A

AIMS EDUCATIONAL ACADEMY Regd. No. 390/09

Register Number

+2 MODEL EXAMINATION

PART III - MATHEMATICS

[English Version]

Time : 3 Hrs.] **[Max. Marks : 200**

SECTION - A

Note : (i) All questions are compulsory.

(ii) Each question carries one mark.

(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. The rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{bmatrix}$ is

a) 1 b) 2 c) 3 d) 4

2. If A is a matrix of order 3, then det (KA)

a) k^3 det (A) b) k^2 det (A) c) k det (A) d) det (A)

3. If the equation $-2x + y + z = 1$; $x - 2y + z = m$; $x + y - 2z = n$ such that $l + m + n = 0$, then the system has

a) a non-zero unique solution b) trivial solution

c) Infinitely many solution d) No Solution

4. Every homogeneous system

a) is always consistent b) has only trivial solution

c) has infinitely many solution d) need not be consistent

5. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then ($\vec{a} + \vec{b}$) is a unit vector if

a) $\theta = \frac{\pi}{3}$ b) $\theta = \frac{\pi}{4}$ c) $\theta = \frac{\pi}{2}$ d) $\theta = \frac{2\pi}{3}$

6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then

a) $\vec{a}$ is parallel to $\vec{b}$ b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $|\vec{a}| = |\vec{b}|$ d) $\vec{a}$ and $\vec{b}$ are unit vectors

7. If $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = 64$ then $[\vec{a}, \vec{b}, \vec{c}]$ is

a) 32 b) 8 c) 128 d) 0

8. The point of intersection of the line $\vec{r} = (\vec{i} - \vec{k}) + t(3\vec{i} + 2\vec{j} + 7\vec{k})$ and the plane $\vec{r} \cdot (\vec{i} + \vec{j} - \vec{k}) = 8$ is

a) (8, 6, 22) b) (-8, -6, -22) c) (4, 3, 11) d) (-4, -3, -11)

9. The distance from the origin to the plane $\vec{r} \cdot (2\vec{i} - \vec{j} + 5\vec{k}) = 7$ is

a) $\frac{7}{\sqrt{30}}$ b) $\frac{\sqrt{30}}{7}$ c) $\frac{30}{7}$ d) $\frac{7}{30}$

10. The vector equation of a sphere whose centre is origin and radius "a" is

a) $r = \vec{a}$ b) $\vec{r} - \vec{c} = \vec{a}$ c) $|\vec{r}| = |\vec{a}|$ d) $\vec{r} = a$

11. The value of $\left[\frac{-1 + i\sqrt{3}}{2}\right]^{100} + \left[\frac{-1 - i\sqrt{3}}{2}\right]^{100}$ is

a) 2   b) 0   c) -1   d) 1

12. If the amplitude of a complex number is $\pi/2$ then the number is

a) purely imaginary   b) purely real

c) 0   d) neither real nor imaginary

13. If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

a) 0   b) 32   c) -16   d) -32

14. Identify the correct statement

a) sum of the moduli of two complex numbers is equal to their modulus of the sum

b) Modulus of the product of the complex numbers is equal to sum of their moduli

c) Arguments of the product of two complex numbers is the product of their arguments

d) Arguments of the product of two complex numbers is equal to sum of their arguments

15. $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents

a) an ellipse   b) a circle   c) a parabola   d) a hyperbola

16. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is

a) 4   b) 6   c) 8   d) 2

17. The co-ordinate of the vertices of the rectangular hyperbola $xy = 16$ are

a) (4, 4), (-4, -4)   b) (2, 8), (-2, -8)   c) (4, 0), (-4, 0)   d) (8, 0), (-8, 0)

