MODEL HIGHER SECONDARY EXAMINATION

KEY FOR MATHS

PART - I

Section - A / B

- 1 mark to write the correct or the corresponding or both.
- If one write both **'option'** and **'answer'** with one of them is wrong, then award zero mark only.
- Instead of **1,2,3,4** if one writes **a,b,c,d** then marks should be awarded.

A

Q. No.	Key	Answer
1.	3	$x e^{2x}$
2.	1	$\frac{dy}{dx} = \lambda y$
3.	3	e^y
4.	3	$xy^1 + y = 0$
5.	4	$\theta = \frac{2\pi}{3}$
6.	3	.5
7.	3	$\frac{25}{51}$
8.	4	$\frac{1}{2}$
9.	4	all the above
10.	1	k^3 det (A)
11.	2	5y - 3x = 2
12.	1	8 cm^2/sec
13.	3	3a + b = 0
14.	4	all the critical numbers are stationary numbers
15.	1	B = O
16.	1	0
17.	2	0
18.	4	9 π
19.	2	f(2a - x) = -f(x)
20.	1	1
21.	4	60^0
22.	4	– u
23.	2	x = - a/2
24.	1	consistent and has unique solution
25.	3	2
26.	2	3 units
27.	1	$\frac{\pi}{4}$
28.	1	$(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7})$
29.	3	-16
30.	4	2 i sinθ

B

Q. No.	Key	Answer
1.	1	0
2.	4	Collinear
3.	3	-1
4.	4	8
5.	4	5
6.	4	16
7.	2	5y - 3x = 2
8.	1	8 cm^2/sec
9.	3	3a + b = 0
10.	4	all the critical numbers are stationary numbers
11.	1	B = O
12.	1	0
13.	2	0
14.	4	9 π
15.	2	f(2a - x) = -f(x)
16.	1	1
17.	3	.5
18.	3	$\frac{25}{51}$
19.	4	$\frac{1}{2}$
20.	4	all the above
21.	1	k^3 det (A)
22.	2	3 units
23.	1	$\frac{\pi}{4}$
24.	1	$(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7})$
25.	3	-16
26.	4	2 i sinθ
27.	4	60^0
28.	4	– u
29.	2	x = - a/2

A			B		
Q. No.	Key	Answer	Q. No.	Key	Answer
31.	4	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$	30.	1	consistent and has unique solution
32.	1	$\sim p \wedge \sim q$	31.	3	2
33.	1	8	32.	4	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
34.	3	(Z , .)	33.	1	$\sim p \wedge \sim q$
35.	1	0	34.	1	8
36.	4	Collinear	35.	3	(Z , .)
37.	3	-1	36.	3	$x e^{2x}$
38.	4	8	37.	1	$\frac{dy}{dx} = \lambda y$
39.	4	5	38.	3	e^y
40.	4	16	39.	3	$xy^1 + y = 0$
			40.	4	$\theta = \frac{2\pi}{3}$

SECTION - B

41 $\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ **- 1 mark**

$|A| = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = 7 - 6 = 1 \neq 0$ **- 1 mark**

$A^{-1} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$ **- 2 marks**

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow x = -1, 2$ **- 2 marks**

---

42 $\Delta = 0$ **- 2 marks**

$\Delta_x \neq 0$ **- 2 marks**

Since $\Delta = 0$ and $\Delta_x \neq 0$ the system is inconsistent

(OR)

i.e. it has no solution **- 2 marks**

---

a) $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$ **- 1 mark**

$(\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta$ **- 1 mark**

sum = $|\vec{a}|^2 |\vec{b}|^2$ **- 1 mark**

Note : Can be done in one line also

b) $[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 3 & 2 & -4 \\ 9 & 8 & -10 \\ \lambda & 4 & -6 \end{vmatrix} = 0$ **- 2 marks**

$\lambda = 5$ **- 1 mark**

44. $[\vec{a} - \vec{b}\ \ \vec{b} - \vec{c}\ \ \vec{c} - \vec{a}] = (\vec{a} - \vec{b}).\{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$ **- 1 mark**

$= (\vec{a} - \vec{b}).\{(\vec{b} \times \vec{c}) - (\vec{b} \times \vec{a}) - (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a})\}$ **- 2 marks**

