MODEL HIGHER SECONDARY EXAMINATION

KEY FOR MATHS

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PART - I

Q. No.	Key	Answer (A)	Q. No.	Key	Answer (B)
1.	a	1	1.	a	$7 / \sqrt{30}$
2.	a	$k^3 \det(A)$	2.	a	1
3.	c	Infinitely many solution	3.	b	8
4.	a	is always consistent	4.	a	is always consistent
5.	d	$\theta = 2\pi/3$	5.	a	$k^3 \det(A)$
6.	b	$\vec{a}$ is perpendicular to $\vec{b}$	6.	d	$\theta = 2\pi/3$
7.	b	8	7.	c	Infinitely many solution
8.	b	(-8, -6, -22)	8.	b	$\vec{a}$ is perpendicular to $\vec{b}$
9.	a	$7 / \sqrt{30}$	9.	b	(-8 , -6 , -22)
10.	c	$\lvert\vec{r}\rvert = \lvert\vec{a}\rvert$	10.	a	cos x
11.	c	-1	11.	c	-1
12.	a	purely imaginary	12.	b	minimum value at x = 0
13.	c	-16	13.	a	purely imaginary
14.	d	Arguments of the product of two complex numbers is equal to sum of their arguments	14.	a	3
15.	d	a hyperbola	15.	d	Arguments of the product of two complex numbers is equal to sum of their arguments
16.	c	8	16.	c	8
17.	a	(4, 4), (-4, -4)	17.	b	y - 5 = 0
18.	b	y - 5 = 0	18.	d	a hyperbola
19.	a	3	19.	a	$-\cot\theta$
20.	a	$-\cot\theta$	20.	c	- 16
21.	b	0	21.	b	0
22.	b	minimum value at x = 0	22.	c	$\lvert\vec{r}\rvert = \lvert\vec{a}\rvert$
23.	b	an asymptote parallel to y-axis	23.	b	an asymptote parallel to y-axis
24.	a	cos x	24.	a	(4, 4), (-4, -4)
25.	d	$3\pi / 8$	25.	d	$3\pi / 8$
26.	a	3/2	26.	a	4
27.	a	48	27.	a	3 / 2
28.	a	f(2a - x) = f(x)	28.	a	f(2a - x) = f(x)
29.	b	$\frac{d^2y}{dx^2} = 0$	29.	a	48
30.	b	ydx - xdy = 0	30.	b	ydx - xdy = 0
31.	a	$xdv + (2v + v^2)dx = 0$	31.	b	$\frac{d^2y}{dx^2} = 0$
32.	b	2, 1	32.	b	24 / 169
33.	c	(Z, .)	33.	a	$xdv + (2v + v^2)dx = 0$
34.	c	$\sim p \vee q$	34.	c	(Z, .)
35.	b	1	35.	b	1
36.	d	[2]	36.	d	[2]
37.	b	$\frac{24}{169}$	37.	d	np, npq
38.	a	4	38.	c	$\sim p \vee q$
39.	d	$\frac{1}{5\sqrt{2\pi}}$	39.	d	$\frac{1}{5\sqrt{2\pi}}$
40.	d	np, npq	40.	b	2 , 1

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SECTION - B

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41. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ **- 1 mark**

$|A| = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = 1 \neq 0$ **- 1 mark**

$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ **- 2 marks**

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 8 \end{bmatrix}$ **- 1 mark**

$x = 1, y = 2$ **- 1 mark**

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45. $A \sim \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{bmatrix}$ $R_2 \leftrightarrow R_2 + 2R_1$, $R_3 \leftrightarrow R_3 + R_1$ **- 3 marks**

$\sim \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$ **- 2 marks**

$\therefore \rho(A) = 2$ **- 1 mark**

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43. (i) $\vec{r} = -\vec{i} - \vec{k}$ **- 1 mark**

$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$ **- 1 mark**

$= 2\vec{i} - 7\vec{j} - 2\vec{k}$ **- 1 mark**

(ii) $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 3 & 2 & -4 \\ 9 & 8 & -10 \\ \lambda & 4 & -6 \end{vmatrix} = 0$ **- 2 marks**

$\Rightarrow \lambda = 5$ **- 1 mark**

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44. $[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}] = \{(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})\} \cdot (\vec{c} \times \vec{a})$ **- 1 mark**

