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+2 MODEL EXAMINATION

PART III - MATHEMATICS

[English Version]

Time : 3 Hrs.] **[Max. Marks : 200**

SECTION - A

Note : (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. The integrating factor of $\frac{dy}{dx} + 2\frac{y}{x} = e^{4x}$ is

   a) $\log x$ b) x^2 c) e^x d) x

2. The amount present in a radio active element disintegrates at a rate propotional to its amount. The differential equation corresponding to the above statement is (k is negative)

   a) $\frac{dp}{dt} = \frac{k}{p}$ b) $\frac{dp}{dt} = kt$ c) $\frac{dp}{dt} = kp$ d) $\frac{dp}{dt} = -kt$

3. Which of the following is not a binary operation on R

   a) $a * b = ab$ b) $a * b = a - b$ c) $a * b = \sqrt{ab}$ d) $a * b = \sqrt{a^2 + b^2}$

4. Which of the following is correct ?

   a) An element of a group can have more than one inverse.

   b) If every element of a group is its own inverse, then the group is abelian.

   c) The set of all 2 x 2 real matrices forms a group under matrix multiplication.

   d) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$

5. The number of rows in the truth table of $\sim[p \wedge (\sim q)]$ is

   a) 2 b) 4 c) 6 d) 8

6. The function $f(x) = x^2 - 5x + 4$ is increasing in

   a) $(-\infty, 1)$ b) $(1, 4)$ c) $(4, \infty)$ d) everywhere

7. The point of inflexion of the curve $y = x^4$ is at

   a) $x = 0$ b) $x = 3$ c) $x = 12$ d) nowhere

8. The curve $9y^2 = x^2(4 - x^2)$ is symmetrical about

   a) y-axis b) x-axis c) $y = x$ d) both the axes

9. If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

   a) 0 b) u c) 2u d) u^{-1}

10. The value of $\int_0^1 x(1-x)^4\,dx$ is

a) $\frac{1}{12}$ b) $\frac{1}{30}$ c) $\frac{1}{24}$ d) $\frac{1}{20}$

11. The quadratic equation whose roots are $+ i\sqrt{7}$ is

a) $x^2 + 7 = 0$ b) $x^2 - 7 = 0$ c) $x^2 + x + 7 = 0$ d) $x^2 - x - 7 = 0$

12. If $a = 3 + i$ and $z = 2 - 3i$ then the points on the Argand diagram representing az, 3 az and - az are

a) Vertices of a right angled trigange b) Vertices of an equilateral triangle

c) Vertices of an isoceles triangle d) Collinear

13. If w is a cube roots of unity then

a) $w^2 = 1$ b) $1 + w = 0$ c) $1 + w + w^2 = 0$ d) $1 - w - w^2 = 0$

14. The eccentricity of the conic $9x^2 + 5y^2 - 54x - 40y + 116 = 0$ is

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{4}{9}$ d) $\frac{2}{\sqrt{5}}$

15. If A is a square matrix of order n then $|\text{adj } A|$ is

a) $|A|^2$ b) $|A|^n$ c) $|A|^{n-1}$ d) $|A|$

16. Inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$

17. If A and B are any two martices such that AB = O and A is non-singular, then

a) B = O b) B is singular c) B is non-singular d) B = A

18. The system of equations $ax + y + z = 0$; $x + by + z = 0$; $x + y + cz = 0$ has a non-trivial solution then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