18. The equation of the latus rectum of the parabola $x^2 = 20y$ is

a) $x - 5 = 0$ b) $y - 5 = 0$ c) $y + 5 = 0$ d) $x + 5 = 0$

19. The slope of the tangent to the curve $y = 3x^2 + 3\sin x$ at $x = 0$ is

a) 3   b) 2   c) 1   d) -1

20. If a normal makes an angle θ with the positive x-axis then the slope of the curve at the point where the normal is drawn is

a) $-\cot\theta$ b) $\tan\theta$ c) $-\tan\theta$ d) $\cot\theta$

21. The Rolle's constant for the function $y = x^2$ on [-2, 2] is

a) $\frac{2\sqrt{3}}{3}$ b) 0   c) 2   d) -2

22. The function $f(x) = x^2$ has

a) a maximum value of $x = 0$ b) minimum value at $x = 0$

c) finite no of maximum values   d) infinite no. of maximum values

23. The curve $y^2(x - 2) = x^2(1 + x)$ has

a) an asymptote parallel to x-axis   b) an asymptote parallel to y-axis

c) asymptotes parallel to both axes   d) no asymptotes

24. If $u = y \sin x$, then $\frac{\delta^2 u}{\delta x\, \delta y}$ is equal to

a) $\cos x$ b) $\cos y$ c) $\sin x$ d) 0

25. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

a) $\frac{3\pi}{16}$ b) $\frac{3}{16}$ c) 0   d) $\frac{3\pi}{8}$

26. The area bounded by the line y = x, the x-axis, the ordinates x = 1, x = 2 is

a) $\frac{3}{2}$ b) $\frac{5}{2}$ c) $\frac{1}{2}$ d) $\frac{7}{2}$

27. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

a) 48   b) 24   c) 12   d) 96

28. $\int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx$ if

a) f(2a - x) = f(x)   b) f(a - x) = f(x)   c) f(x) = -f(x)   d) f(-x) = f(x)

29. The differential equation of all non-vertical lines in a plane is

a) $\frac{dy}{dx} = 0$ b) $\frac{d^2y}{dx^2} = 0$ c) $\frac{dy}{dx} = m$ d) $\frac{d^2y}{dx^2} = m$

30. The differential equation of the family of lines y = mx is

a) $\frac{dy}{dx} = m$ b) ydx - xdy = 0   c) $\frac{d^2y}{dx^2} = 0$ d) ydx + xdy = 0

31. On putting y = vx, the homogeneous differential equation $x^2dy + y(x + y)dx = 0$ becomes

a) $xdv + (2v + v^2)dx = 0$ b) $vdx + (2x + x^2)dv = 0$

c) $v^2dx - (x + x^2)dv = 0$ d) $vdv + (2x + x^2)dx = 0$

32. The order and degree of the differential equation $y^{11} + 3{y^1}^2 + y^3 = 0$ are

a) 2, 2   b) 2, 1   c) 1, 2   d) 3, 1

33. Which of the following is not a group ?

a) $(Z_n, +_n)$ b) (Z, +)   c) (Z, .)   d) (R, +)

34. The conditional statement $p \rightarrow q$ is equivalent to

a) $p \vee q$ b) $p \wedge \sim q$ c) $\sim p \vee q$ d) $p \wedge q$

35. In the set of integers under the operation * defined by a * b = a + b - 1, the identity element is

a) 0   b) 1   c) a   d) b

36. In congruence modulo 5, $\{x \in z / x = 5k + 2, k \in z\}$ represents

a) [0]   b) [5]   c) [7]   d) [2]

37. X is a discrete random variable which takes the values 0, 1, 2 and $P(X = 0) = \frac{144}{169}$, $P(X = 1) = \frac{1}{169}$ then the value of P(X = 2) is

a) $\frac{145}{169}$ b) $\frac{24}{169}$ c) $\frac{2}{169}$ d) $\frac{143}{169}$

38. In 16 throws of a die getting an even number is considered a success. Then the variance of the successes is

a) 4   b) 6   c) 2   d) 256

39. The random variable X follows normal distribution $f(x) = ce^{\frac{-1/2\,(x - 100)^2}{25}}$ Then the value of c is

a) $\sqrt{2\pi}$ b) $\frac{1}{\sqrt{2\pi}}$ c) $5\sqrt{2\pi}$ d) $\frac{1}{5\sqrt{2\pi}}$

40. Mean and variance of binomial distribution are

a) nq, npq   b) np, $\sqrt{npq}$ c) np, np   d) np, npq

SECTION - B

Note : (i) Answer any *ten* questions.

