Register Number

+2 MODEL EXAMINATION

PART III - MATHEMATICS

[ENGLISH VERSION]

Time : 3 Hrs. **Max. Marks : 200**

SECTION - A **40 x 1 = 40**

Note :- (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives.

1. The integrating factor of $\frac{dy}{dx} + 2\frac{y}{x} = e^{4x}$ is

a) $\log x$ b) x^2 c) e^x d) x

2. A particular integral of $(D^2 - 4D + 4)y = e^{2x}$ is

a) $\frac{x^2}{2}e^{2x}$ b) xe^{2x} c) xe^{-2x} d) $\frac{x}{2}e^{-2x}$

3. The differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is

a) $\frac{d^2y}{dx^2} + ay = 0$ b) $\frac{d^2y}{dx^2} - 9y = 0$ c) $\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0$ d) $\frac{d^2y}{dx^2} + 9x = 0$

4. The order and degree of the differential equation are $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$

a) (2, 1) b) (1, 2) c) (2, 1/2) d) (2, 2)

5. In the multiplicative group of cube root of unity, the order of w^2 is

a) 4 b) 3 c) 2 d) 1

6. Which of the following is correct ?

a) An element of a group can have more than one inverse

b) If every element of a group is its own inverse, then the group is abelian

c) The set of all 2 x 2 real matrices forms a group under matrix multiplication

d) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$

7. Which of the following is a tautology ?

a) $p \vee q$ b) $p \wedge q$ c) $p \vee \sim p$ d) $p \wedge \sim p$

8. "÷" is a binary operation on

a) N b) R c) Z d) C - {0}

9. If $u = y \sin x$, then $\frac{\delta^2 u}{\delta x\, \delta y}$ is equal to

a) $\cos x$ b) $\cos y$ c) $\sin x$ d) 0

10. If $u = \sin^{-1}\left(\frac{x^4 + y^4}{x^2 + y^2}\right)$ and $f = \sin u$ then f is a homogeneous function of degree

a) 0 b) 1 c) 2 d) 4

11. If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

a) 0 b) 32 c) -16 d) -32

12. The conjugate of $i^{13} + i^{14} + i^{15} + i^{16}$ is

a) 1 b) -1 c) 0 d) $-i$

13. If the amplitude of a complex number is $\pi/2$ then the number is

a) purely imaginary b) purely real

c) 0 d) neither real nor imaginary

14. The cube roots of unity are

a) in G.P with common ratio ω b) in G.P with common difference ω^2

c) in A.P with common diffence ω d) in A.P with common difference with w^2

15. If A = [2 0 1], then rank of AA^T is

a) 1 b) 2 c) 3 d) 0

16. The inverse of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17. If A and B are any two matrices such that AB = O and A is non-singular, then

a) B = O b) B is singular c) B is non-singular d) B = A

18. Every homogeneous system

a) is always consistent b) has only trivial solution

c) has infinitely many solution d) need not be consistent

19. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$, then

a) u is a unit vector b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$ c) $\vec{u} = \vec{0}$ d) $\vec{u} \neq \vec{0}$

20. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then

a) $\vec{a}$ is parallel to $\vec{b}$ b) $\vec{a}$ is perpenticular to $\vec{b}$

c) $|\vec{a}| = |\vec{b}|$ d) $\vec{a}$ and $\vec{b}$ are unit vectors

21. The value of $[\vec{i} + \vec{j}, \vec{j} + \vec{k}, \vec{k} + \vec{i}]$ is equal to

a) 0 b) 1 c) 2 d) 4

22. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

a) (0, 0, -4) b) (1, 0, 0) c) (0, 2, 0) d) (1, 2, 0)

23. The vector equation of a sphere whose centre is origin and radius "a" is

a) $r = \vec{a}$ b) $\vec{r} - \vec{c} = \vec{a}$ c) $|\vec{r}| = |\vec{a}|$ d) $\vec{r} = a$

24. The work done by the force $\vec{F} = \vec{ai} + \vec{j} + \vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight lines is given to be 5 units. The value of a is

a) -3 b) 3 c) 8 d) -8

25. The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at $x = 0$ is

a) 3 b) 2 c) 1 d) -1

26. What is the surface area of a sphere when the volume is increasing at the same rate as its radius ?

a) 1 b) $1/2\pi$ c) 4π d) $4\pi/3$

27. The value of c in Rolle's Theorem for the function $f(x) = \cos x/2$ on $[\pi, 3\pi]$ is

a) 0 b) 2π c) $\pi/2$ d) $3\pi/2$

28. $\lim_{x \to 0} \frac{x}{\tan x}$ is

a) 1 b) -1 c) 0 d) $\propto$

29. The directrix of the parabola $y^2 = x + 4$ is

a) $x = \frac{15}{4}$ b) $x = -\frac{15}{4}$ c) $x = -\frac{17}{4}$ d) $x = \frac{17}{4}$

30. If the normal at the end of the latus rectum of the ellipse $x^2 + 3y^2 = 12$ intersect the major axis at G, then CG is

a) $\frac{16}{3}$ b) $\frac{8\sqrt{2}}{3}$ c) $\frac{4\sqrt{2}}{3}$ d) $\frac{32}{3}$