$= \vec{a}.(\vec{b} \times \vec{c}) - \vec{a}.(\vec{b} \times \vec{a}) + \vec{a}.(\vec{c} \times \vec{a}) - \vec{b} \times (\vec{b} \times \vec{c}) - \vec{b} \times (\vec{c} \times \vec{a})$ **- 2 marks**

$= 0$ **- 1 mark**

---

45. $(1 + \cos\theta + i\sin\theta)^n = (2\cos^2\frac{\theta}{2} + i\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2})^n$ **- 1 mark**

$= 2^n \cos^n\frac{\theta}{2}(\cos n\frac{\theta}{2} + i\sin\frac{\theta}{2})$ **- 2 marks**

$(1 + \cos\theta + i\sin\theta)^n = 2^n \cos^n\frac{\theta}{2}(\cos n\frac{\theta}{2} - i\sin\frac{\theta}{2})$ **- 1 mark**

$(1 + \cos\theta + i\sin\theta)^n + (1\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos^n\frac{\theta}{2}$ **- 2 marks**

---

46. $\sqrt{-7 + 24i} = x + iy$ **- 1 mark**

$x^2 - y^2 = -7$ and $2xy = 24$ **- 2 marks**

$x = \pm 3$ **- 1 mark**

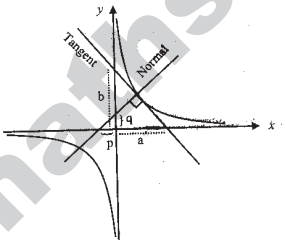
$y = \pm 4$ **- 1 mark**

Ans $(3 - 4i))$ or $(-3 + 4i)$ **- 1 mark**

Note : Different method can be adopted

---

47. Rough Diagram **- 1 mark**



Intercept on the axes are $a = 2ct$, $b = \frac{2c}{t}$ **- 1 mark**

$p = -\frac{1}{t^2}(\frac{c}{t} - ct^3)$, $q = \frac{c}{t} - ct^3$ **- 2 marks**

$\therefore ap + bq = 0$ **- 2 marks**

---

48. $y = x^{\frac{1}{x-1}}$ **- 1 mark**

$\log y = \frac{1}{x-1}\log_e x$ **- 1 mark**

$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{\log_e x}{x-1}\left(= \frac{0}{0}\right)$ **- 1 mark**

i.e. $\lim_{x\to 1} \log y = 1$ **- 1 mark**

By composite function theorem

$\Rightarrow \lim_{x\to 1} x^{\frac{1}{x-1}} = e$ **- 2 marks**

---

49. The function is constinuous on [1, 2] **- 1 mark**

It has no critical point **- 2 marks**

The values of the function at these values are f (1), f(2), i.e. $\frac{1}{2}$, $\frac{2}{3}$ **- 2 marks**

The absolute minimum is $\frac{1}{2}$ and

absolute maximum is $\frac{2}{3}$ **- 1 mark**

---

50. $\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u}$ **- 1 mark**

$\frac{\partial w}{\partial u} = \frac{2u}{\sqrt{1-(u^2 - v^2)^2}}$ **- 2 marks**

$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v}$ **- 1 mark**

$\frac{\partial w}{\partial u} = \frac{2v}{\sqrt{1-(u^2 - v^2)^2}}$ **- 2 marks**

---

51. $dv = e^{-4x}\, dx$

$u = x$ $\qquad v = -\frac{1}{4} e^{-4x}$

$u' = 1$ $\qquad v_1 = -\frac{1}{16} e^{-4x}$ **- 2 marks**

using Bernoullis formula we get

$\int_0^1 x e^{-4x}\, dx = [(x) (-\frac{1}{4} e^{-4x}) - (1) (\frac{1}{16} e^{-4x})]$ **- 2 marks**

$= \frac{1}{16} - \frac{5}{16} e^{-4}$ **- 2 marks**

---

52. Truth table for $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

3rd column **- 1 mark**

4th column **- 1 mark**

Truth table for $(\sim p) \vee (\sim q)$

p	q	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

3rd column **- 1 mark**

4th column **- 1 mark**

5th column **- 1 mark**

Both the truth tables have identical last columns

$\therefore \sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$ **- 1 mark**

