PIMS EDUCATIONAL ACADEMY Regd. No. : 130/10

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+2 MODEL EXAMINATION

BUSINESS MATHEMATICS

[English Version]

Time : 3 Hrs.] **[Max. Marks : 200**

SECTION - A

Note : (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. If $A = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$ then A^{-1} is

a) $\begin{bmatrix} -0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$ b) $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$ c) $\begin{bmatrix} 0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}$ d) $\begin{bmatrix} 0.2 & 0.4 \\ -0.4 & 0.2 \end{bmatrix}$

2. If $|A| = 0$ then $|\text{Adj } A|$ is

a) 0 b) 1 c) -1 d) + 1

3. The rank of a zero matrix is

a) 0 b) 1 c) -1 d) ∞

4. The rank of an n x n matrix each of whose elements is 1 is

a) 1 b) 2 c) n d) n^2

5. If the minor of a_{23} equals the cofactor of a_{23} in $|a_{ij}|$ then the minor of a_{23} is

a) 1 b) 2 c) 0 d) 3

6. Eccentricity of the rectangular hyperbola is

a) 2 b) $\frac{1}{2}$ c) $\sqrt{2}$ d) $\frac{1}{\sqrt{2}}$

7. Eccentricity of the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$ is

a) $\frac{3}{2}$ b) $\frac{9}{4}$ c) $\frac{5}{4}$ d) 4

8. Focus of $y^2 = -4ax$ is

a) (a, 0) b) (0, a) c) (0, -a) d) (-a, 0)

9. Latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is

a) $\frac{2a^2}{b}$ b) $\frac{a^2}{2b}$ c) $\frac{2b^2}{a}$ d) $\frac{b^2}{2a}$

10. The average fixed cost of the function $C = 2x^3 - 3x^2 + 4x + 8$ is

a) $\frac{2}{x}$ b) $\frac{4}{x}$ c) $\frac{-3}{x}$ d) $\frac{8}{x}$

11. For cost function $C = \frac{1}{10} e^{2x}$, the martinal cost is

a) $\frac{1}{10}$ b) $\frac{1}{5} e^{2x}$ c) $\frac{1}{10} e^{2x}$ d) $\frac{1}{10} e^{x}$

12. If $y = 2x^2 + 3x$, the instantaneous rate of change of y at $x = 4$ is

a) 16 b) 19 c) 30 d) 4

13. The slope of the normal to the curve $\sqrt{x} + \sqrt{y} = 5$ at $(9, 4)$ is

a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $\frac{3}{2}$ d) $-\frac{3}{2}$

14. The slope of the tangent to the curve $y = \cos t, x = \sin t$ at $t = \frac{\pi}{4}$ is

a) 1 b) 0 c) $\frac{1}{\sqrt{2}}$ d) -1

15. If $u = \log (e^x + e^y)$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is equal to

a) $\frac{1}{e^x + e^y}$ b) $\frac{e^x}{e^x + e^y}$ c) 1 d) $e^x + e^y$

16. The maximum value of $f(x) = \cos x$ is

a) 0 b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) 1

17. $f(x, y) = \frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}$ is a homogeneous function of degree

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{5}$

18. If $R = 5000$ units / year, $C_1 = 20$ paise, $C_3 =$ Rs. 20 then EOQ is

a) 1000 b) 5000 c) 200 d) 100

19. $\int_{-3}^{3} x \, dx$ is

a) 0 b) 2 c) 1 d) -1

20. The area bounded by $y = x$, y-axis and $y = 1$ is

a) 1 b) ½ c) log 2 d) 2

21. The area under the curve $x = g(y)$, the y - axis and the lines $y = c$ and $y = d$ is

a) $\int_{c}^{d} y \, dy$ b) $\int_{c}^{d} x \, dy$ c) $\int_{c}^{d} y \, dx$ d) $\int_{c}^{d} x \, dx$

