A

Register Number | | | | | |

+2 MODEL EXAMINATION

PART III – MATHEMATICS

[English Version]

Time allowed: 3 Hours] **[Maximum Marks: 200**

SECTION – A

Note: (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. The number of rows in the truth table of $\sim[p \wedge (\sim q)]$ is www.mathstimes.com

   1) 2 2) 4 3) 6 4) 8

2. A monoid becomes a group if it also satisfies the

   1) closure axiom 2) associative axiom 3) identity axiom 4) inverse axiom

3. In the multiplicative group of nth roots of unity, the inverse of ω^k is $(k < n)$

   1) $\omega^{1/k}$ 2) ω^{-1} 3) ω^{n-k} 4) $\omega^{n/k}$

4. Equivalent matrices are obtained by

   1) taking inverses 2) taking transposes

   3) taking adjoints 4) taking finite number of elementary transformations

5. The spherical snow ball is melting in such a way that its volume is decreasing at a rate of 1 cm^3 / min. The rate at which the diameter is decreasing when the diameter is 10 cms is

   1) $\frac{-1}{50\pi} cm/\min$ 2) $\frac{1}{50\pi} cm/\min$ 3) $\frac{-11}{75\pi} cm/\min$ 4) $\frac{-2}{75\pi} cm/\min$

6. If the length of the diagonal of a square is increasing at the rate of 0.1 cm / sec. What is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}} cm$?

   1) 1.5 cm^2/sec 2) 3 cm^2/sec 3) $3\sqrt{2}$ cm^2/sec 4) 0.15 cm^2/sec

7. The value of 'c' of Lagranges Mean Value Theorem for f(x) = $\sqrt{x}$ when a = 1 and b = 4 is

   1) $\frac{9}{4}$ 2) $\frac{3}{2}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

8. The projection of $\vec{i} - \vec{j}$ on the z – axis is

   1) 0 2) 1 3) −1 4) 2

9. The fourth roots of unity are

   1) $1 \pm i,\ -1 \pm i$ 2) $\pm i,\ 1 \pm i$ 3) $\pm 1,\ \pm i$ 4) $1, -1$

10. The true statements of the following are

 i) Two tangents and 3 normal can be drawn to a parabola from a point

 ii) Two tangents and 4 normal can be drawn to an ellipse from a point

 iii) Two tangents and 4 normal can be drawn to an hyperbola from a point

 iv) Two tangents and 4 normal can be drawn to an R.H. from a point

 1) (i) , (ii), (iii) and (iv) 2) (i) , (ii) only 3) (iii) , (iv) only 4) (i) , (ii), and (iii)

11. A missile fired from ground level rises x metres vertically upwards in "t" seconds and x = (100 − 12.5t). Then the maximum height reached by the missiles is

 1) 100m 2) 150 m 3) 250 m 4) 200m

12. If A = $\begin{bmatrix} 2 & 0 & 1 \end{bmatrix}$, then rank of AA^T is

 1) 1 2) 2 3) 3 4) 0

13. If I is the unit matrix of order n, where k ≠ 0 is a constant, then adj(kI) =

 1) k^n (adj I) 2) k (adj I) 3) k^2 (adj (I)) 4) k^{n-1} (adj I)

14. The system of equations $ax+y+z=0; x+by+z=0; x+y+cz=0$ has a non-trivial solution then

 $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$ www.mathstimes.com

 1) 1 2) 2 3) −1 4) 0

15. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is

 1) $\frac{2}{\sqrt{3}}$ 2) $\frac{1}{\sqrt{6}}$ 3) $\frac{2}{3}$ 4) $\frac{1}{2\sqrt{6}}$

16. The point of intersection of the line $\vec{r}=(\vec{i}-\vec{k})+t(3\vec{i}+2\vec{j}+7\vec{k})$ and the plane $\vec{r}.(\vec{i}+\vec{j}-\vec{k})=8$ is

 1) (8, 6, 22) 2) (−8, −6, −22) 3) (4, 3, 11) 4) (−4, −3, −11)

