PIMS® EDUCATIONAL ACADEMY
Regd. No. : 130/10

MODEL HIGHER SECONDARY EXAMINATION

KEY FOR MATHS

PART - I

Section - A / B

- 1 mark to write the correct or the corresponding or both.
- If one write both **'option'** and **'answer'** with one of them is wrong, then award zero mark only.
- Instead of **1,2,3,4** if one writes **a,b,c,d** then marks should be awarded.

A

Q. No.	Key	Answer
1.	b	x^2
2.	c	$\frac{dp}{dt} = kp$
3.	c	$a * b = \sqrt{ab}$
4.	b	If every element of a group is its own inverse, then the group is abelian.
5.	b	4
6.	c	$(4, \infty)$
7.	d	nowhere
8.	d	both the axes
9.	a	0
10.	b	$\frac{1}{30}$
11.	a	$x^2 + 7 = 0$
12.	d	Collinear
13.	c	$1 + w + w^2 = 0$
14.	c	$\frac{2}{3}$
15.	c	$\lvert A \rvert^{n-1}$
16.	a	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$
17.	a	B = O
18.	a	1
19.	a	1
20.	d	60^0
21.	d	Z
22.	c	3
23.	a	5/3
24.	a	1
25.	a	- 2
26.	a	48
27.	d	9π
28.	c	$\frac{\angle 5}{4^6}$
29.	d	(1, 3)
30.	a	$(y')^2 - xy' + y = 0$

B

Q. No.	Key	Answer
1.	a	- 2
2.	a	48
3.	d	9π
4.	c	$\frac{\angle 5}{4^6}$
5.	d	(1, 3)
6.	a	$(y')^2 - xy' + y = 0$
7.	c	$\lvert A \rvert^{n-1}$
8.	a	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$
9.	a	B = O
10.	a	1
11.	a	1
12.	d	60^0
13.	c	$(4, \infty)$
14.	d	nowhere
15.	d	both the axes
16.	a	0
17.	b	$\frac{1}{30}$
18.	c	40
19.	b	3 units
20.	d	abc
21.	c	(3, -4, 5), 7
22.	a	$3\sqrt{7}$
23.	3	$\sqrt{3}$
24.	a	$x^2 + 7 = 0$
25.	d	Collinear
26.	c	$1 + w + w^2 = 0$
27.	c	$\frac{2}{3}$
28.	d	Z
29.	c	3
30.	a	5/3
31.	a	1

A

Q. No.	Key	Answer
31.	a	$(6t^2, 8t)$
32.	d	$\frac{\sqrt{5}}{2}$
33.	c	$\frac{2\pi}{3}$
34.	d	$0 < \theta < 1$
35.	c	40
36.	b	3 units
37.	d	abc
38.	c	(3, -4, 5), 7
39.	a	$3\sqrt{7}$
40.	3	$\sqrt{3}$

B

Q. No.	Key	Answer
32.	a	$(6t^2, 8t)$
33.	d	$\frac{\sqrt{5}}{2}$
34.	c	$\frac{2\pi}{3}$
35.	d	$0 < \theta < 1$
36.	b	x^2
37.	c	$\frac{dp}{dt} = kp$
38.	c	$a * b = \sqrt{ab}$
39.	b	If every element of a group is its own inverse, then the group is abelian.
40.	b	4

SECTION - B

41 $\Delta = 0$ **- 1 mark**

$\Delta x = 0$ **- 1 mark**

$\Delta y = 0$ **- 1 mark**

The above system is reduced to a single equation $4x + 5y = 9$ **- 1 mark**

$y = k \,;\, k \in R$ **- 1 mark**

$x = \dfrac{9 - 5\kappa}{4}$ **- 1 mark**

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42. $A \sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \\ 1 & 5 & -7 & 2 \end{bmatrix}$ $R_2 \leftrightarrow R_2$ **- 1 mark**

$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 7 & -8 & 7 \end{bmatrix}$ $\begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$ **- 2 marks**

$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 0 & 0 & -7 \end{bmatrix}$ $R_3 \rightarrow R_3 - R_2$ **- 2 marks**

$\rho(A) = 3$ **- 1 mark**

Note : The sequence of transformation need not be be same as in the above scheme. If he finds the rank by any other transformations or determinant method, full mark should be given.