22. The order and degree of $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{2}{3}} = \frac{d^2y}{dx^2}$ are

a) 3 and 2 b) 2 and 2 c) 3 and 3 d) 2 and 2

23. The solution of $x\,dx + y\,dy = 0$ is

a) $x^2 + y^2 = c$ b) $\frac{x}{y} = c$ c) $x^2 - y^2$ d) $xy = c$

24. The integrating factor of $x\frac{dy}{dx} - y = e^x$ is

a) $\log x$ b) $e^{\frac{-1}{x}}$ c) $\frac{1}{x}$ d) $\frac{-1}{x}$

25. The solution of $\frac{d^2y}{dx^2} - y = 0$

a) $(A + B)\,e^x$ b) $(Ax + B)\,e^x$ c) $Ae^x + \frac{B}{c^x}$ d) $(A + Bx)\,e^{-x}$

26. E =

a) $1 + \Delta$ b) $1 - \Delta$ c) $\nabla + 1$ d) $\Delta - 1$

27. Five data relating to x and y are to be fit in a straight line. It is found that $\sum x = 0$ and $\sum y = 15$. Then the y-intercept of the line of best fit is.

a) 1 b) 2 c) 3 d) 4

28. If $X \sim N(\mu, \sigma)$, the standard Normal variate is distributed as

a) N (0, 0) b) N (1, 0) c) N (0, 1) d) N (1, 1)

29. The random variables X and Y are independent if

a) E (X Y) =1 b) E (XY) = 0 c) E (X Y) = E(X) E(Y) d) E (X+Y) = E(X) +E(Y)

30. If $X \sim N(8, 64)$, the standard normal variage Z will be

a) $\frac{X - 64}{8}$ b) $\frac{X - 8}{64}$ c) $\frac{X - 8}{8}$ d) $\frac{X - 8}{\sqrt{8}}$

31. The normal distribution curve is

a) Bimodal b) Unimodal c) Skewed d) none of these

32. The critical region for Z at 1% level is

a) $|Z| < 1.96$ b) $|Z| > 2.58$ c) $|Z| < 1.96$ d) $|Z| > 2.58$

33. The theory of sampling is based on

a) sample size b) sample unit c) principle of statistical regularity d) population size

34. Which of the following statements is true ?

a) point estimate gives a range of values

b) Sampling is done only to estimate a statistic

c) Sampling is done to estimate the population parameter

d) Sampling is not possible for an infinite population

35. Probability of rejecting the null hypothesis when it is true is

a) Type I error b) Type II error c) Sampling error d) Standard error

36. An additive model of time series with the components T, S, C and I is

a) $Y = T + S + C - 1$ b) $Y = T + S \times C + 1$ c) $Y = T + S + C + 1$ d) $Y = T + S + C \times 1$

37. Index number is a

a) measure of relative changes b) a special type of an average

c) a percentage relative d) all the above

38. Variation in the items produced in a factory may be due to

a) chance causes b) assignable causes c) both (a) and (b) d) neither (a) or (b)

39. The range of correlation co-efficient is

a) 0 to ∞ b) $-\infty$ to ∞ c) -1 to 1 d) none of these

40. The lines of regression interest at the point

a) (X, Y) b) ($\overline{X}$, $\overline{Y}$) c) (0, 0) d) none of these

SECTION - B

Note : (i) Answer any 10 questions.

(iii) Each question carries six marks. **10 x 6 = 60**

41. Verify $(AB)^{-1} = B^{-1} A^{-1}$, when $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 0 \\ 0 & 9 \end{bmatrix}$

42. Find the rank of the matrix $A \begin{bmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{bmatrix}$

43. Find the equation of the ellipse whose focus is (1,2) directrix is $2x - 3y + 6 = 0$ and eccentricity is $\frac{2}{3}$

44. Find the marginal revenue for the revenue function $R(x) = 100x + \frac{x^2}{2}$, where $x = 10$.

45. Find the equation of the tangent and normal to the demand curve $y = 10 - 3x^2$ at (1, 7)

46. Using Euler's theorem if $u = \log \frac{x^4 + y^4}{x - y}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$.

47. The marginal cost function of manufacturing x units of a commodity is $3x^2 - 2x + 8$. If there is no fixed cost find the total cost and average cost functions.