a) 1 b) 2 c) -1 d) 0

19. The rank of the matrix $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ is

a) 1 b) 2 c) 0 d) 8

20. If a line makes 45^0, 60^0 with positive direction of axes x and y then the angle it makes with the z axis is

a) 30^0 b) 90^0 c) 45^0 d) 60^0

21. $\ast$ is a binary operation on

a) N b) Q - { O } c) R - { O } d) Z

22. In a Poisson distribution of $P(X = 2) = P(X = 3)$ then the value of its parameter λ is

a) 6 b) 2 c) 3 d) 0

23. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is

a) $\frac{5}{3}$ b) $\frac{3}{5}$ c) $\frac{5}{9}$ d) $\frac{9}{5}$

24. If $f(x)$ is a p.d.f. of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)\,dx$ is

a) 1 b) 0.5 c) 0 d) 0.25

25. Given E (X + c) = 8 and E(X - c) = 12 then the value of c is

a) -2 b) 4 c) -4 d) 2

26. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

a) 48 b) 24 c) 12 d) 96

27. The volume generated by rotating the triangle with vertices at (0,0), (3,0) and (3,3) about x-axis is

a) 18π b) 2π c) 36π d) 9π

28. $\int_{0}^{\infty} x^5\ c^{-4x}\ dx$

a) $\frac{\angle 6}{4^6}$ b) $\frac{\angle 6}{4^5}$ c) $\frac{\lfloor 5}{4^6}$ d) $\frac{\lfloor 5}{4^5}$

29. The order and degree of the differential equation are $\left(\frac{dy}{dx}\right)^2 + x = \frac{dx}{dy} + x^2$

a) (2,2) b) (2,1) c) (1,2) d) (1,3)

30. $y = cx - c^2$ is the general solution of the differential equation

a) $(y')^2 - xy' + y = 0$ b) $y'' = 0$ c) $y' = c$ d) $(y')^2 + xy' + y - 0$

31. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is

a) $(6t^2, 8t)$ b) $(8t, 6t^2)$ c) $(t^2, 4t)$ d) $(4t, t^2)$

32. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{5}{3}$ c) $\frac{3}{2}$ d) $\frac{\sqrt{5}}{2}$

33. The angle between asymptotes of the hyperbola $24x^2 - 8y^2 = 27$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{-2\pi}{3}$

34. In the law of mean, the value of 'θ" statisfies the condition

a) $\theta > 0$ b) $\theta < 0$ c) $\theta < 1$ d) $0 < \theta < 1$

35. The least possible perimeter of a rectangle of area 100 m^2 is

a) 10 b) 20 c) 40 d) 60

36. The work done by the force $\vec{F} = \vec{i} + \vec{j} + \vec{k}$ acting on a particle, if the particle is displaced from A(3,3,3) to the point B (4,4,4) is

a) 2 units b) 3 units c) 4 units d) 7 units

37. If $\vec{a}, \vec{b}, \vec{c}$ are a right handed triad of mutually perpendicular vectors of magnitude, a, b, c then the value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

a) $a^2b^2c^2$ b) 0 c) ½ abc d) abc

38. The centre and radius of the sphere given by $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$ is

a) (-3, 4, -5), 49 b) (-6, 8, - 10), 1 c) (3, -4, 5), 7 d) (6, -8, 10), 7

39. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 9$ then $|\vec{a} \times \vec{b}|$ is

a) $3\sqrt{7}$ b) 63 c) 69 d) $\sqrt{69}$

40. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpenticular unit vectors, then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ =

a) 3 b) 9 c) $3\sqrt{3}$ d) $\sqrt{3}$

SECTION - B

Note : (i) Answer any *ten* questions.
(ii) Question No.55 in compulsory and choose any nine questions from the remaining.
(iii) Each question carries six marks. 10 x 6 = 60

41. Solve the following non-homogeneous system of linear equations by determinant method :
$4x + 5y = 9$
$8x + 10y = 18$

42. Find the rank of the matrix : $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

43. i) If $\vec{x} \cdot \vec{a} = 0$, $\vec{x} \cdot \vec{b} = 0$, $\vec{x} \cdot \vec{c} = 0$ and $\vec{x} \neq \vec{0}$ then show that $\vec{a}, \vec{b}, \vec{c}$ are coplanar

ii) Find the angle between $2x - y + z = 4$ and $x + y + 2z = 4$

44. Show that diameter of a sphere subtends a right angle at a point on the surface.

45. Find the square root of $(-7 + 24i)$

46. Find all the values of $(8i)^{1/3}$

47. Prove that the product of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its sympototes is constant and the value is $\frac{a^2 b^2}{a^2 + b^2}$

48. (i) Find the equation of the tangent to the parabola $y^2 + 2x + 1 = 0$ at (-1, 1)

