

HSC MATHS QUARTERLY EXAM ANSWER KEY-2015

SEC-A

1.c	2.c	3.c	4.b	5.d	6.d	7.d	8.c	9.b	10.b
11.b	12.b	13.b	14.c	15.a	16.a	17.d	18.c	19.b	20.c
21.c	22.b	23.d	24.d	25.d	26.c	27.b	28.b	29.a	30.b
31.b	32.a	33.c	34.b	35.b	36.c	37.d	38.c	39.b	40.c

SEC-B

41. $|A| = -1$ ----- 1

$$\text{Adj} = \begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix} \quad \text{-----} 2+2$$

$A^{-1} = A$ -----1

42. $\sim \begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ -2 & 1 & 3 & 4 \end{bmatrix} R_{1 \rightarrow R_3}$ -----1

$\sim \begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 11 & 18 \end{bmatrix} R_{3 \rightarrow R_3 + 2R_1}$ -----2

$\sim \begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 4 \end{bmatrix} R_{3 \rightarrow R_3 - 7R_2}$ -----2

$\rho(A) = 3$ ----- 1

43. $|A| = 1$ -----1

$\text{adj } A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ ----- 1+2

$x=1, y=2$ -----2

44.(i) $a+3b=10i+7j-k$ -----1

$2a-b=-i+5k$ -----1

$(a+3b).(2a-b)=-15$ -----1

(ii) $[a \ b \ c] = \begin{vmatrix} \lambda & 0 & 3 \\ 1 & 3 & -1 \\ -5 & -3 & 7 \end{vmatrix} = 0$ -----2

$\lambda = -2$ -----1

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45. $r = -i + 2j + 3k$ -----1

$M = r \times F$ -----1

$M = 21i - 7k$ -----2

Moment = $7\sqrt{10}$ -----1

D.C.S = $\left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right)$ -----1

46. Diagram -----1

$PA = PO + OA$ -----1

$PB = PO - OA$ -----1

$PA \cdot PB = 0$ -----2

$PA \perp PB$ (or) $\angle APB = \frac{\pi}{2}$ -----1

47. $Z_1 = r_1 e^{i\theta_1} \Rightarrow |Z_1| = r_1, \arg(Z_1) = \theta_1$ -----1

$Z_2 = r_2 e^{i\theta_2} \Rightarrow |Z_2| = r_2, \arg(Z_2) = \theta_2$ -----1

$z_1 z_2 = r_1 e^{i\theta_1} r_2 e^{i\theta_2}$ -----1

$= r_1 r_2 e^{i(\theta_1 + \theta_2)}$ -----1

(i) $|z_1 z_2| = r_1 r_2$
 $= |z_1| |z_2|$ -----1

(ii) $\arg(z_1 z_2) = \theta_1 + \theta_2$
 $= \arg(Z_1) + \arg(Z_2)$ -----1

48. $\sqrt{-7 + 24i} = x + iy$ -----1

$x^2 + y^2 = -7, 2xy = 24$ -----1

$x^2 - y^2 = 25$ -----1

$x^2 = 9, y^2 = 16$ -----1

$x = \pm 3, y = \pm 4$ -----1

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- $\sqrt{-7 + 24i} = (3+4i)$ (or) $(-3-4i)$ ---1
49. Diagram ----2
- Parabola equation is $y^2 = 4ax$ -----2
- It passes P(4,6) ----1
- $a = 9/4 = 2.25$ -----1
50. centre (1,3) ----1
- Equation of R.H is $(x-1)(y-3) = c^2$ -----2
- $c^2 = 16$ -----1
- R.H is $(x-1)(y-3) = 16$ ----1
- Asymptotes $(x-1) = 0$ and $(y-3) = 0$ -----1
51. $m_1 = \frac{x_1}{y_1}$ -----2
- $m_2 = -\frac{c^2}{x_1^2}$ -----2
- To prove = -1 -----2
52. (i) continuous in $[0,1]$, diff in $(0,1)$ ----1
- $f(a) \neq f(b)$ -----1
- Rolls theorem fails. -----1
- (ii) $\lim_{x \rightarrow \infty} \frac{\log x}{x} = \lim_{x \rightarrow \infty} \frac{1/x}{1}$ -----2
- $= 0$ -----1
53. $dy = \frac{1}{3} x^{-\frac{2}{3}} dx$ -----2
- $x=64$, $dx=1$, $dy = \frac{1}{48}$ -----2
- $\sqrt[3]{65} = 4 + 0.021 \approx 4.021$ -----2
- 54.
- $\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$ -----1