48. Solve the differential equation $\frac{dy}{dx} + y \cot x = \text{cosec } x$

49. Solve $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 4y = 5 + 3e^x$

50. From the following data, find f(3)

x :	1	2	3	4	5
f(x) :	2	5	-	14	32

51. Fit the line of best fit if $\Sigma x = 75$, $\Sigma y = 115$, $\Sigma x^2 = 1375$ $\Sigma xy = 1875$, and $n = 6$.

52. A random variable X has the following probability function

Values of X, x :	0	1	2	3
p(x) :	$\frac{1}{16}$	$\frac{3}{8}$	k	$\frac{5}{16}$

(i) Find the value of k (ii) Construct the c.d.f. of X

53. Out of 1000 TV viewers, 320 watched a particular programme. Find 95% confidence limits for TV viewers who watched this programme.

54. Calculate the correlation co-efficient from the following data.

X :	12	9	8	10	11	13	7
Y :	14	8	6	9	11	12	3

55. Obtain the trend values by the method of Semi-Average

Year	1987	1988	1989	1990	1991	1992	1993
production (in Tonnes)	90	110	130	150	100	150	200

SECTION - C

Note : (i) Answer any 10 questions.
(iii) Each question carries ten marks. **10 x 10 = 100**

56. Solve by Cramer's rule : $x + y = 2$, $y + z = 6$, $Z + x = 4$.

57. Suppose the inter-relationship between the production of two industries P and Q in a year (in lakhs of ruppes) is

Producer	User		Final Demand	Total Output
	P	Q		
P	15	10	10	35
Q	20	30	15	65

Find the outputs when the final demand changes to (i) 12 for P and 18 for Q (ii) 8 for P and 12 for Q

58. Find the centre, eccentricity, foci and latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 64y - 199 = 0$

59. Find the elasticity of demand when the demand is $q = \frac{20}{p+1}$ and $p = 3$. Interpret the result.

60. $R = 21x - x^2$ and $C = \frac{x^3}{3} - 3x^2 + 9x + 16$ are respectively the sales revenue and cost function of x units sold.

Find (i) At what output the revenue is maximum ? What is the total revenue at this point ?

(ii) What is the marginal cost at a minimum ?

(iii) What output will maximise the profit ?

61. The demand for a quantity A is $q_1 = 16 - 3p_1 - 2p_2^2$ Find

(i) the partial elasticities $\frac{Eq_1}{Ep_1}$, $\frac{Eq_1}{Ep_2}$

(ii) the partial elasticities for $p_1 = 2$ and $p_2 = 1$.

62. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

63. The demand and supply functions under pure competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price.

64. Suppose that the quantity demanded

$Q_d = 42 - 4p - 4\frac{dp}{dt} + \frac{d^2p}{dt^2}$ and quantity supplied $Q_s = -6 + 8p$ where p is the price. Find the

equilibrium price for market clearance.

65. Fit a straight line to the following data :

x :	4	8	12	16	20	24
y :	7	9	13	17	21	25

66. For the following probability distribution of X

X :	0	1	2	3
p(x) :	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Find : (i) $P(X < 1)$ (ii) $P(X < 2)$ (iii) $P(0 < X < 2)$

67. The mean yield for one-acre plot is 663 kgs with a S.D. 32 kgs. Assuming normal distribution, how many one-acre plot in a batch of 1000 plots would you expect to have yield (i) over 700 kgs (ii) below 650 kgs.

68. To Test the conjecture of the management that 60 percent employees favour a new bonus scheme, a sample of 150 employees was drawn and their opinion was taken whether they favoured it or not. Only 55 employees out of 150 favoured the new bonus scheme Test the conjecture at 1% level of significance.

69. Calculate Fisher's Ideal Index from the following data and verify that it satisfies both Times Reversal and Factor Reversal test

Commodity	Price		Quantity	
	1985	1986	1985	1986
A	8	20	50	60
B	2	6	15	10
C	1	2	20	25
D	2	5	10	8
E	1	5	40	30

70. The following are the $\overline{X}$ and R values for 20 samples of 5 readings. Draw $\overline{X}$ chart and R chart and write your conclusion.

Samples	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	34	31.6	30.8	33	35	33.2	33	32.6	33.8	37.8
R	4	4	2	3	5	2	5	13	19	6

Samples	11	12	13	14	15	16	17	18	19	20
$\overline{X}$	35.8	38.4	34	35	38.8	31.6	33	28.2	31.8	35.6
R	4	4	14	4	7	5	5	3	9	6

(Given for n = 5, $A_2 = 0.58$, $D_3 = 0$ $D_4 = 2.12$)