(ii) Find the angle between the curves $y = e^x$ and $y = e^{-x}$

49. It took 14 sec for a thermometer to rise from -19^0C to 100^0C when it was taken from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at exactly 8.5^0 C/sec

50. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ if $w = \log(x^2 + y^2)$ where $x = r\cos\theta$, $y = r\sin\theta$

51. Evaluate : $\int_0^3 \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{3 - x}}$

52. Solve : $(1 + x^2)\ \frac{dy}{dx} + 2xy = \cos x$

53. Use the truth table to determine whether the statement $((\sim p) \vee q) \vee (p \wedge (\sim q))$ is a tautology.

54. 20% of the bolts produced in a factory are found to be defective. Find the probability that in a sample of 10 bolts chosen at random exactly 2 will be defective using. i) Binomial distribution
(ii) Poisson distribution [$e^{-2} = 0.1353$]

55. (a) State and prove cancellation laws on groups

(OR)

(b) Find the mean and variance of the distribution

$$f(x) = \begin{cases} 3e^{-3x} & , 0 < x < \infty \\ 0 & , \text{elesewhere} \end{cases}$$

SECTION - C

Note : (i) Answer any *ten* questions.
(ii) Question No.70 is compulsory and choose any nine questions from the remaining.
(iii) Each question carries ten marks. **10 x 10 = 100**

56. Discuss the solutions of the system of equations for all values of λ (use rank method) :

$x + y + z = 2$, $2x + y - 2z = 2$, $\lambda x + y + 4z = 2$

57. Verify $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\ \vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\ \vec{d}$ where $\vec{a} = \vec{i} + \vec{j} + \vec{k}$; $\vec{b} = 2\vec{i} + \vec{k}$;

$\vec{c} = 2\vec{i} + \vec{j} + \vec{k}$; $\vec{d} = \vec{i} + \vec{j} + 2\vec{k}$

58. Find the vector and cartesian equation of the plane containing the line $\frac{x - 2}{2} = \frac{y - 2}{3} = \frac{z - 1}{-2}$ and passing through the point (-1, 1, -1).

59. If α and β are the roots of $x^2 - 2x + 2 = 0$ Prove that $\alpha^n + \beta^n = 2^{\frac{n+2}{2}} \cos n\frac{\pi}{4}$ and deduct $\alpha^8 + \beta^8$

60. A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of the cable on the towers are 200 ft above the road way and the lowest point on the cable is 70 ft above the roadway. Find the vertical distance to the cable (parallel to the roadway) from a pole whose height is 122 ft.

61. Find the equation of the rectangular hyperbola which has one of its asymptotes $x + 2y - 5 = 0$ and passes through the points (6, 0) and (-3, 0)

62. At noon, ship A is 100 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4.00 p.m.

63. A man is at a point P on a bank of straight river, 3 km wide, and wants to reach point Q, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point R and then run to Q, or he could row directly to Q, or he could row to some point S between Q and R and then run to Q. If he can row at 6 km/h and run at 8 km/h where should he land to reach Q as soon as possible ?

64. Using Euler's therorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ if $u = \sin^{-1}\left(\frac{x - y}{\sqrt{x} + \sqrt{y}}\right)$

65. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

66. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r(b - a)$ and hence deduct the surface area of the sphere. $(b > a)$.

67. In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the first substance was there initially ?

68. Show that $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \omega & 0 \\ 0 & \omega^2 \end{bmatrix}, \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \omega^2 \\ \omega & 0 \end{bmatrix}, \begin{bmatrix} 0 & \omega \\ \omega^2 & 0 \end{bmatrix}\right\}$ Where $\omega^3 = 1$, $\omega \neq 1$ form a group with respect to matrix multiplication.

69. The probability density function of a random variable x is

$$f(x) = \begin{cases} kx^{\alpha - 1} e^{-\beta x^{\alpha}}, & x, \alpha, \beta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find : (i) k (ii) $P(X > 10)$

70. (a) Find the eccentricity, centre, foci and vertices of the hyperbola $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ and also trace the curve.

(OR)

(b) Solve : $(x^2 + y^2)\,dx + 3xy\,dy = 0$