**A**

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# +2 MODEL EXAMINATION

## PART III - MATHEMATICS

**[English Version]**

**Time : 3 Hrs. ]** **[ Max. Marks : 200**

**SECTION - A**

**Note :** **(i) All questions are compulsory.**
**(ii) Each question carries one mark.**
**(iii) Choose the most suitable answer from the given four alternatives.** **40 x 1 = 40**

1. The P.I. of $(3D^2 + D - 14)y = 13e^{2x}$ is

   1) $26x\,e^{2x}$ 2) $13x\,e^{2x}$ 3) $x\,e^{2x}$ 4) $x^2/2e^{2x}$

2. If $y = ke^{\lambda x}$ then its differential equation is

   1) $\frac{dy}{dx} = \lambda y$ 2) $\frac{dy}{dx} = ky$ 3) $\frac{dy}{dx} + ky = 0$ 4) $\frac{dy}{dx} = e^{\lambda x}$

3. The integrating factor of $dx + xdy = e^{-y}\sec^2 y\, dy$ is

   1) $e^x$ 2) $e^{-x}$ 3) $e^y$ 4) $e^{-y}$

4. The differnetial equation corresponding to $xy = c^2$ where C is arbitary contant, is

   1) $xy^{11} + x = 0$ 2) $y^{11} = 0$ 3) $xy^1 + y = 0$ 4) $xy^{11} - x = 0$

5. If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\theta$ is the angle between them, then $(\vec{a} + \vec{b})$is a unit vector is

   1) $\theta = \frac{\pi}{3}$ 2) $\theta = \frac{\pi}{4}$ 3) $\theta = \frac{\pi}{2}$ 4) $\theta = \frac{2\pi}{3}$

6. If f(x) is a p.d.f. of a normal variate X and $X \sim N(\mu,\sigma^2)$ then $\int_{-\infty}^{\mu} f(x)\,dx$ is

   1) undefined 2) 1 3) .5 4) - .5

7. If 2 cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours without replacement, is

   1) $\frac{1}{2}$ 2) $\frac{26}{51}$ 3) $\frac{25}{51}$ 4) $\frac{25}{102}$

8. A random variable X has the following probability distribution

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(X=x) | 1/4 | 2a | 3a | 4a | 5a | 1/4 |

   Then P (1 < x < 4 ) is

   1) $\frac{10}{21}$ 2) $\frac{2}{7}$ 3) $\frac{1}{14}$ 4) $\frac{1}{2}$

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9. If x is continuous random variable then P ( a < x < b)



1) p (a < x < b) 2) p (a < x < b) 3) p (a < x < b) 4) all the above

10. If A is a matrix of order 3, then det (kA)

1) $k^3$ det (A) 2) $k^2$ det (A) 3) k det (A) 4) det (A)

11. The equation of the tangent to the curve $y = \frac{x^3}{5}$ at the point $(-1, {}^{-1}/5)$ is

1) 5y + 3x = 2 2) 5y - 3x = 2 3) 3x - 5y = 2 4) 3x + 3y = 2

12. If the volume of an expanding cube is increasing at the rate of 4 $cm^3$/sec then the rate of change of surface area when the volume of the cube is 8 cubic on is

1) 8 $cm^2$/sec 2) 16 $cm^2$/sec 3) 2 $cm^2$/sec 4) 4 $cm^2$/sec

13. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at x = 1 then

1) a + b = 0 2) a + 3b = 0 3) 3a + b = 0 4) 3a + b =1

14. Identify the false statement

1) all the stationary numbers are critical numbers

2) at the stationary point the first derivative is zero

3) at critical numbers the first derivative need not exist

4) all the critical numbers are stationary numbers

15. If A and B are any two matrices such that AB = O and A is non-singular, then

1) B = O 2) B is singular 3) B is non-singular 4) B = A

16. The value of $\int_{-\pi/2}^{\pi/2} \left(\frac{\sin x}{2 + \cos x}\right) dx$ is

1) 0 2) 2 3) log 2 4) log 4

17. The value of $\int_{0}^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is

1) $\frac{\pi}{2}$ 2) 0 3) $\frac{\pi}{4}$ 4) $\pi$

18. The volume generated by rotating the triangle with vertices at (0, 0), (3,0) and (3,3) about x-axis is

1) 18 $\pi$ 2) 2 $\pi$ 3) 36 $\pi$ 4) 9 $\pi$

19. $\int_{0}^{2a} f(x)\, dx = 0$ if

1) f (2a - x) = f(x) 2) f(2a - x) = -f(x) 3) f(x) = -f(x) 4) f (-x) = f(x)