$= \{[\vec{a}\ \vec{b}\ \vec{c}]\vec{b} - [\vec{a}\ \vec{b}\ \vec{b}]\vec{c}\} \cdot (\vec{c} \times \vec{a})$ **- 2 marks**

$= [\vec{a}\ \vec{b}\ \vec{c}]\{\vec{b} \cdot (\vec{c} \times \vec{a})\}$ **- 1 mark**

$= [\vec{a}\ \vec{b}\ \vec{c}][\vec{a}\ \vec{b}\ \vec{c}]$ **- 1 mark**

$= [\vec{a}\ \vec{b}\ \vec{c}]^2$ **- 1 mark**

Note : Different method can be adopted

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45. $\sqrt{-8 - 6i} = x + iy$ **- 1 mark**

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$x^2 - y^2 = -8$ and $2xy = -6$ **- 2 marks**

$x = \pm 1$ **- 1 mark**

$y = \pm 3$ **- 1 mark**

Ans (1 - 3i)) or (-1 + 3i) **- 1 mark**

Note : Different method can be adopted

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46. $z_1 = r_1 (\cos \theta_1 + i\sin \theta_1)$ and $z_2 = r_2 (\cos \theta_2 + i\sin \theta_2)$ **- 1 mark**

$|z_1| = r_1$, $\arg z_1 = \theta_1$; $|z_2| = r_2$, $\arg z_2 = \theta_2$ **- 1 mark**

$z_1 . z_2 = r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$ **- 2 marks**

$\therefore |z_1 z_2| = r_1 r_2 = |z_1| . |z_2|$ and **- 1 mark**

$\arg (z_1 z_2) = \theta_1 + \theta_2 = \arg z_1 + \arg z_2$ **- 1 mark**

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47. Combined equationof asymptotes is $3x^2 - 5xy - 2y^2 + 17x + y + k = 0$ **- 1 mark**

$\therefore$ Separate equations are $x - 2y + l = 0$, $3x + y + m = 0$ **- 2 marks**

$m_1 = 1/2$, $m_2 = -3$ **- 1 mark**

Angle between the lines is $\tan\theta = 7$ **- 1 mark**

$\theta = \tan^{-1} (7)$ **- 1 mark**

Another method :

$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$ **- 2 marks**

we have $a = 3$, $b = -2$, $2h = -5$ **- 2 marks**

$\tan \theta = 7$ **- 1 mark**

$\theta = \tan^{-1} (7)$ **- 1 mark**

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48. $f(0) = 1$ **- 1 mark**

$f'(0) = -1$ **- 1 mark**

$f''(0) = 2$ **- 1 mark**

$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + ..$ **- 1 mark**

$\frac{1}{1 + x} = 1 - x + x^2 - ......$ **- 2 marks**

Note : The sign of the 4th term may be put as + or -

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49. $\frac{\delta u}{\delta x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$ **- 2 marks**

$\sin 2x \frac{\delta u}{\delta x} = \frac{2\tan x}{\tan x + \tan y + \tan z}$ **- 1 mark**

$\sin 2y \frac{\delta u}{\delta y} = \frac{2\tan y}{\tan x + \tan y + \tan z}$ **- 1 mark**

$\sin 2z \frac{\delta u}{\delta y} = \frac{2\tan z}{\tan x + \tan y + \tan z}$ **- 1 mark**

$\Sigma \sin 2x \frac{\delta u}{\delta x} = 2$ **- 1 mark**

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50. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}}$ **- 1 mark**

$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$ **- 1 mark**

$2I = \int_{\pi/6}^{\pi/3} dx$ **- 2 marks**

$= \frac{\pi}{6}$ **- 1 mark**

$I = \frac{\pi}{12}$ **- 1 mark**

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51. (i) The characteristic equation is $p^2 + 6p + 9 = 0$

$(p + 3)^2 = 0 \Rightarrow p = -3, -3$ **- 1 mark**

The C.F is $(Ax + B)e^{-3x}$ **- 1 mark**

The general solution is $y = (Ax + B)e^{-3x}$ **- 1 mark**

(ii) First differentiation **- 1 mark**

Second differentiation **- 1 mark**

$y'' - 4y' + 4y = 0$ is the required differential equation **- 1 mark**

52.