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43. i) $\vec{x} = \lambda\ (\vec{a} \times \vec{b})$ - 1 mark

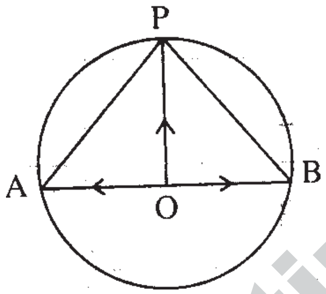
$\vec{x} \cdot \vec{c} \Rightarrow 0,\ \lambda (\vec{a} \times \vec{b}) \cdot \vec{c} = 0,\ [\vec{a}\ \vec{b}\ \vec{c}\ ] = 0$ - 1 mark

$\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}$ are coplanar - 1 mark

(ii) $\vec{n_1} = 2\vec{i} - \vec{j} + \vec{k}\ ,\ \vec{n_2} = \vec{i} + \vec{j} + 2\vec{k}$ - 1 mark

$\vec{n_1} \cdot \vec{n_2} = 3\ ,\ |\vec{n_1}| = \sqrt{6},\ |\vec{n_2}| = \sqrt{6}$ - 1 mark

$\theta = \dfrac{\pi}{3}$ - 1 mark



44. Diagram - 1 mark

$\vec{PB} = \vec{OB} - \vec{OP}$ - 1 mark

$\vec{AP} = \vec{OP} - \vec{OA} = \vec{OP} + \vec{OB}$ - 1 mark

$\vec{AP} \cdot \vec{PB} = (\vec{OP} + \vec{OB}) \cdot (\vec{OB} - \vec{OP}) = |\vec{OB}|^2 - |\vec{OP}|^2$ - 1 mark

$= 0$ - 1 mark

∴ AB subtends a right angle at P on the surface - 1 mark

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45. $\sqrt{-7 + 24i} = x + iy$ - 1 mark

$x^2 - y^2 = -7$ and $2xy = 24$ - 2 marks

$x = \pm 3$ - 1 mark

$y = \pm 4$ - 1 mark

Ans (3 - 4i)) or (-3 + 4i) - 1 mark

Note : Different method can be adopted

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46. $(8i) = 8^{1/3}\ (\text{cis}\ \pi/2)^{1/3}$ - 2 marks

$= 2\ [\ \text{Cis}\ (4k + 1)\ \pi/6]\ ,\ K = 0, 1, 2$ - 2 marks

∴ The values are $2\ \text{cis}\ \pi/6,\ \ 2\ \text{cis}\ 5\pi/6\ ,\ 2\ \text{cis}\ 9\pi/6$ - 2 marks

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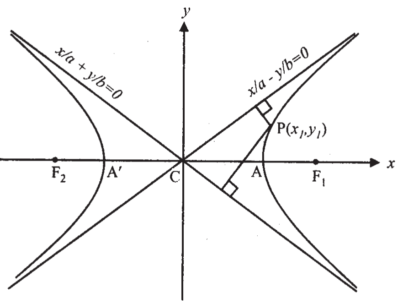
47. Diagram 

- **2 marks**

The perpendicular distance from (x_1, y_1)

to the asymptate $\frac{x}{a} - \frac{y}{b} = 0$ is $\dfrac{\frac{x_1}{a} - \frac{y_1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$ **- 1 mark**

and to $\frac{x}{a} + \frac{y}{b} = 0$ is $\dfrac{\frac{x_1}{a} + \frac{y_1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$ **- 1 mark**