17. The shortest distance of the point (2, 10, 1) from the plane $\vec{r}.(3\vec{i}-\vec{j}+4\vec{k})=2\sqrt{26}$

 1) $2\sqrt{26}$ 2) $\sqrt{26}$ 3) 2 4) $\frac{1}{\sqrt{26}}$

18. If $\vec{p},\vec{q}$ and $\vec{p}+\vec{q}$ are vectors of magnitude λ then the magnitude of $|\vec{p}-\vec{q}|$ is

 1) 2λ 2) $\sqrt{3}\lambda$ 3) $\sqrt{2}\lambda$ 4) 1

19. The centre and radius of the sphere $\left|\overrightarrow{r}-\left(2\vec{i}-\vec{j}+4\vec{k}\right)\right|=5$

 a) (2 , −1, 4) and 5 2) (2, 1, 4) and 5 3) (−2, 1, 4) and 6 4) (2, 1, −4) and 5

20. The area bounded by the curve x = f (y) , y-axis and the lines y = c and y = d is rotated about y-axis. Then the volume of the solid is

 1) $\pi\int_c^d x^2dy$ 2) $\pi\int_c^d x^2dx$ 3) $\pi\int_c^d y^2dx$ 4) $\pi\int_c^d y^2dy$

21. One of the foci of the rectangular hyperbola xy = 18 is

 1) (6, 6) 2) (3, 3) 3) (4, 4) 4) (5, 5)

22. The line $5x - 2y + 4k = 0$ is a tangent to $4x^2 - y^2 = 36$ then k is

1) $\frac{4}{9}$ 2) $\frac{2}{3}$ 3) $\frac{9}{4}$ 4) $\frac{81}{16}$

23. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is

1) 8 2) 6 3) 4 4) 2

24. If $(m - 5) + i(n + 4)$ is the complex conjugate of $(2m + 3) + i(3n - 2)$ then (n, m) are

1) $\left(-\frac{1}{2}, -8\right)$ 2) $\left(-\frac{1}{2}, 8\right)$ 3) $\left(\frac{1}{2}, -8\right)$ 4) $\left(\frac{1}{2}, 8\right)$

25. The polar form of the complex number $(i^{25})^3$ is

1) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ 2) $\cos\pi + i\sin\pi$ 3) $\cos\pi - i\sin\pi$ 4) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

26. If ω is the nth root of unity then

1) $1 + \omega^2 + \omega^4 + \ldots = \omega + \omega^3 + \omega^5 + \ldots$ 2) $\omega^n = 0$

3) $\omega^n = 1$ 4) $\omega = \omega^n - 1$

27. The order and degree of the differential equation are $\frac{dy}{dx} + y = x^2$

1) 1, 1 2) 1, 2 3) 2, 1 4) 0, 1

28. The value of $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is

1) π/2 2) 0 3) π/4 4) π

29. The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x-axis is

1) 18π 2) 2π 3) 36π 4) 9π

30. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

1) 20π 2) 40π 3) 10π 4) 30π

31. '+' is not a binary operation on www.mathstimes.com

1) N 2) Z 3) C 4) Q – {0}

32. In a Poisson distribution if P(X = 2) = P(X = 3) then the value of its parameter λ is

1) 6 2) 2 3) 3 4) 0

33. If 2 cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours without replacement, is

1) $\frac{1}{2}$ 2) $\frac{26}{51}$ 3) $\frac{25}{51}$ 4) $\frac{25}{102}$

34. X is a discrete random variable which takes the values 0, 1, 2 and $P(X = 0) = \frac{144}{169}$, $P(X = 1) = \frac{1}{169}$ then the value of P(X = 2) is

1) $\frac{145}{169}$ 2) $\frac{24}{169}$ 3) $\frac{2}{169}$ 4) $\frac{143}{169}$

35. A continuous random variable takes

1) only a finite number of values
2) all possible values between certain given limits
3) infinite number of values
4) a finite or countable number of values

36. If $\frac{dy}{dx} = \frac{x-y}{x+y}$ then

1) $2xy + y^2 + x^2 = c$ 2) $x^2 + y^2 - x + y = c$ 3) $x^2 + y^2 - 2xy = c$ 4) $x^2 - y^2 - 2xy = c$

37. If $y = ke^{\lambda x}$ then its differential equation is

1) $\frac{dy}{dx} = \lambda y$ 2) $\frac{dy}{dx} = ky$ 3) $\frac{dy}{dx} + ky = 0$ 4) $\frac{dy}{dx} = e^{\lambda x}$

38. The integrating factor of the differential equation $\frac{dy}{dx} + py = Q$ is

1) $\int pdx$ 2) $\int Qdx$ 3) $e^{\int Qdx}$ 4) $e^{\int pdx}$

39. The curve $y^2 (x - 2) = x^2 (1 + x)$ has

1) an asymptote parallel to x-axis
2) an asymptote parallel to y-axis
3) asymptotes parallel to both axes
4) no asymptotes