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Column 1 to 4 (each 1 mark) **- 4 marks**

Column 5 **- 2 marks**

Note : The order of the rows need not be same as in the scheme.

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52. $E(x) = \frac{1}{3}$ **- 2 marks**

$E(x^2) = \frac{2}{9}$ **- 2 marks**

$Var(x) = \frac{1}{9}$ **- 2 marks**

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54. $f(x) = ke^{-2(x^2 - 2x + 1)}$ - 1 mark

$= k\, e^{-2(x-1)^2}$ - 1 mark

$= k \cdot e^{-\frac{1}{2}\left(\frac{x-1}{1/2}\right)^2}$ - 1 mark

$\sigma = \frac{1}{2}$, $m = 1$ - 2 marks

$k = \sqrt{\frac{2}{\pi}}$ - 1 mark

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55. (a) Point (x, y) = (0, 1) - 1 mark

$m_1 = \log a$ - 1 mark

$m_2 = \log b$ - 1 mark

If θ be the angle between them then

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ - 1 mark

$\theta = \tan^{-1}\left[\left|\frac{\log a - \log b}{1 + \log a \log b}\right|\right]$ - 2 marks

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55. (b) Stating

$a * b = a * c \Rightarrow b = c$ - 1 mark

$b * a = c * a \Rightarrow b = c$ - 1 mark

Proving

$a * b = a * c \Rightarrow b = c$ - 2 marks

$b * a = c * a \Rightarrow b = c$ - 2 marks

Note : For LCL the left elements must be same. For RCL the right elements must be same. One may take any three different elements.

SECTION - C

56. $[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$ - 2 marks

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{bmatrix}$ $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_2$ - 2 marks

Case (i) $\lambda = 3$ and $\mu \neq 10$, $\therefore \rho(A) \neq \rho[A, B]$

$\therefore$ the given system is inconsistent and has no solution - 2 marks

Case (ii) $\lambda \neq 3$, and $\mu \neq 10$, $\rho(A) = \rho\,[A, B] = 3$ = number of unknowns

$\therefore$ the given system is consistent and has a unique solution - 2 marks

Case (iii) $\lambda = 3$ and $\mu = 10$, $\rho(A) = \rho[A, B] = 2 <$ number of unknowns

$\therefore$ the given system is consistent but has an infinite number of solutions - 2 marks

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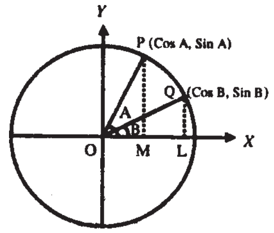
57. Rough diagram - **3 marks**



$\vec{OP} = \cos A\, \vec{i} + \sin A\, \vec{j}$ - **2 marks**

$\vec{OQ} = \cos B\, \vec{i} + \sin B\, \vec{j}$ - **2 marks**

$\vec{OP} \cdot \vec{OQ} = \cos A \cos B + \sin A \sin B$ - **1 mark**

$\vec{OP} \cdot \vec{OQ} = \cos(A - B)$ - **1 mark**

$\cos(A - B) = \cos A \cos B + \sin A \sin B$ - **1 mark**

Note : Instead of P and Q they can use any letters

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58. $\vec{a} = -\vec{i} + \vec{j} + \vec{k}$

$\vec{b} = \vec{i} - \vec{j} + \vec{k}$

$\vec{v} = \vec{i} + 2\vec{j} + 2\vec{k}$ - **2 marks**

Vector Form

$\vec{r} = (\vec{1} - \vec{s})(-\vec{i} + \vec{j} + \vec{k}) + s(\vec{i} - \vec{j} + \vec{k}) + t(\vec{i} + 2\vec{j} + 2\vec{k})$ - **3 marks**

(OR) $\vec{r} = (-\vec{i} + \vec{j} + \vec{k}) + s(2\vec{i} - 2\vec{j}) + t(\vec{i} + 2\vec{j} + 2\vec{k})$

Cartesian Form :

$$\begin{vmatrix} x+1 & y-1 & z-1 \\ 2 & -2 & 0 \\ 1 & 2 & 2 \end{vmatrix}$$ - **3 marks**

$-4x - 4y + 6z - 6 = 0$

(OR)