20. If A = [ 2 0 1], then rank of $AA^T$ is

1) 1 2) 2 3) 3 4) 0

21. If a line makes $45^0$, $60^0$ with positive direction of axes x and y then the angle it makes with the z axis is

1) $30^0$ 2) $90'$ 3) $45^0$ 4) $60^0$



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22. If $u = \dfrac{1}{\sqrt{x^2 + y^2}}$ , then $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y}$ is equal to mathstimes.com

1) $\frac{1}{2}$ u  2) u  3) $\frac{3}{2}$ u  4) $-$u

23. An asymptote to the curve $y^2 (a + 2x) = x^2 (3a - x)$ is

1) x = 3a  2) x = -a/2  3) x = a/2  4) x = 0

24. If $\Delta \neq 0$ then the system is

1) consistent and has unique solution  2) consistent and has infinitely many solutions

3) Inconsistent  4) Either consistent or inconsistent

25. The shortest distance of the point (2, 10, 1) from the plane $\vec{r} . (3\vec{i} - \vec{j} + 4\vec{k}) = 2\sqrt{26}$ is

1) $2\sqrt{26}$  2) $\sqrt{26}$  3) 2  4) $\frac{1}{\sqrt{26}}$

26. The work done by the force $\vec{F} = \vec{i} + \vec{j} + \vec{k}$ acting on a particle, if the particle is displaced from A(3,3,3) to the point B (4,4,4) is

1) 2 units  2) 3 units  3) 4 units  4) 7 units

27. The angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} . \vec{b}$ is

1) $\frac{\pi}{4}$  2) $\frac{\pi}{3}$  3) $\frac{\pi}{6}$  4) $\frac{\pi}{2}$

28. The d.c.s. of a vector whose direction ratios are 2,3, - 6 are

1) $(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7})$  2) $(\frac{2}{49}, \frac{3}{49}, \frac{-6}{49})$  3) $(\frac{\sqrt{2}}{7}, \frac{\sqrt{3}}{7}, \frac{-\sqrt{6}}{7})$  4) $(\frac{2}{7}, \frac{3}{7}, \frac{6}{7})$

29. If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

1) 0  2) 32  3) -16  4) -32

30. The value of $e^{i\theta} - e^{-i\theta}$ is

1) sin θ  2) 2 sin θ  3) i sin θ  4) 2isinθ

31. The polar form of the complex number $(i^{25})^3$ is

1) $\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}$  2) $\cos \pi + i \sin \pi$  3) $\cos \pi - i \sin \pi$  4) $\cos\frac{\pi}{2} - i \sin\frac{\pi}{2}$

32. If *p* stands for the statement "sita likes reading" and *q* for the statement "sita likes playing". " sita likes neither rading nor playing" stands for

1) $\sim p \wedge \sim q$  2) $p \wedge \sim q$  3) $\sim p \wedge q$  4) $p \wedge q$