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$$\sin 2x \frac{\partial u}{\partial x} = \frac{2 \tan x}{\tan x + \tan y + \tan z} \quad \text{-----}2$$

$$\sin 2y \frac{\partial u}{\partial y} = \frac{2 \tan y}{\tan x + \tan y + \tan z} \quad \text{-----}1$$

$$\sin 2z \frac{\partial u}{\partial z} = \frac{2 \tan z}{\tan x + \tan y + \tan z} \quad \text{-----}1$$

$$\text{By adding } \sum \sin 2x \frac{\partial u}{\partial x} = 2. \quad \text{-----}1$$

55. (a)

$$\text{other root is } 2 - i, \quad \text{-----}1$$

$$\text{factor is } x^2 - 4x + 5 \quad \text{-----}1$$

$$\therefore 6x^4 - 25x^3 + 32x^2 - 3x - 10 \equiv (x^2 - 4x + 5)(6x^2 + px - 2)$$

$$\Rightarrow p = -1. \quad \text{-----}1$$

$$\text{other factor is } 6x^2 - x - 2. \text{ m} \quad \text{-----}1$$

$$x = -2, \frac{2}{3} \quad \text{-----}1$$

$$\text{Roots are } 2 \pm i, -2, \frac{2}{3}. \quad \text{-----}1$$

$$(b). f^1(x) = 3x^2 - 6x \quad \text{-----}1$$

$$f^1(x) = 0, x = 0, 2 \quad \text{-----}1$$

$$\text{Max val} = 17, \quad \text{-----}2$$

$$\text{Min val} = -3 \quad \text{-----}2$$

SEC -C

$$56. \Delta = 24 \quad \text{-----}1$$

$$\Delta_a = 24 \quad \text{-----}2$$

$$\Delta_b = 12 \quad \text{-----}2$$

$$\Delta_c = 24 \quad \text{-----}2$$

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Solution $x=1, y=2, z=1, \dots-3$

$$57. [A, B] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -2 & 2 \\ \lambda & 1 & 4 & 2 \end{bmatrix} \quad \dots\dots\dots 1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & -4 & -2 \\ 0 & 1-\lambda & 4-\lambda & 2-2\lambda \end{bmatrix} \quad \dots\dots\dots 2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & -4 & -2 \\ 0 & -\lambda & -\lambda & -2\lambda \end{bmatrix} \quad \dots\dots\dots 1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & -4 & -2 \\ 0 & 0 & 3\lambda & 0 \end{bmatrix} \quad \dots\dots\dots 2$$

Case(i) $\lambda=0, \rho(A)=\rho(A,B)=2$, The system has infinitely many solutions. $\dots\dots\dots 2$

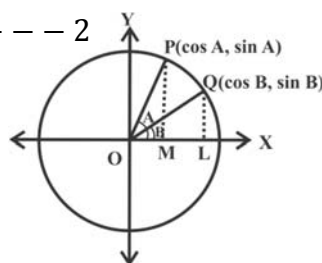
Case(ii) $\lambda \neq 0, \rho(A)=\rho(A,B)=3$, The system has unique solution $\dots\dots\dots 2$

58. Diagram $\dots\dots\dots 3$,

$$\vec{OP} = \cos A \vec{i} + \sin A \vec{j} \quad \dots\dots\dots 2$$

$$\vec{OQ} = \cos B \vec{i} + \sin B \vec{j} \quad \dots\dots\dots 2$$

$$\vec{OP} \cdot \vec{OQ} = \cos(A - B) \quad \dots\dots\dots 1$$



$$\vec{OP} \cdot \vec{OQ} = \cos A \cos B + \sin A \sin B \quad \dots\dots\dots 1$$

$$\boxed{\cos(A - B) = \cos A \cos B + \sin A \sin B} \quad \dots\dots\dots 1$$

$$59. \vec{a} = -\vec{i} - 2\vec{j} + \vec{k}, \vec{u} = \vec{i} + 2\vec{j} + 4\vec{k}, \vec{v} = 2\vec{i} - \vec{j} + 3\vec{k} \quad \dots\dots\dots 3$$

Vector equation is $\therefore \vec{r} = \vec{a} + s\vec{u} + t\vec{v}$,

$$\boxed{\vec{r} = -\vec{i} - 2\vec{j} + \vec{k} + s(\vec{i} + 2\vec{j} + 4\vec{k}) + t(2\vec{i} - \vec{j} + 3\vec{k})} \quad \dots\dots\dots 2$$

$$(x_1, y_1, z_1) = (-1, -2, 1); (l_1, m_1, n_1) = (1, 2, 4); (l_2, m_2, n_2) = (2, -1, 3)$$