$2x + 2y - 3z + 3 = 0$ - **2 marks**

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59. $x = 1 \pm i\sqrt{3}$ - **2 marks**

$\alpha^n = 2^n (\cos n\frac{\pi}{3} + i\sin n\frac{\pi}{3})$ - **2 marks**

$\beta^n = 2^n (\cos n\frac{\pi}{3} - i\sin n\frac{\pi}{3})$ - **2 marks**

$\alpha^n - \beta^n = i2^{n+1} \sin n\frac{\pi}{3}$ - **2 marks**

$\alpha^9 - \beta^9 = 0$ - **2 marks**

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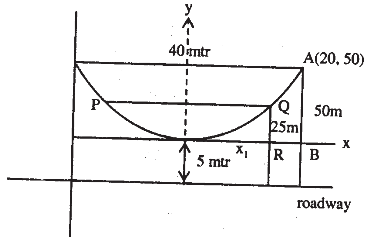
60. Diagram - **3 marks**



$x^2 = 4ay$ - **1 mark**

$\therefore\ a = 2$ - **2 marks**

$x^2 = 8y$ - **1 mark**

The point Q is $(x_1, 25)$ lies on the parabola - **1 mark**

$\Rightarrow x_1 = 10\sqrt{2}$ - **1 mark**

$PQ = 2x_1 = 20\sqrt{2}$ mts - **1 mark**

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61. Diagram **- 2 marks**

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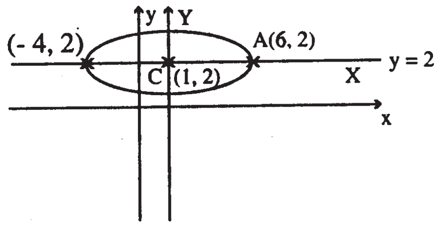

$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$ **- 2 marks**

$e = \frac{4}{5}$ **- 1 mark**

centre C (1, 2) **- 1 mark**

Foci F_1 (5, 2) **- 1 mark**

F_2 (-3 , 2) **- 1 mark**

Vertices A (6, 2) **- 1 mark**

A' (-4, 2) **- 1 mark**

Note : Only a rough diagram is enough with indication of proper type

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62. Other asymptote is $2x - y + k = 0$ **- 1 mark**

Equation of RH is $(x + 2y - 5)(2x - y + k) + c = 0$

Passing through (6, 0) **- 2 marks**

$k + c = -12$ **- 2 marks**

Passing through (-3, 0)

$-8k + C = -48$ **- 2 marks**

$k = 4, c = -16$ **- 2 marks**

Equation of RH is

$(x + 2y - 5)(2x - y + 4) - 16 = 0$ **- 1 mark**

Note : One may take the unknown k, c in a different manner. Further the last stage need not be simplified.

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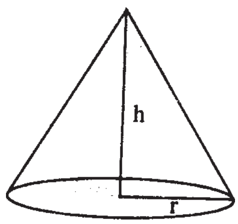
63. Diagram



- 1 mark

$\frac{dV}{dt} = 30\ ft^3 / min$ **- 1 mark**

Volume of cone $V = \frac{1}{3}\pi r^2 h$ **- 1 mark**

$V = \frac{\pi}{12}(h^3)$ **- 2 marks**

$\frac{dV}{dt} = \frac{\pi}{12} \times 3h^2 \frac{dh}{dt}$ **- 2 marks**

$\therefore$ the height of the cone is increased at the rate of $\frac{6}{5\pi}$ ft/ min **- 3 marks**

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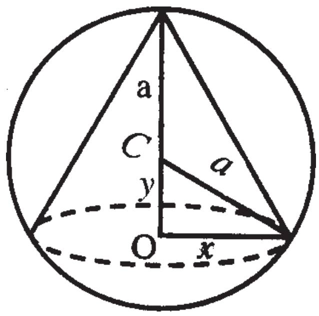
64. Rough diagram **- 2 marks**



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Volume of the cone

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$V = \frac{1}{3}\pi x^2 (a + y)$ **- 1 mark**

