SHORT CUT METHODS @ ALTERNATIVE METHODS IN +2 MATHS

Note: This methods are only to make interest and also based on entrance exams.

1. Find the square root of $-8-6i$.

Solution: $-8-6i = 1-9-6i$

$$= 1+(3i)^2-6i$$

$$= (1-3i)^2$$

$$\Rightarrow \sqrt{-8-6i} = \pm(1-3i)$$

$$= 1-3i \ (or) \ -1+3i.$$

2. *Find the square root of* $-7+24i$

solution: $-7+24i = 9-16+24i$

$$= 3^2+24i+(4i)^2$$

$$= (3+4i)^2$$

$$\Rightarrow \sqrt{-7+24i} = \pm(3+4i)$$

$$= 3+4i \ (or) -3-4i.$$

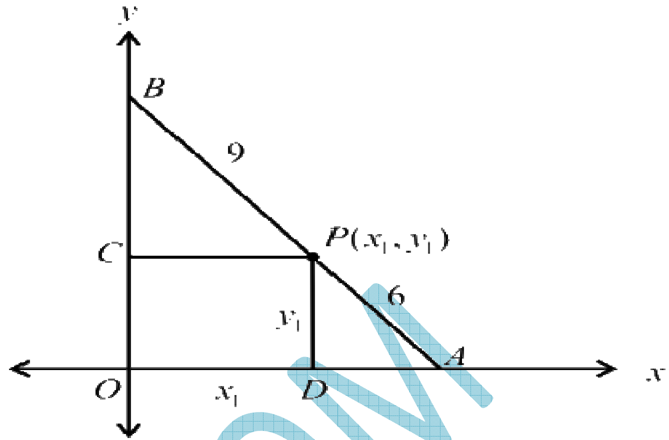
3. If $Z^2=(0,1)$ find Z

Solution: $Z^2=i$, $Z=\sqrt{i}$, $Z=\sqrt{\frac{2i}{2}}=\sqrt{\frac{(1+i)^2}{2}}=\pm\frac{(1+i)}{\sqrt{2}}$

4. A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.

solution:

By the similar triangle property



$$\text{from right angled triangle } ADP, \qquad \sin\theta = \frac{PD}{PA} = \frac{y_1}{6}$$

$$\text{from right angled triangle } PCB, \qquad \cos\theta = \frac{PC}{PB} = \frac{OD}{PB} = \frac{x_1}{9}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \left(\frac{x_1}{9}\right)^2 + \left(\frac{y_1}{6}\right)^2 = 1$$

$$\Rightarrow \frac{x_1^2}{81} + \frac{y_1^2}{36} = 1$$

$\therefore$ *the locus of* $P(x_1, y_1)$ is

$$\frac{x^2}{81} + \frac{y^2}{36} = 1,$$ Which is an ellipse.

5. Use differentials to find the an approximate value for the given number

Solution:

$y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$ $\quad y = f(x) = \sqrt[3]{x} + \sqrt[4]{x} = x^{\frac{1}{3}} + x^{\frac{1}{4}}$

$$\frac{dy}{dx} = f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}}$$

$$x = 1, dx = \Delta x = 0.02$$

$$dy = \left(\frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}}\right)dx$$

$$x = 1, dx = \Delta x = 0.02; f(1) = 2$$

$$dy = \left(\frac{1}{3} + \frac{1}{4}\right)(0.02) = \left(\frac{4+3}{2}\right)(0.02)$$

$$= \frac{0.07}{2} = 0.01167$$

$$\sqrt[3]{1.02} + \sqrt[4]{1.02} = 2 + 0.01167 = 2.011$$

6.A bag contains 3 types of coins namely Re.1, Rs. 2 and Rs. 5. There are 30 coins amounting to Rs. 100 in total. Find the number of coins in each category.

Solution :(Rank method)

By given,x+2y+z=30,x+2y+5z=100,we have two equations but three unknown

Let z=k

$$[A,B] = \begin{bmatrix} 1 & 1 & 1 & 30 \\ 1 & 2 & 5 & 100 \\ 0 & 0 & 1 & k \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 30 \\ 0 & 1 & 4 & 70 \\ 0 & 0 & 1 & k \end{bmatrix} \; R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 70 \\ k \end{bmatrix}$$

Z=k,y+4z=70 , y+4k=70,we get y=70−4k

x+y+z=30,x+70−4k+k=30,we get x=3k−40

(x,y,z)=(3k−40,70−4k,k), where k=14,15,16,17

Solution:(matrix inversion method)

By given,x+2y+z=30,x+2y+5z=30,we have two equations but three unknown

Let z=k

X+2y=30-k,x+2y=100-5k

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$$

The solution is x=$A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 30-k \\ 100-5k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3k-40 \\ 70-4k \end{bmatrix}$$

(x,y,z)=(3k-40,70-4k,k), where k=14,15,16,17

7.A small seminar hall can hold 100 chairs. Three different colours(red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chair is Rs. 25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased.