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Catesian equation is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ -----2

$\begin{vmatrix} x + 1 & y + 2 & z - 1 \\ 1 & 2 & 4 \\ 2 & -1 & 3 \end{vmatrix} = 0$ -----1

$10x + 5y - 5z + 25 = 0$ (or) $2x + y - z + 5 = 0$ -----2

60. $\alpha = p + iq; \beta = p - iq$ -----2

$(y + \alpha)^n = q^n \frac{(\cos\theta + i\sin\theta)^n}{\sin^n\theta} = \frac{q^n}{\sin^n\theta} (\cos n\theta + i\sin n\theta)$ -----2

$(y + \beta)^n = \frac{q^n}{\sin^n\theta} (\cos n\theta - i\sin n\theta)$ -----2

$\alpha - \beta = p + iq - p + iq = 2iq$ -----2

$\therefore \frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\frac{q^n}{\sin^n\theta}(2i\sin n\theta)}{2iq} = q^{n-1} \frac{\sin n\theta}{\sin^n\theta}$ -----2

61.

$x^4 - x^3 + x^2 - x + 1 = 0$

Multiply by $x + 1 \Rightarrow x^5 + 1 = 0 \Rightarrow x^5 = -1$ -----1

$\Rightarrow x = (-1)^{\frac{1}{5}} = (\cos\pi + i\sin\pi)^{\frac{1}{5}}$ -----2

$= \cos(2k + 1)\frac{\pi}{5} + i\sin(2k + 1)\frac{\pi}{5}, k = 0, 1, 2, 3, 4$ -----3

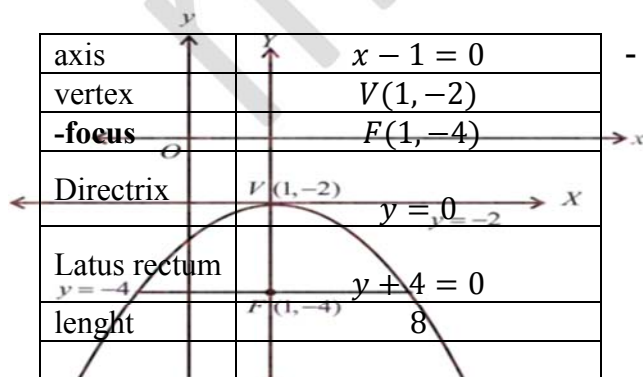
$\boxed{\text{cis } \frac{\pi}{5}, \text{cis } \frac{3\pi}{5}, \text{cis } \frac{5\pi}{5} = \text{cis } \pi = -1, \text{cis } \frac{7\pi}{5}, \text{cis } \frac{9\pi}{5}}$ -----2

the root $x = -1$, then the solutions are

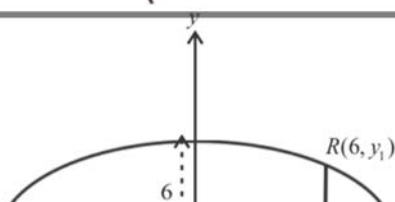
$\boxed{\text{cis } \frac{\pi}{5}, \text{cis } \frac{3\pi}{5}, \text{cis } \frac{7\pi}{5}, \text{cis } \frac{9\pi}{5}}$ -----2

62. Rough diagram -----2

$(x - 1)^2 = -8(y + 2)$ -----2



#each point carries 1 mark-----6



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63. Rough diagram -----2

$$AA' = 2a = 20 \Rightarrow a = 10 \text{ -----1}$$

$$b = 18 - 12 = 6 \text{ -----1}$$

Ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{10^2} + \frac{y^2}{6^2} = 1 \text{ -----2}$

It passes $R(6, y_1)$ -----1

$$\therefore \frac{6^2}{100} + \frac{y_1^2}{36} = 1 \text{ -----1}$$

$$y_1 = 4 \cdot 8 \text{ -----1}$$

Required height = $16 \cdot 8$ ft -----1

64.