31. The product of the perpendicular drawn from the point (8, 0) on the hyperbola to its asymptotes is $\frac{x^2}{64} - \frac{y^2}{36} = 1$ is

a) $\frac{25}{576}$ b) $\frac{576}{25}$ c) $\frac{6}{25}$ d) $\frac{25}{6}$

32. The equations of the L.R of $25x^2 + 9y^2 = 225$ are

a) $y = \pm 5$ b) $x = \pm 5$ c) $y = \pm 4$ d) $x = \pm 4$

33. The volume, when the curve $y = \sqrt{3 + x^2}$ from $x = 0$ to $x = 4$ is rotated about x-axis is

a) 100π b) $\frac{100\pi}{9}$ c) $\frac{100\pi}{3}$ d) $\frac{100}{3}$

34. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

a) 20π b) 40π c) 10π d) 30π

35. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

a) $3\pi/16$ b) 3/16 c) 0 d) $3\pi/8$

36. $\int_0^\infty x^5 e^{-4x} dx$ is

a) $\frac{\lfloor 6}{4^6}$ b) $\frac{\lfloor 6}{4^5}$ c) $\frac{\lfloor 5}{4^6}$ d) $\frac{\lfloor 5}{4^5}$

37. If $f(x) = \frac{A}{\pi} \frac{1}{16 + x^2}$, $-\infty < x < \infty$ is a p.d.f of a continuous random variable X, then the value of A is

a) 16 b) 8 c) 4 d) 1

38. Var (4x + 3) is

a) 7 b) 16 Var (X) c) 19 d) 0

39. If f(x) is a p.d.f of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)\, dx$ is

a) 1 b) 0.5 c) 0 d) 0.25

40. For a standard normal distribution the mean and variance are

a) μ, σ^2 b) μ , σ c) 0, 1 d) 1, 1

SECTION - B

10 x 6 = 60

Note :- (i) Answer any ten questions.

(ii) Question No.55 in compulsory and choose any nine questions from the remaining.

(iii) Each question carries six marks.

41. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$

42. Find the rank of $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

43. Angle in a semi-circle is a right angle. Prove by vector method.

44. (a) If A (-1, 4, -3) is one end of a diameter AB of the sphere $x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0$, then find the coordinates of B.

(b) Prove that $|[\vec{a}\ \vec{b}\ \vec{c}]| = abc$ if and only if $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular.

45. Solve $x^4 + 4 = 0$ if $1 + i$ is one of the roots

46. Find the square root of $(-7 + 24i)$

47. Verify Lagrange's law of mean for the function $f(x) = x^3 - 5x^2 - 3x$, on [1, 3]

48. (a) Obtain the Maclaurin's Series for e^x

(b) Evaluate : $\lim_{x \to \infty} \frac{x^2}{e^x}$

49. If u = log (tan x + tan y + tan z), prove that $\sum \sin 2x \frac{\delta u}{\delta x} = 2$

50. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

51. Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

52. Construct the truth table for the following statements (a) $p \vee (\sim q)$ (b) $(p \wedge q) \vee (\sim q)$

53. Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x} & , 0 < x < \infty \\ 0 & , \text{elsewhere} \end{cases}$

54. 20% of the bolts produced in a factory are found to be defective. Find the probability that in a sample of 10 bolts chosen at random exactly 2 will be defective using (i) Binomial distribution (ii) Poisson distribution. $[e^{-2} = 0.1353]$

55. (a) Show that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ **(OR)**

(b) A standard rectangular hyperbola has its vertices at (5, 7) and (-3, -1). Find its equation and equations of asymptotes.

SECTION - C

10 x 10 = 100

Note :- (i) Answer any ten questions.

(ii) Question No.70 is compulsory and choose any nine questions from the remaining.

(iii) Each question carries ten marks.

56. A bag contains 3 types of coins namely Re.1, Rs.2 and Rs.5. There are 30 coins amounting to Rs.100 in total. Find the number of coins in each category.

57. Prove that sin (A - B) = sin A cos B - cos A sin B.

58. Find the vector and cartesian equations of the plane passing through the points (2, 2, -1), (3, 4, 2) and (7, 0, 6)

59. If $a = \cos 2\alpha + i \sin 2\alpha$, $b = \cos 2\beta + i \sin 2\beta$ and $c = \cos 2\gamma + i \sin 2\gamma$ prove that

(i) $\sqrt{abc} + \dfrac{1}{\sqrt{abc}} = 2 \cos (\alpha + \beta + \gamma)$

(ii) $\dfrac{a^2 b^2 + c^2}{abc} = 2 \cos 2(\alpha + \beta - \gamma)$

60. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground ?

61. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun ? (ii) the greatest possible distance between mercury and sun.

62. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$.

63. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

64. Trace the curve $y = x^3 + 1$

65. Find the area of the region enclosed by $y^2 = x$ and $y = x - 2$

66. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r (b - a)$ and hence deduct the surface area of the sphere (b > a).

67. Solve $(D^2 - 13D + 12)y = x + 5e^x$

68. A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm. [$\log_e 2 = 0.6931$]

69. Show that $\left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}\right\}$

Where $\omega^3 = 1$, $w \neq 1$ form a group with respect to matrix multiplication

70. (a) A random variable X has the following probability mass function

x	0	1	2	3	4	5	6
P(X = x)	k	3k	5k	7k	9k	11k	13k

(i) Find k,

(ii) Evaluate $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X < 6)$,

(iii) What is the smallest value of x for which $P (X < x) > 1/2$.

(OR)

(b) Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points (6, 0) and (-3, 0).