(ii) Question No.55 in compulsory and choose any nine questions from the remaining.

(iii) Each question carries six marks. 10 x 6 = 60

41. Solve by matrix inversion method x + y = 3, 2x + 3y = 8

42. Find the rank of the matrix : $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

43. (i) A force given by $3\vec{i} + 2\vec{j} - 4\vec{k}$ is applied at the point (1, -1, 2). Find the moment of the force about the point (2, -1, 3).
(ii) Find the value of λ if the points (3, 2, -4), (9, 8, -10) and (λ, 4, -6) are collinear.

44. Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

45. Find the square root of $(-8 - 6i)$

46. For any two complex numbers z_1 and z_2 prove that (i) $|z_1 z_2| = |z_1| . |z_2|$ (ii) $\arg(z_1 . z_2) = \arg z_1 + \arg z_2$

47. Find the angle between the asymptotes to the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$

48. Obtain the Maclaurin's series for $\frac{1}{1+x}$

49. If u = log (tan x + tan y + tan z), prove that $\sum \sin 2x \frac{\delta u}{\delta x} = 2$

50. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

51. (i) Solve : $(D^2 + 6D + 9)y = 0$
(ii) Form the differential equation from the following equation $y = e^{2x}(A + Bx)$

52. Construct the truth table for $(p \wedge q) \vee r$

53. Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$

54. Find k, μ and σ of the normal distribution whose probability function is given by $f(x) = ke^{-2x^2 + 4x - 2}$

55. (a) At what angle θ do the curves $y = a^x$ and $y = b^x$ intersect ($a \neq b$) ? **(OR)**
(b) State and prove cancellation laws on group.

SECTION - C

Note : (i) Answer any *ten* questions.

(ii) Question No.70 is compulsory and choose any nine questions from the remaining.

(iii) Each question carries ten marks. 10 x 10 = 100

56. Investigate for what values of λ, μ the simultaneous equations x + y + z = 6, x + 2y + 3z = 10, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

57. Prove that cos (A - B) = cos A cos B + sin A sin B.

58. Find the vector and cartesian equations of the plane passing through the points (-1, 1, 1), (1, -1, 1) and perpendicular to the plane x + 2y + 2z = 5

59. If α and β are the roots of $x^2 - 2x + 4 = 0$ Prove that $\alpha^n - \beta^n = i2^{n+1} \sin \frac{n\pi}{3}$ and deduct $\alpha^9 - \beta^9$

60. A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level, find the length of the support if the height of the pillars are 55 mts.

61. Find the eccentricity, vertices, foci, centre and draw the diagram for the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$

62. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points (6, 0) and (-3, 0).

63. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min and its coarsened such that it forms a pile in the shape of the cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high ?

64. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere).

65. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ prove that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = \sin 2u$.

66. Find the area between the curves $y = x^2 - x - 2$, x-axis and the lines $x = -2$ and $x = 4$

67. Find the perimeter of the circle with radius a.

68. Solve $(D^2 - 1)y = \cos 2x - 2\sin 2x$

69. The sum of Rs.1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal ? ($\log_e 2 = 0.6931$)

70. (a) Show that the set G of all matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$, where $x \in R - \{0\}$, is a group under matrix multiplication.

(OR)

(a) A random variable X has the following probability mass function

x	0	1	2	3	4	5	6
P(X = x)	k	3k	5k	7k	9k	11k	13k

(i) Find k,

(ii) Evaluate $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X \leq 6)$,

(iii) What is the smallest value of x for which $P(X \leq x) > \frac{1}{2}$.