Note : (1) Instead of T and F one may use 0 & 1 (or) 1 & 0

(2) The order of the rows and columns need not be as in the scheme

---

53. (i) $n = 10 \quad p = \frac{1}{6}, \; q = \frac{5}{6}$ **- 1 mark**

$p(4) = p(x = 4) = 10c_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^6$ ..... **- 2 marks**

(ii) $p(0 < x < 1) = F(1) - F(0)$ **- 2 marks**

$= \frac{1}{4}$ **- 1 mark**

---

54. When $x = 12, \; z = 2$ **- 1 mark**

$p(x < 12) = p(z < 2)$

$= p(-\infty z < 2) = p(-\infty < z < o) + p(o < z < 2)$ **- 2 marks**

$p(x < 12) = 0.9772$ **- 1 mark**

pair of shoes = 4886 **- 2 marks**

---

55. (a) The characteristic equation is

$p^2 - 2p - 3 = 0 \Rightarrow (p - 3)(p + 1) = 0$

$\Rightarrow p = 3$ and -1 **- 1 mark**

The CF is $Ae^{3x} + Be^{-x}$ **- 1 mark**

PI $= \frac{1}{D^2 - 2D - 3} \sin x \cos x$ **- 1 mark**

$= \frac{1}{130}(4\cos 2x - 7\sin 2x)$ **- 2 marks**

$y = CF + PI$

$y = Ae^{3x} + Be^{-x} + \frac{1}{130}(4\cos 3x - 7\sin 2x)$ **- 1 mark**

(or)

(b) Let G be a group a, b $\in$ G

Then $(a * b)^{-1} = b^{-1} * a^{-1}$ **- 2 marks**

$(a * b) * (b^{-1} * a^{-1}) = e$ **- 2 marks**

$(b^{-1} * a^{-1}) * (a * b) = e$ **- 2 marks**

SECTION - C

56. $[A, B] = \begin{bmatrix} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{bmatrix}$ **- 1 mark**

$\sim \begin{bmatrix} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & 0 & -199 & -398 \end{bmatrix}$ **- 3 marks**

$\rho(A, B) = \rho(A) = 3$ **- 2 marks**

The system is consistent (or) has a unique solution **- 1 mark**

$x = 4$ **- 1 mark**

$y = -1$ **- 1 mark**

$z = 2$ **- 1 mark**

Note : The echelon from need not be the same as in same as in the scheme. Full credit should be given for such echelon form

---

57. $\vec{a} = 2\vec{i} + 2\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 4\vec{j} + 2\vec{k}$, $\vec{c} = 2\vec{i} + 6\vec{k}$ **- 2 marks**

Vector form

$\therefore \vec{r} = (1 - s - t)(2\vec{i} + 2\vec{j} - \vec{k}) + s(3\vec{i} + 4\vec{j} + 2\vec{k}) + t(7\vec{i} + 6\vec{k})$ **- 3 marks**

Cartesian form

The equation of the plane is $\begin{vmatrix} x-2 & y-2 & y+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ **- 3 marks**

$5x + 2y - 3z = 17$ **- 2 marks**

Note : other vector form equation (or) cartesian equation may be adopted

---

58. $\arg\left(\frac{z-1}{z+3}\right) = \arg(z-1) - \arg(z+3) = \frac{\pi}{2}$ **- 2 marks**

$\tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+3}\right) = \frac{\pi}{2}$ **- 2 marks**

$\tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+3}}{1 + \frac{y}{x-1} \cdot \frac{y}{x+3}}\right) = \frac{\pi}{2}$ **- 2 marks**

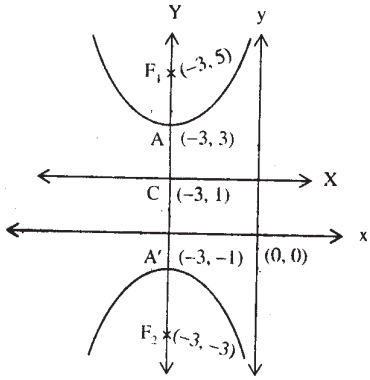
$1+\frac{y}{x-1}\cdot\frac{y}{x+3}=0$ **- 2 marks**

