1.Solve: $\boldsymbol{dx + xdy = e^{-y}\, sec^2x}$

Solution: multiply by e^y

$e^y dx + xe^y dy = sec^2y$ dy

$d(xe^y) = sec^2y$dy

On integrating

Solution is $xe^y = tany$+c

2.Solve: $\boldsymbol{D^2y = -9sin3x}$

Solution: on integrating

$D = 9\frac{cos3x}{3}$+B

Again integrating

$y = 3\frac{sin3x}{3} + Bx$ +A

Solution is $y = sin3x + Bx$+A

3.solve: $\boldsymbol{\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0}$

Solution: dx+$e^{\frac{x}{y}}\, dy + e^{\frac{x}{y}}\, \left(dx - \frac{x}{y}\, dy\right) = 0$

$dx + e^{\frac{x}{y}} + ye^{\frac{x}{y}}\left(\frac{ydx - xdy}{y^2}\right)$ =0

$dx + d(ye^{\frac{x}{y}})$+c=0

On integrating,$x + ye^{\frac{x}{y}}) + c = 0$

4.solve: $\boldsymbol{(1 + x^2)\frac{dy}{dx} + 2xy = cosx}$

Solution:$(1 + x^2)$dy+2xydx=cosx dx

$dy + x^2dy + 2xy\, dx = cosx$ dx

$dy + d(x^2y) = cosx$ dx

On integrating, $y + x^2y = sinx + c$

5.solve: $\boldsymbol{(x^3+3xy^2)dx+(y^3+3x^2y)dy=0}$

Solution: $x^3dx+y^3dy+3(xy^2dx+x^2y\,dy)=0$

Multiply by 4, we get

$4x^3dx+4y^3dy+6(2xy^2dx+2x^2y\,dy)=0$

$4x^3dx+4y^3dy+6d(x^2y^2)=0$

On integrating, $x^4+y^4+6x^2y^2=c$

6.solve: $\boldsymbol{(1-x^3)\frac{dy}{dx}-3x^2y=sec^2x}$

Solution: $(1-x^3)dy-3x^2ydx=sec^2x\,dx$

$dy-x^3dy-3x^2ydx=sec^2x$

$dy-(x^3dy+3x^2ydx)=sec^2x$

$dy-d(yx^3)=d(tanx)$

Integrating, $y(1-x^3)=tanx+c$

7.solve: $\boldsymbol{(1+2x^3)\frac{dy}{dx}+6x^2y=cosec^2x}$

solution: $dy+2x^3dy+6x^2y\,dx=cosec^2x\,dx$

$dy+2d(x^3y)=d(cotx)$

Integrating, $y(1+2x^3)=cotx+c$

8.solve: $\boldsymbol{(x+y)^2\frac{dy}{dx}=1}$

Solution: Let $x+y=u$, diff w.r.t y

$\frac{dx}{dy}=\frac{du}{dy}-1,$

From given, $\frac{du}{dy}=1+u^2$, $dy=\frac{du}{1+u^2}$

Integrating, $y=tan^{-1}u+c$

$y=tan^{-1}(x+y)+c$