$$y = x + 4 \text{ -----1}$$

$$m = 1, c = 4 \text{ -----2}$$

Ellipse is $x^2 + 3y^2 = 12$

$$\frac{x^2}{12} + \frac{y^2}{4} = 1 \text{ -----1}$$

$$a^2 = 12, b^2 = 4 \text{ -----1}$$

$$\text{Condition } c^2 = a^2m^2 + b^2 \text{ -----2}$$

Given line touches the ellipse -----1

$$\text{Point of contact} = \left(\frac{-a^2m}{c}, \frac{b^2}{c} \right) = (-3, 1) \text{ -----2}$$

65. The other asymptote is $2x - y + k = 0$ -----2

Combined equation of the asymptotes $(x + 2y - 5)(2x - y + k) = 0$ -----2

Equation of R.H $(x + 2y - 5)(2x - y + k) + c = 0$ -----2

$$k=4, \quad c=-16 \text{ -----2}$$

the equation of R.H is

$$(x + 2y - 5)(2x - y + 4) - 16 = 0 \text{ -----2}$$

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66. $\frac{dx}{d\theta} = -4a\cos^3\theta \sin\theta$ -----2

$\frac{dy}{d\theta} = 4a\sin^3\theta \cos\theta$ -----2

Slope $= \frac{dy}{dx} = -\frac{\sin^2\theta}{\cos^2\theta}$ -----2

Equation of the tangents $x\sin^2\theta + y\cos^2\theta = a\sin^2\theta\cos^2\theta$ (or) Intercept form-----2

Sum of the intercepts= a , -----2

67. Rough Diagram -----2

Volume of the cone, $V = \frac{1}{3}\pi x^2(a+y)$ -----2

$V = \frac{1}{3}\pi(a^2 - y^2)(a+y)$ -----2

$V^1 = 0, y = \frac{a}{3}$ -----1

When $y = \frac{a}{3}, V^{11} < 0$ -----1

$V = \frac{8}{27}$ (volume of the sphere) -----2

68(i). Domain, extent, intercept and origin -----1

Domain is $(-\infty, \infty)$, *horizontal Extent* $(-\infty, \infty)$, and *the vertical extent* $(-\infty, \infty)$, It does not pass through origin. -----1

(ii) symmetry: does not possess any symmetrical -----1

(iii) Asymptotes: The curve does not admit any asymptotes -----1

(iv) Monotonicity: the curve is increasing in $(-\infty, \infty)$ -----1

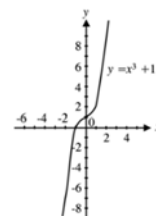
(v) $(0, 1)$ is a point of inflection (or) writing the concavity -----2

Rough Diagram -----3

69. $\frac{\partial w}{\partial u} = 2ue^v$ -----1 $\frac{\partial w}{\partial v} = u^2e^v$ -----1

$\frac{\partial u}{\partial x} = \frac{1}{y}$ -----1 $\frac{\partial u}{\partial y} = \frac{-x}{y^2}$ -----1

$\frac{\partial v}{\partial x} = \frac{y}{x}$ -----1 $\frac{\partial v}{\partial y} = \log x$ -----1



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$$\frac{\partial w}{\partial x} = \frac{x}{y} x^y \left(\frac{2}{y} + 1 \right) \text{-----}2$$

$$\frac{\partial w}{\partial y} = \frac{x^2}{y^3} x^y (y \log x - 2) \text{-----}2$$

70.(a)

$$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 5 \\ 0 & 1 & -3 \end{vmatrix} \text{-----}1$$

$$= -5\vec{i} - 6\vec{j} - 2\vec{k} \text{-----}1$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ -5 & -6 & -2 \end{vmatrix} \text{-----}1$$

$$= -12\vec{i} + 9\vec{j} + 3\vec{k} \text{-----}2$$

$$(\vec{a} \cdot \vec{c}) = 6 \text{-----}1$$

$$(\vec{a} \cdot \vec{b}) = -9 \text{-----}1$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = -12\vec{i} + 9\vec{j} + 3\vec{k} \text{-----}2$$

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}} \text{-----}1$$

$$(b) y^1 = -2xe^{x^2} \text{-----}1$$

$$y^{11} = -2e^{x^2}(2x^2 - 1) \text{-----}2$$

$$y^{11} = 0, x = \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \text{-----}2$$

For $(-\infty, -\frac{1}{\sqrt{2}}]$ and $[\frac{1}{\sqrt{2}}, \infty)$ the curve is concave upward -----2

For $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ the curve is concave downward. -----1

The point of inflections are $(\frac{-1}{\sqrt{2}}, e^{\frac{-1}{2}})$ and $(\frac{1}{\sqrt{2}}, e^{\frac{-1}{2}})$ -----2