Product of perpendicular distances

$$= \frac{\frac{x_1}{a} + \frac{y_1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \cdot \frac{\frac{x_1}{a} - \frac{y_1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$ **- 1 mark**

$$= \frac{a^2 \; b^2}{a^2 + b^2}$$, a constant **- 1 mark**

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48. i) At (x_1, y_1) the tangent equation is $yy_1 + (x+x_1) + 1 = 0$ **- 1 mark**

At (-1, 1) the equation is $y(1) + (x-1) + 1 = 0$ **- 1 mark**

$x + y = 0$ **- 1 mark**

ii) The point of intersection is (0,1) **- 1 mark**

$m_1 = 1, m_2 = -1$ **- 1 mark**

The angle between the curve is $\frac{\pi}{2}$ **- 1 mark**

Note : Different method can be adopted

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49. By law of the mean there exists a 't_o' in (0,4) such that $\dfrac{T(t_2) - T(t_1)}{t_2 - t_1} = T^1(t_o)$ **- 3 marks**

$t_2 - t_1 = 14$, $T(t_2) = 100$; $T(t_1) = -19$ **- 1 mark**

$T^1(t_o) = \dfrac{100 + 19}{14} = \dfrac{119}{14} = 8.5^0$ C/sec **- 2 marks**

50. $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r}$ **- 1 mark**

$\frac{\partial w}{\partial r} = \frac{2}{r}$ **- 2 marks**

$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial \theta}$ **- 1 mark**

$\frac{\partial w}{\partial \theta} = 0$ **- 2 marks**

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51. $I = \int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}}\, dx$ **- 1 mark**

$I = \int_0^3 \frac{\sqrt{3 - x}}{\sqrt{3 - x} + x}\, dx$ **- 2 marks**

$2I = \int_0^3 dx$ (or) $[x]_0^3$ **- 2 marks**

$I = \frac{3}{2}$ **- 1 mark**

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52. I. F = $(1 + x^2)$ **- 2 marks**

Solution is $y(1 + x^2) = \int \frac{\cos x}{1 + x^2}(1 + x^2)\, dx$ **- 2 marks**

$y(1 + x^2) = \sin x + c$ **- 2 marks**

53.

p	q	~ p	~ q	$(\sim P) \vee q$	$p \wedge (\sim q)$	$[(\sim P) \vee q] \vee [p \wedge (\sim q)]$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

3rd column **- 1 mark**

4th column **- 1 mark**

5th column **- 1 mark**

6th column **- 1 mark**

7th column **- 1 mark**

The given statement is a tautology **- 1 mark**

Note : The order of the rows need not be same as in the scheme.

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54. $P = \frac{1}{5}$ $q = \frac{4}{5}$, $n = 10$ **- 1 mark**

(i) using Binomial distribution **- 2 marks**

$P(x = 2) = 10C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8 = 45\left(\frac{4^8}{5^{10}}\right)$

(ii) using poisson distribution

$\lambda = 2$ **- 1 mark**

$p(x=2) = 0.2706$ **- 2 marks**

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55. (a) **Stating**

$a * b = a * c \Rightarrow b = c$ **- 1 mark**

$b * a = c * a \Rightarrow b = c$ **- 1 mark**

Proving

$a * b = a * c \Rightarrow b = c$ **- 2 marks**

$b * a = c * a \Rightarrow b = c$ **- 2 marks**

Note : For LCL the left elements must be same. For RCL the right elements must be same. One may take any three different elements.

[OR]

b) $E(x) = \frac{1}{3}$ **- 2 marks**

$E(x^2) = \frac{2}{9}$ **- 2 marks**

$Var(x) = \frac{1}{9}$ **- 2 marks**

SECTION - C

56. $[A, B] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -2 & 2 \\ \lambda & 1 & 4 & 2 \end{bmatrix}$ **- 1 mark**

$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & 1-\lambda & 4-\lambda & 2-2\lambda \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - \lambda R_1 \end{matrix}$ **- 3 marks**

$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & -\lambda & -\lambda & -2\lambda \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3 + R_2 \end{matrix}$ **- 1 mark**

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & 0 & 3\lambda & 0 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3 - \lambda R_2 \end{matrix}$$ **- 1 mark**

Case (i) $\lambda = 0$ and $\rho(A) = \rho(A, B) = 2$ **- 1 mark**

the system has infinitely many solutions **- 1 mark**

Case (ii) $\lambda \neq 0$, $\rho(A) = \rho(A, B) = 3$ **- 1 mark**

the system has unique solution **- 1 mark**

Note : Elementary transformation need not be same as in the above scheme.