$\therefore$ locus of p is $x^2+y^2+2x-3=0$ **- 2 marks**

Note : Different method can be adopted

---

59. Rough Diagram **- 2 marks**



$\frac{(y-1)^2}{4}-\frac{(x-3)^2}{12}=1$ **- 2 marks**

$e=2$ **- 1 mark**

Centre (-3, 1) **- 1 mark**

Foci F_1 (-3, 5) **- 1 mark**

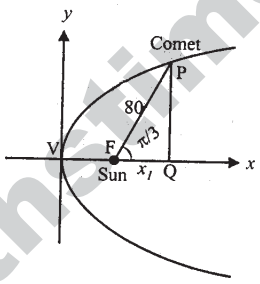
F_2 (-3, -3) **- 1 mark**

Vertices A (-3, 3) **- 1 mark**

A_1 (-3, -1) **- 1 mark**

---

60. Rough Diagram **- 2 marks**



$PQ=40\sqrt{3}$ **- 2 marks**

$FQ=40$ **- 2 marks**

$P(a+40, 40\sqrt{3})$ **- 1 mark**

$a=20$ **- 1 mark**

$y^2=80x$ **- 1 mark**

Shortest distance = 20 million kms **- 1 mark**

---

61. Equations of the two asymptotes are of the form $x+2y+l=0$ and $x-2y+m=0$ **- 2 marks**

$l=-10$, $m=6$ **- 2 marks**

Equation of the asymptotes are $x+2y-10=0$ and $x-2y+6=0$ **- 1 mark**

$(x+2y-10)(x-2y+6)=0$ **- 1 mark**

The equation of the hyperbola is of the form $(x+2y-10)(x-2y+6)+K=0$ **- 1 mark**

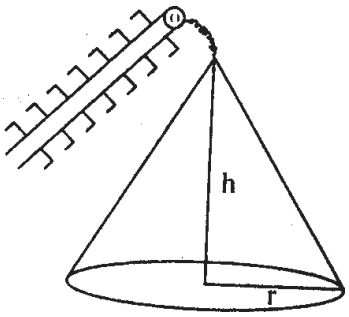
$K=64$ **- 2 marks**

Equation of the hyperbola is $(x+2y-10)(x-2y+6)+64=0$

OR equivalent form **- 1 mark**

---

62. Rough Diagram



$V = \frac{1}{3}\pi r^2 h$ **- 1 mark**

$r = \frac{h}{2}$ **- 1 mark**

$V = \frac{\pi}{12} h^3$ **- 2 marks**

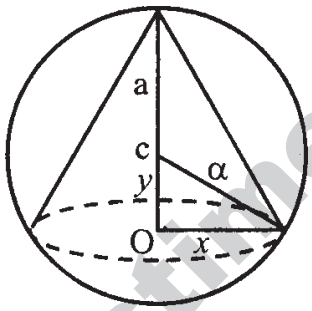
$\frac{dv}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$ **- 2 marks**

$\frac{dh}{dt} = \frac{6}{5\pi}$ **- 3 marks**

Note : π value need not be subtituted

---

63. Rough Diagram **- 2 marks**



Voulume of the cone

$V = \frac{1}{3}\pi x^2 (a + y)$ **- 1 mark**

$= \frac{1}{3}\pi (a^2 - y^2)(a + y)$ **- 1 mark**

$V_1 = 0, \Rightarrow y = \frac{a}{3}$ **- 3 marks**

When $y = \frac{a}{3}$, $V^{11} < 0$ **- 2 marks**

$V = \frac{8}{27}$(volume of the sphere) **- 1 mark**

Note : This problem can be done by different method also. If the method is correct full credit should be given.