Solution :(Rank method)

By given,x+y+z=100,240x+260y+300z=25000 →12x+13y+15z=1250

we have two equations but three unknown , Let z=k

$$[A,B] = \begin{bmatrix} 1 & 1 & 1 & 100 \\ 12 & 13 & 15 & 1250 \\ 0 & 0 & 1 & k \end{bmatrix}$$

$$[A,B] = \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 1 & 3 & 50 \\ 0 & 0 & 1 & k \end{bmatrix} R_2 \rightarrow R_2 - 12R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ k \end{bmatrix}$$

Z=k,y+3z=50,we get y =50-3k

x+y+z =100,x+50-3k+k=100, we get x=50+2k

(x,y,z)=(50+2k,50-3k,k), where k=0.......16

Solution: (matrix inversion method)

By given, x+y+z=100, 240x+260y+300z=25000 →12x+13y+15z=1250

we have two equations but three unknown , Let z=k

x+y=100-k, 12x+13y=1250-15k

$$A^{-1} = \begin{bmatrix} 13 & -1 \\ -12 & 1 \end{bmatrix}$$

The solution is x=$A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 & -1 \\ -12 & 1 \end{bmatrix} \begin{bmatrix} 100-k \\ 1250-15k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50+2k \\ 50-3k \end{bmatrix}$$

(x,y,z)=(50+2k,50-3k,k), where k=0........16

8. Derive the equation of the plane in the intercept form.

Let the vertices be A (a,0,0),(0,b,0) and C(0,0,C)

Required Equation of the plane is Ax+By+Cz+D =0 ………(1)

It passes A(a,0,0), Aa+D=0 , $A = \frac{-D}{a}$

It passes B(0,b,0), Bb+D=0 , $B = \frac{-D}{b}$

It passes C(0,0,c), Cc+D=0 , $C = \frac{-D}{c}$

Putting the values of A,B and C in (1)

$-\frac{Dx}{a} - \frac{Dy}{b} - \frac{Dz}{c} + D = 0$

On dividing throughout by -D

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Which is the required equation of the plane in the intercept form.

9.Points A and B are 10km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

Given that 2a=6 , a=3 2ae=10, (6)e=10 , e=5/3

For hyperbola $b^2 = a^2(e^2 - 1)$, $b^2 = 9\left(\frac{25}{9} - 1\right)$, $b^2 = 16$

The required equation of hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

10.The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts a, b and the normal at the point makes intercepts p,q on the axes. Prove that $ap + bq = 0$

By given equation of the tangent is $\frac{x}{a} + \frac{y}{b} = 1$

Slope is $m_1 = -\frac{\frac{1}{a}}{\frac{1}{b}}$ $m_1 = -\frac{b}{a}$

By given equation of the normal is $\frac{x}{p} + \frac{y}{q} = 1$

Slope is $m_2 = -\frac{\frac{1}{p}}{\frac{1}{q}}$ $m_1 = -\frac{q}{p}$,

By Condition $m_1 \text{x} m_2 = -1$, we get $ap + bq = 0$

11.Find c , μ and σ^2 of the normal distribution whose probability function is given by

$f(x) = ce^{-x^2+3x}$ $-\infty < X < \infty.$

$$\mu = -\frac{co\ efficient\ of\ x}{co\ efficient\ of\ 2x^2} = \frac{3}{2}$$

$$\sigma^2 = -\frac{1}{co\ efficient\ of\ 2x^2} = \frac{1}{2}$$

$$f(x) = ce^{-x^2+3x} = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Putting x=0 then we get the value of C

$$C = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\mu^2}{2\sigma^2}}$$

$\sigma^2 = \frac{1}{2}$, $\sigma = \frac{1}{\sqrt{2}}$, $C = \frac{1}{\sqrt{\pi}} e^{\frac{-9}{4}}$

12. Obtain k , μ and σ^2 of the normal distribution whose probability distribution function is given by

$$f(x) = ke^{-2x^2+4x} \qquad -\infty < X < \infty.$$

$$\mu = -\frac{co\ efficient\ of\ x}{co\ efficient\ of\ 2x^2} = \frac{4}{4} = 1$$

$$\sigma^2 = -\frac{1}{co\ efficient\ of\ 2x^2} = \frac{1}{4}$$

$$f(x) = Ke^{-2x^2+4x} = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Putting x=0 then we get the value of K

$$K = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\mu^2}{2\sigma^2}}$$

$\sigma^2 = \frac{1}{4}$, $\sigma = \frac{1}{2}$, $K = \frac{\sqrt{2}}{\sqrt{\pi}} e^{-2}$