One may obtain different Echelon form.

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57. $\vec{a} \times \vec{b} = \vec{i} + \vec{j} - 2k$ **- 2 marks**

$\vec{c} \times \vec{d} = \vec{i} - 3\vec{j} + \vec{k}$ **- 2 marks**

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = -5\vec{i} - 3\vec{j} - 4\vec{k}$ **- 2 marks**

$[\vec{a} \ \vec{b} \ \vec{d}] = 1$ **- 1 marks**

$[\vec{a} \ b \ \vec{d}] = -2$ **- 1 marks**

$[\vec{a} \ \vec{b} \ \vec{d}]\vec{c} - [\vec{a} \ \vec{b} \ \vec{c}] \ \vec{d} = -5\vec{i} - 3\vec{j} - 4\vec{k}$ **- 2 marks**

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58. $\vec{a} = (-\vec{i} + \vec{j} - \vec{k})$ $\vec{b} = (2\vec{i} + 2\vec{j} + \vec{k})$, $\vec{v} = 2\vec{i} + 3\vec{j} - 2\vec{k}$ **- 2 marks**

$\vec{r} = (1 - S)(-\vec{i} + \vec{j} - \vec{k}) + s \ (2\vec{i} + 2\vec{j} + \vec{k}) + t \ (2\vec{i} + 3\vec{j} - 2\vec{k})$ **- 2 marks**

(OR)

$\vec{r} = (-\vec{i} + \vec{j} - \vec{k}) + s \ (3\vec{i} + \vec{j} + 2\vec{k}) + t \ (2\vec{i} + 3\vec{j} - 2\vec{k})$ **- 3 marks**

Cartesian form :

The equation of the plane is $\begin{vmatrix} x+1 & y-1 & z+1 \\ 3 & 1 & 2 \\ 2 & 3 & -2 \end{vmatrix} = 0$ **- 3 marks**

$\Rightarrow 8x - 10y - 7z + 11 = 0$ **- 2 marks**

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59. $x = 1 \pm i$ **- 2 marks**

$\alpha^n = 2^n (\cos n \frac{\pi}{4} + i\sin n \frac{\pi}{4})$ **- 2 marks**

$\beta^n = 2^n (\cos n \frac{\pi}{4} - i \sin n \frac{\pi}{4})$ **- 2 marks**

$\alpha^n + \beta^n = 2^{\frac{n+2}{2}} \cos n \frac{\pi}{4}$ **- 2 marks**

$\alpha^8 + \beta^8 = 32$ **- 2 marks**

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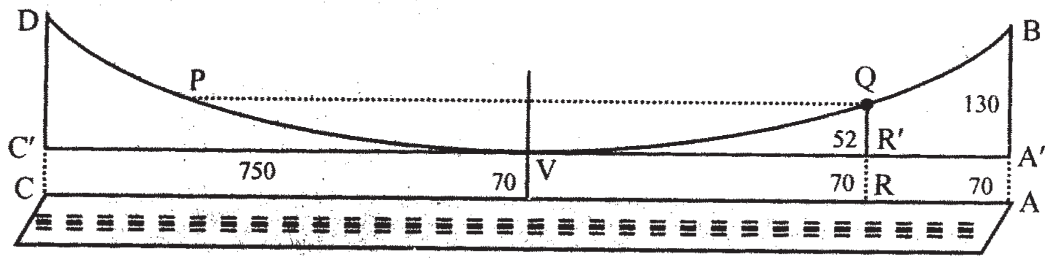
60. Diagram - 2 marks