33. If a compound statement is made up of three simple statements, then the number of rows in the truth table is

1) 8  2) 6  3) 4  4) 2

34. Which of the following is not a group ?

1) $(Z_n, +_n)$ 2) $(Z, +)$ 3) $(Z, .)$ 4) $(R, +)$

35. In the set of integers under the operation * defined by a * b = a + b - 1, the identity element is

1) 0 2) 1 3) a 4) b

36. If a = 3 + i and z = 2 - 3i then the points on the Argand diagram representing az, 3az and - az are

1) Vertices of a right angled triangel 2) Vertices of an equilateral triangle

3) Vertices of an isosceles triangle 4) Collinear

37. If the normal to the R.H. $xy = c^2$ at '$t_1$' meets the curve again at '$t_2$' then $t_1^3\, t_2$ =

1) 1 2) 0 3) -1 4) -2

38. The length of the latus rectum of the parabola whose vertex is (2, -3) and the directrix $x = 4$ is

1) 2 2) 4 3) 6 4) 8

39. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is

1) $\sqrt{7}$ 2) 4 3) 3 4) 5

40. The length of the latus rectum of the rectangular hyperbola xy = 32 is

1) $8\sqrt{2}$ 2) 32 3) 8 4) 16

## SECTION - B

**Note : (i) Answer any *ten* questions.**

**(ii) Question No.55 in compulsory and choose any nine questions from the remaining.**

**(iii) Each question carries six marks. 10 x 6 = 60**

41. Solve by matrix iversion method 7x + 3y = -1, 2x + y=0

42. Solve the following non-homogeneous equations of three unknown

$2x + 2y + z = 5$

$x - y + z = 1$

$3x + y + 2z = 4$

43. a) If $\vec{a}, \vec{b}$ are any two vectors, then $|\vec{a} \times \vec{b}|^2 + (\vec{a} . \vec{b})^2 = |\vec{a}|^2 \; |\vec{b}|^2$

b)Find the value of $\lambda$ if the points ( 3, 2, -4), (9, 8, -10) and ( $\lambda$ , 4, -6) are collinear.

44. Prove that $[\vec{a} - \vec{b} \;\; \vec{b} - \vec{c} \;\; \vec{c} - \vec{a}] = 0$

45. Prove that : $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n (\theta/2) \cos \frac{n\theta}{2}$

46. Find the square root of ( - 7 + 24i)

47. The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts a, b and the normal at the point makes intercepts p, q on the axes. Prove that ap + bq = 0.

48. Evaluate $\lim_{x \to 1} x^{\frac{1}{x-1}}$

49. Find the absolute maximum and absolute minimum values of *f* on the given interval : $f(x) = \frac{x}{x+1}$ , [1,2]

50. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = \sin^{-1}xy$ where $x = u + v$ , $y = u - v$.

51. Evaluate : $\int_0^1 x\, e^{-4x}\, dx$

52. Show that $\sim (p \wedge q) \equiv ((\sim p) \vee (\sim q))$

53. a) A pair of dice is thrown 10 times. If getting a doublet is considered a success find the probability of 4 success.

b) If $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right)$ $-\infty < x < \infty$ is a distribution function of a continuous variable X,

find $P(0 < x < 1)$

54. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

55. (a) Solve $(D^2 - 2D - 3)\, y = \sin x \cos x$

**(OR)**

(b) State and prove reversal law.

## SECTION - C

**Note : (i) Answer any *ten* questions.**
**(ii) Question No.70 is compulsory and choose any nine questions from the remaining.**
**(iii) Each question carries ten marks. 10 x 10 = 100**

56. Examine the consistency of the following system of equations. If it is consistent then solve the same.

$4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$

57. Find the vector and cartesian equations of the plane passing through the points (2,2 -1), (3,4,2) and (7,0,6)

58. P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$

59. Find the eccentricity, centre, foci and vertices of the following hyperbola and draw their diagrams.

$x^2 - 3y^2 + 6x + 6y + 18 = 0$

60. A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet come nearer to the sun ? (Take the orbit as open rightward)

61. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, (2,4) is the centre of the hyperbola and it passes through (2, 0)

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62. Gravel is being dumped from a conveyor belt at a rate of 30 $ft^3$/min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high ?

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63. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius *a* is $\frac{8}{27}$ (volume of the sphere)

64. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ when $u = \frac{x}{y^2} - \frac{y}{x^2}$

65. Find the length of the curve x = a ( t - sin t), y = a ( 1 - cos t) between t = 0 and $\pi$ .

66. Solve $(1 - x^3) \frac{dy}{dx} - 3x^2 y = \sec^2 x$

67. A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10mgm to 5 mgm. [ $\log_e 2 = 0.6931$]

68. Show that the set G of all matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$, where $x \in R - \{ 0 \}$, is a group under matrix multiplication.

69. A random variable X has the following probability mass function

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |

1) Find K

2) Evaluate P (x < 4), P (X > 5) and P (3 < X < 6)

3) What is the smallest value of x for which $P ( X < x ) > \frac{1}{2}$ .

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70. (a) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -2\vec{i} + 5\vec{k}$, $\vec{c} = \vec{j} - 3\vec{k}$

Verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

**(OR)**

(b) The area of the region enclosed by $y^2 = x$ and $y = x - 2$.