **- 1 mark**

$a^2 = 9, \ b^2 = 1$ **- 1 mark**

$c^2 = \frac{9}{16}, \ a^2 m^2 - b^2 = \frac{9}{16}$

$c^2 = a^2 m^2 - b^2$ **- 2 marks**

it touches the hyperbola

The point of contact is $\left(\frac{-a^2 m}{c}, \frac{-b^2}{c}\right)$ **- 1 mark**

The point of contact is $\left(5, \frac{-4}{3}\right)$ **- 2 marks**