$= \frac{1}{3}\pi(a^2 - y^2)(a + y)$ **- 1 mark**

$V' = 0 \Rightarrow y = \frac{a}{3}$ **- 3 marks**

When $y = \frac{a}{3}$, $V'' < 0$ **- 2 marks**

$V = \frac{8}{27}$ (Volume of the sphere) **- 1 mark**

Note : This problem can be done by different method also. If the method is correct full credit should be given

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65. $f = \tan u$ **- 2 marks**

degree of $f = 2$ **- 2 marks**

$x\frac{\delta f}{\delta x} + y\frac{\delta f}{\delta y} = 2f$ **- 3 marks**

$x\sec^2 u\frac{\delta u}{\delta x} + y\sec^2 u\frac{\delta u}{\delta y} = 2\tan u$ **- 2 marks**

$x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = \sin 2u$ **- 1 mark**

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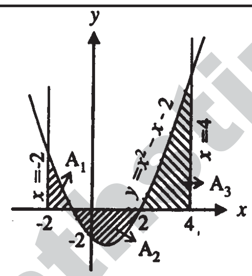
66. Diagram **- 3 marks**



This curve intersects x axis

at x = -1 and x = 2 **- 1 mark**

$= \int_{-2}^{-1} y\,dx + \int_{-1}^{+2} (-y)\,dx + \int_{2}^{4} y\,dx$ (or)

$= \int_{-2}^{-1} (x^2 - x - 2)\,dx + \int_{-1}^{2} -(x^2 - x - 2)\,dx + \int_{2}^{4} (x^2 - x - 2)\,dx$ **- 3 marks**

= 15 sq units **- 3 marks**

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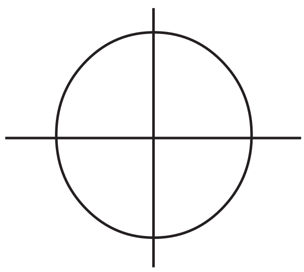
67. Rough Diagram **- 1 mark**



$x^2 + y^2 = a^2$

x = 0 and x = a **- 1 mark**

Perimeter $= 4\int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ **- 2 marks**

$\frac{dy}{dx} = \frac{-x}{y}$ **- 1 mark**

$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{a}{y} = \frac{a}{\sqrt{a^2 - x^2}}$ **- 1 mark**

Perimeter $= 4\int_0^a \frac{a}{\sqrt{a^2 - x^2}}\,dx$ **- 2 marks**

$= 2\pi a$ **- 2 marks**

Note : Different method can be adopted.

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68. $P^2 - 1 = 0$ **- 1 mark**

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$p = \pm 1$ **- 1 mark**

$CF = Ae^{x} + Be^{-x}$ **- 3 marks**

$PI_1 = -\frac{1}{5}\cos 2x$ **- 2 marks**

$PI_2 = \frac{2}{5}\sin 2x$ (OR) $\frac{-2}{5}\sin 2x$ **- 2 marks**

$y = Ae^{x} + Be^{-x} - \frac{1}{5}\cos 2x + \frac{2}{5}\sin 2x$ **- 1 mark**

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69. $\frac{dA}{dt} \propto A$ **- 1 mark**

$\frac{dA}{dt} = KA$ **- 1 mark**

$\Rightarrow \frac{dA}{dt} = 0.04\,t$ **- 1 mark**

$A = Ce^{0.04t}$ **- 2 marks**

when $t = 0$, $c = 1000$ **- 1 mark**

when $A = 2000$,

$2000 = 1000\, e^{0.04t}$ **- 2 marks**

$t = 17$ years (app...) **- 2 marks**

70.a (i) **Closure axiom**

$A = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \in G, B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in G$

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in G$ **- 2 marks**

(ii) **Associative axiom**

Matrix multiplication is always associative **- 1 mark**

(iii) **Identity axiom**

$E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in G$ **- 3 marks**

(iv) **Inverse axiom**

$A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \in G$ **- 3 marks**

G is a group under matrix multiplication **- 1 mark**

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70. b $K = \frac{1}{49}$ **- 2 marks**

$P(x < 4) = \frac{16}{49}$ **- 2 marks**

$P(x \geq 5) = \frac{24}{49}$ **- 2 marks**

$P(3 < x \leq 6) = \frac{33}{49}$ **- 2marks**

$\therefore$ the smallest value of x for which

$P(X \leq x) > \frac{1}{2}$ is 4 **- 2 marks**

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