The point B is (750, 130) - 2 marks

The equation of the parabola is $x^2 = 4\,ay$ - 1 mark

$\therefore$ Tthe equation is $x^2 = \dfrac{75 \times 750}{12}\ y$ - 2 marks

Q is $(x_1 , 52)$ - 1 mark

$v_1 = 150\sqrt{10}$ - 1 mark

$PQ = 2x_1 = 300\sqrt{10}$ ft - 1 mark

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61. The other asymptote is $2x - y + k = 0$ combined equation of the asymptotes is - 2 marks

$(x + 2y - 5)\ (2x - y + k) = 0$ - 2 marks

Equation of the rectangular hyperbola

$(x + 2y - 5)\ (2x - y + k)\ + c\ = 0$ - 2 marks

$K = 4,\ C = -16$ - 2 marks

The equation of R.H. is

$(x + 2y - 5)\ (2x - y + 4)\ - 16 = 0$ - 2 marks

Note : i) Name of the variable and the process may be different

ii) Final answer need not be further simplified

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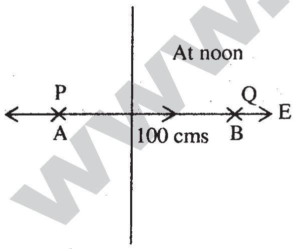
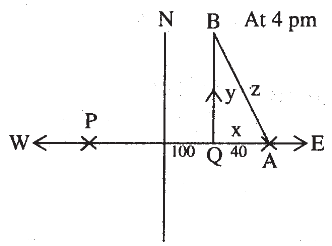
62. Diagram - 3 marks





$z^2 = x^2 + y^2$ - 2 marks

$2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$ - 1 mark

$Z = 2O\sqrt{29}$ - 2 marks

$\dfrac{dz}{dt} = \dfrac{195}{\sqrt{29}}$ - 2 marks

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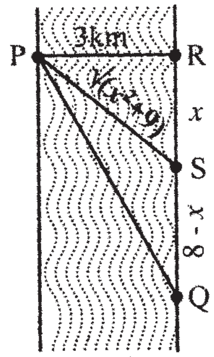
63. Diagram



- 2 marks

Rowing time $R_t = \dfrac{\sqrt{x^2+9}}{6}$ - 1 mark

Running time $r_t = \dfrac{8 - x}{8}$ - 1 mark

$T = \dfrac{\sqrt{x^2+9}}{6} + \dfrac{8 - x}{8}$ - 1 mark

$T^1_{(x)} = 0 \Rightarrow x = \dfrac{9}{\sqrt{7}}$ - 2 marks

$T(o) = 1.5, \quad T\left(\frac{9}{\sqrt{7}}\right) = 1 + \frac{\sqrt{7}}{8} = 1.33$ and $T(8) = \frac{\sqrt{73}}{6}$ - 2 marks

The smallest of these values of T occurs when $x = \dfrac{9}{\sqrt{7}}$ - 1 mark

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64. $f = \sin u$ - 2 marks

degree = ½ - 2 marks

$x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = \frac{1}{2} f$ - 3 marks

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \frac{1}{2}\tan u$ - 3 marks

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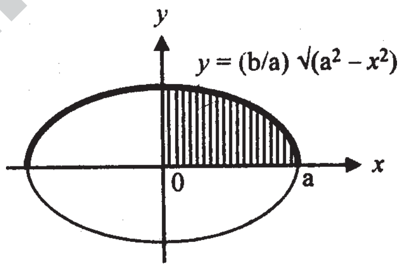
65. Diagram - 1 marks



Area $= 4\int_0^a y\,dx$ - 2 marks

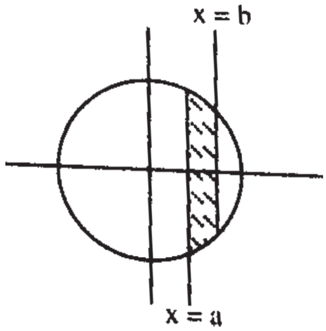
$= 4\int_0^a \dfrac{b}{a}\sqrt{a^2 - x^2}\,dx$ - 2 marks