40. If $u = f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

1) 0 2) 1 3) 2u 4) u

SECTION - B

Note: (i) Answer any *ten* questions.
(ii) Question No.55 is compulsory and choose any nine questions from the remaining.
(iii) Each question carries six marks. **10 x 6 = 60**

41. If A = $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

42. Solve the following non-homogeneous system of linear equations by determinant method:

$4x + 5y = 9$

$8x + 10y = 18$

43. i) Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$, $-3\vec{i} + 4\vec{j} + 4\vec{k}$ form the sides of a right angled triangle.

ii) A force given by $3\vec{i} + 2\vec{j} - 4\vec{k}$ is applied at the point (1, −1, 2). Find the moment of the force about the point (2, −1, 3).

44. Find the Vector and Cartesian equation of the sphere on the join of the points A and B having position vectors $2\vec{i} + 6\vec{j} - 7\vec{k}$ and $-2\vec{i} + 4\vec{j} - 3\vec{k}$ respectively as a diameter. Find also the centre and radius of the sphere.

45. If n is a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$.

46. If $(a_1 + ib_1)(a_2 + ib_2).....(a_n + ib_n) = A + iB,$ prove that

i) $(a_1^2 + b_1^2)(a_2^2 + b_2^2)....(a_n^2 + b_n^2) = A^2 + B^2$

ii) $\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + ... + \tan^{-1}\left(\frac{b_n}{a_n}\right) = k\pi + \tan^{-1}\left(\frac{B}{A}\right), k \in Z$

47. Show that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact.

48. Obtain the Maclaurin's Series for $\frac{1}{1+x}$.

49. Evaluate: $\int_0^3 \frac{\sqrt{x}dx}{\sqrt{x} + \sqrt{3-x}}$

50. Solve: $\frac{dy}{dx} + y\cot x = 2\cos x$

51. Construct the truth table for $(p \vee q) \wedge r$.

52. State and prove cancellation laws.

53. i) Verify that the function $f(x) = \begin{cases} \frac{2x}{9}, & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$ is probability density function.

ii) The mean of a binomial distribution is 6 and its standard deviation is 3. Is this statement true or false? Comment.

54. Find K, μ and σ of the normal distribution whose probability function is given by $f(x) = Ke^{-2x^2+4x-2}$

55. a) Find the equations of the tangent and normal to the curve $y = x^2 - x - 2$ at the point $(1, -2)$

(OR)

b) The time of swing T of a pendulum is given by $T = k\sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of the pendulum l changes from 32.1 cm to32.0 cm.

SECTION - C

Note : (i) Answer any *ten* questions.
(ii) Question No.70 is compulsory and choose any nine questions from the remaining.
(iii) Each question carries ten marks. 10 x 10 = 100

56. Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and solve them.

57. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and hence find the point of intersection.

58. Find the Vector and Cartesian equation to the plane through the point $(-1, 3, 2)$ and perpendicular to the planes $x+2y+2z = 5$ and $3x+y+2z = 8$.

59. P represents the variable complex number Z, find the locus of P, if Re $\left(\frac{z+1}{z+i}\right) = 1$

60. Find the axis, vertex, focus, equation of directrix, latus rectum, length of the latus rectum for the parabola and draw the graph $x^2 - 6x - 12y - 3 = 0$.

61. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun? (ii) the greatest possible distance between mercury and sun.

62. Evaluate: $\lim_{x\to 0+} x^{\sin x}$

63. Find the intervals of concavity and the points of inflection of the functions : $f(\theta) = \sin 2\theta$ in $(0, \pi)$

64. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the function $u = \sin 3x \cos 4y$

65. Using integration find the area bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$

66. Solve $(D^2 - 6D + 9)y = x + e^{2x}$

67. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

68. Show that the set $G = \{ a + b\sqrt{2} \ / \ a, b \in Q\}$ is an infinite Abelian group with respect to addition.

69. If the number of incoming buses per minute at a bus terminus is a random variable having a Poisson distribution with $\lambda=0.9$, find the probability that there will be

   (i) Exactly 9 incoming buses during a period of 5 minutes

   (ii) Fewer than 10 incoming buses during a period of 8 minutes.

   (iii) Atleast 14 incoming buses during a period of 11 minutes.

70. a) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Find the co-ordinates of the point of contact.

(OR)

   b) Find the length of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$

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