---

64. $\frac{\partial u}{\partial x} = \frac{1}{y^2} + \frac{2y}{x^3}$ **- 2 marks**

$\frac{\partial u}{\partial x} = \frac{-2x}{y^3} - \frac{1}{x^2}$ **- 2 marks**

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{-2}{y^3} + \frac{2}{x^3}$ **- 2 marks**

$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{-2}{y^3} + \frac{2}{x^3}$ **- 2 marks**

$\Rightarrow \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ **- 2 marks**

Note :If the last stage is not written, then split the two marks to the earlier two stages

65. $\frac{dx}{dt} = a(1 - \cos t)$ **- 2 marks**

$\frac{dy}{dt} = a \sin t$ **- 2 marks**

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4a^2 \sin^2 \frac{t}{2}$

(OR)

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2a \sin \frac{t}{2}$ **- 1 mark**

Length $= \int_0^{\pi} 2a \sin \frac{t}{2} dt$ **- 3 marks**

correct integration **- 1 mark**

Length = 4a **- 1 mark**

---

66. $p = \frac{-3x^2}{1 - x^3}$, $Q = \frac{1}{1 - x^3} \sec^2 x$ **- 3 marks**

IF $= (1 - x^3)$ **- 3 marks**

The solution is $y(1 - x^3) = \int (1 - x^3) \frac{1}{1 - x^3} \sec^2 x dx + C$ **- 2 marks**

$y(1 - x^3) = \tan x + C$ **- 2 marks**

---

67. $\frac{dA}{dt} = k$ **- 1 mark**

$A = Ce^{kt}$ **- 1 mark**

$C = 10$ **- 1 mark**

$A = 10 e^{kt}$ **- 1 mark**

$K = - 0.0051$ **- 2 marks**

$A = 10 e^{- 0.0051 t}$ **- 1 mark**

$2 = e^{0.0051 t}$ **- 2 marks**

$t \approx 136$ days **- 1 mark**

68. (i) closure axiom

$A = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \in G, \quad B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in G$ **- 2 marks**

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in G$

(ii) matrix multiplication is always associative **- 1 mark**

(iii) $E = \begin{pmatrix} ½ & ½ \\ ½ & ½ \end{pmatrix} \in G$ **- 3 marks**

(iv) $A^{-1} = \begin{bmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{bmatrix} \in G$ **- 3 marks**

G is a group under matrix multiplication **- 1 mark**

---

69. $K = \frac{1}{49}$ **- 2 marks**

$p\ (x < 4) = \frac{16}{49}$ **- 2 marks**

$p\ (x > 5) = \frac{24}{49}$ **- 2 marks**

$p\ (3 < x < 6) = \frac{33}{49}$ **- 2 marks**

The smallest value of x for which $P(x < x) > \frac{1}{2}$ is 4 **- 2 marks**

---

70. (a) $\vec{b} \times \vec{c} = -5\vec{i} - 6\vec{j} - 2\vec{k}$ **- 2 marks**

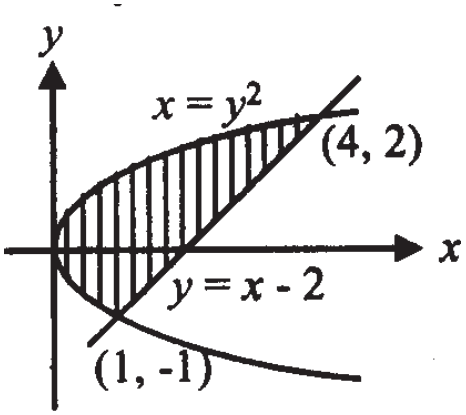
$\vec{a} \times (\vec{b} \times \vec{c}) = -12\vec{i} + 9\vec{j} + 3\vec{k}$ **- 2 marks**

$(\vec{a} \cdot \vec{c})\ \vec{b} = -12\vec{i} + 30\vec{k}$ **- 2 marks**

$(\vec{a} \cdot \vec{b})\ \vec{c} = -9\vec{j} + 27\vec{k}$ **- 2 marks**

$(\vec{a} \cdot \vec{c})\ \vec{b} - (\vec{a} \cdot \vec{b})\ \vec{c} = -12\vec{i} + 9\vec{j} + 3\vec{k}$ **- 2 marks**

(b) Rough Diagram **- 3 marks**



The points of intersection are (-1, 1) and (4, 2) **- 2 marks**

Required area $= \int_{-1}^{2} [(y + 2) - y^2]\, dy$ **- 2 marks**

$= \frac{9}{2}$ sq units **- 2 marks**