$= \dfrac{4b}{a}\left[\dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x}{a}\right)\right]_0^a$ - 2 marks

$= \pi ab$ sq. units - 3 mark

Note : For any other method, full marks may be given

66. Diagram - 2 marks



$x^2 + y^2 = r^2$

$\Rightarrow y^1 = -\dfrac{x}{y}$ - 1 mark

$y \,.\, \sqrt{1 + (y'^2)} = r$ - 2 marks

$S = 2\pi \int_b^a r\,dx$ - 1 mark

$= 2\pi r\,(b - a)$ Sq. units - 2 marks

Deduction :

$S = 4\pi r^2$ Sq. units - 2 marks

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67. $\dfrac{dA}{dt} = KA$ - 1 mark

$A = Ce^{kt}$ - 1 mark

When $t = 1$, $A = 60 \Rightarrow Ce^k = 60$ - 2 marks

When $t = 4$, $A = 21 \Rightarrow Ce^{4k} = 21$ - 2 marks

$C^3 = \dfrac{60^4}{21}$ - 2 marks

$C = 85.15$ - 1 mark

When $t = 0$, $A = C = 85.15$ gms (app.) - 1 mark

68.

•	I	A	B	C	D	E
I	I	A	B	C	D	E
A	A	B	I	E	C	D
B	B	I	A	D	E	C
C	C	D	E	I	A	B
D	D	E	C	B	I	A
E	E	C	D	A	B	I

(Each row or column 6 x 1) - 6 marks

Closure axim is true - 1 mark

'.' is associative - 1 mark

I is the identity element in G **- 1 mark**

Inverse of I, A, B, C, D, E are I, B, C, D, E **- 1 mark**

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69. Since f(x) is a probability density function

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \Rightarrow k \int_{0}^{\infty} x^{\alpha-1} \; e^{-\beta x^{\alpha}} \; dx = 1$$ **- 2 marks**

$K = \alpha\beta$ **- 3 marks**

(ii) $p(x > 10) = \int_{10}^{\infty} f(x)\, dx = \int_{10}^{\infty} \alpha\beta x^{\alpha-1} \; e^{-\beta x^{\alpha}} \; dx$ **- 2 marks**

$p(x > 10) = e^{-\beta(10)^{\alpha}}$ **- 3 marks**

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70. a) $\dfrac{(x-1)^2}{16} - \dfrac{(y+2)^2}{9} = 1$ **- 2 marks**

$C = \dfrac{5}{4}$ **- 1 mark**

Centre is C (1, - 2) **- 1 mark**

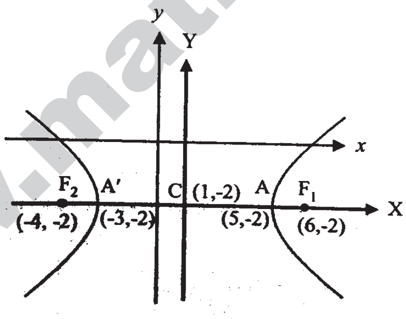
Foci : F_1 (6, -2) **- 1 mark**

F_2 (-4, -2) **- 1 mark**

Vertices : A (5, -2) **- 1 mark**

A_1 (-3, -2) **- 1 mark**

Rough Diagram : **- 2 marks**



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b) put y = vx **- 1 mark**

$V + x\dfrac{dv}{dx} = -\left(\dfrac{1+v^2}{3v}\right)$ **- 2 marks**

$\dfrac{3v}{1+4v^2}\, dv = -\dfrac{dx}{x}$ **- 1 mark**

$3\log(1+4v^2) + 8\log x = \log c$ **- 2 marks**

$(1+4v^2)^3 \,.\, x^8 = C$ **- 2 marks**

$(x^2 + 4y^2)^3\, x^2 = C$ **- 2 marks**