FIVE MARKS QUESETIONS

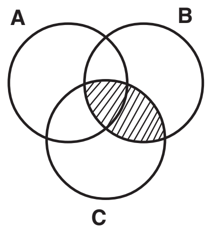
1. SETS AND FUNCTIONS

1. Use Venn diagrams to verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution :

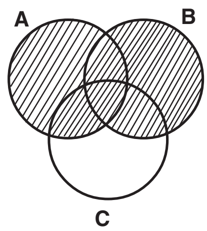
LHS:

$B \cap C$

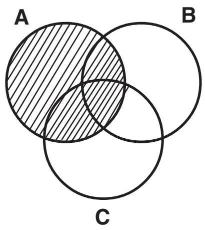


RHS

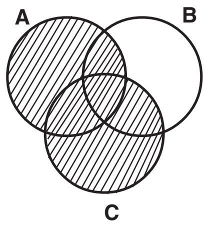
$A \cup B$



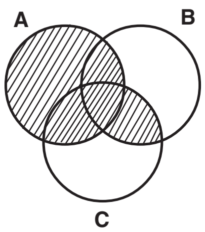
A



$A \cup C$

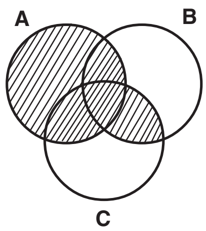


$A \cup (B \cap C)$



------ (I)

$(A \cup B) \cap (A \cup C)$



--------- (II)

I = II

LHS = RHS

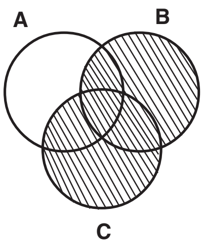
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

2. Use Venn diagrams to verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution:

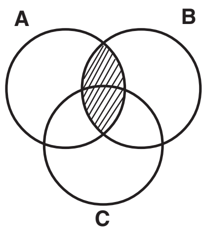
LHS

$B \cup C$

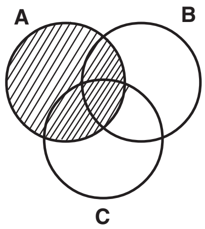


RHS

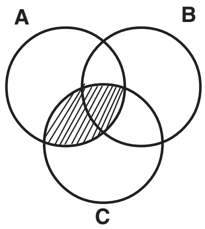
$A \cap B$



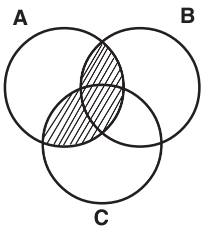
A



$A \cap C$

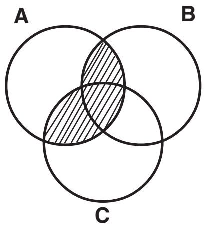


$A \cap (B \cup C)$



------ (I)

$(A \cap B) \cup (A \cap C)$



-------- (II)

I = II

LHS = RHS

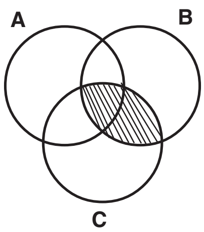
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

3. Use Venn diagrams to verify $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

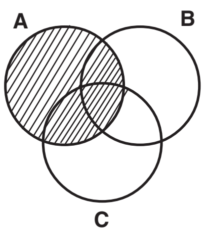
Solution:

LHS

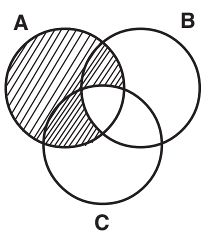
$B \cap C$



A



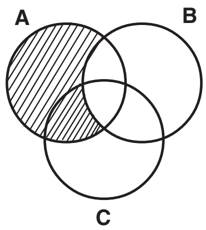
$A \setminus (B \cap C)$



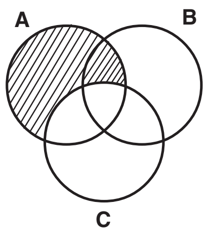
------ (I)

RHS

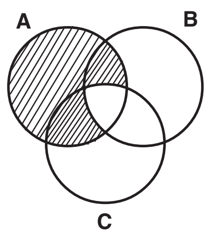
$A \setminus B$



$A \setminus C$



$(A \setminus B) \cup (A \setminus C)$



-------- (II)

I = II

LHS = RHS

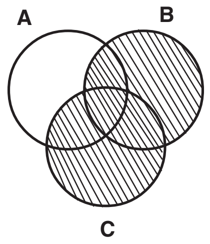
$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

4. Use Venn diagrams to verify $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$

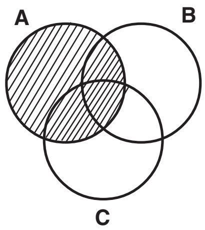
Solution:

LHS

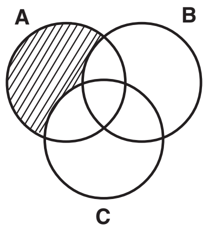
$(B \cup C)$



A



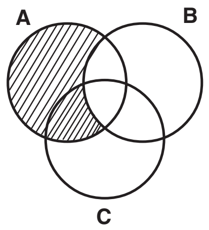
$A \backslash (B \cup C)$



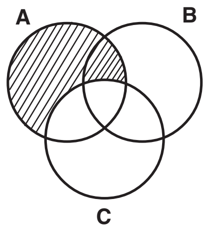
---- (I)

RHS

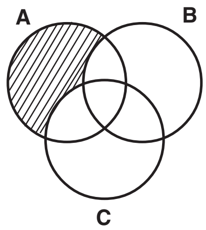
$(A \backslash B)$



$(A \backslash C)$



$(A \backslash B) \cap (A \backslash C)$



------ (II)

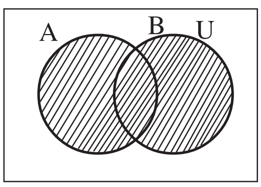
$$I = II$$
$$LHS = RHS$$
$$A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$$

5. Use Venn diagrams to verify De Morgan's law for complementation $(A \cup B)' = A' \cap B'$ ★

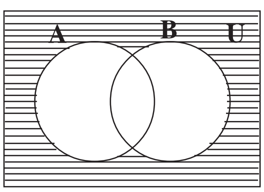
Solution :

LHS

$A \cup B$



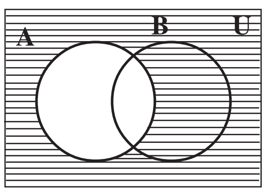
$(A \cup B)' = U - A \cup B$



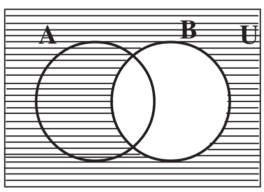
--- (I)

RHS

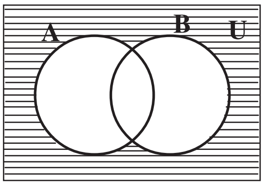
$A' = U - A$



$B' = U - B$



$A' \cap B'$



-------- (II)

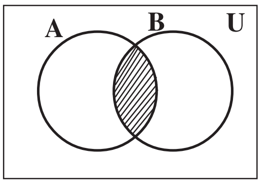
$$I = II$$
$$LHS = RHS$$
$$(A \cup B)' = A' \cap B'$$

6. Use Venn diagrams to verify $(A \cap B)' = A' \cup B'$

Solution:

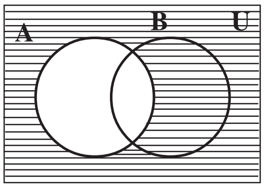
LHS

$A \cap B$

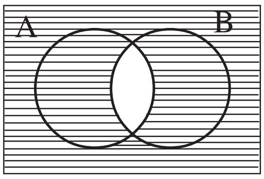


RHS

$A' = U - A$

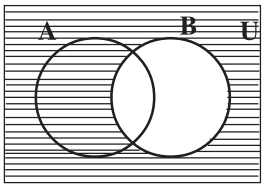


$(A \cap B)' = U - A \cap B$

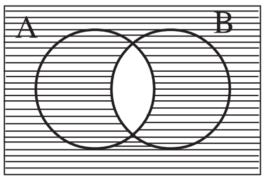


---- (I)

$B' = U - B$



$A' \cup B'$



-------- (II)

$$I = II$$
$$LHS = RHS$$
$$(A \cap B)' = A' \cup B'$$

7. U = {-2, -1, 0, 1, 2, 3, ... 10}, A = {-2, 2, 3, 4, 5} B = {1, 3, 5, 8, 9} P.T.

i) $(A \cup B)' = A' \cap B'$ ii) $(A \cap B)' = A' \cup B'$

i) **L.H.S.** = $(A \cup B)'$

$A \cup B$ = {-2, 2, 3, 4, 5} $\cup$ {1, 3, 5, 8, 9}

= {-2, 1, 2, 3, 4, 5, 8, 9}

$(A \cup B)' = U \setminus (A \cup B)$

= {-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} \ {-2, 1, 2, 3, 5, 8, 9}

= {-1, 0, 6, 7, 10} ------- (I)

R.H.S. $= A' \cap B'$

$A' = U \backslash A$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \backslash \{-2,2,3, 4, 5\}$

$= \{-1, 0, 1, 6, 7, 8, 9, 10\}$

$B' = U \backslash B$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \backslash \{1, 3, 5, 8, 9\}$

$= \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$A' \cap B' = \{-1, 0, 1, 6, 7, 8, 9, 10\} \cap \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$= \{-1, 0, 6, 7, 10\}$ ----- (II)

I = II

$(A \cup B)'\ A' \cap B'$

ii) $(A \cup B)' = A' \cup B'$

L.H.S. $= (A \cap B)'$

$(A \cap B) = \{-2, 2, 3, 4, 5\} \cap \{1, 3, 5, 8, 9\}$

$= \{3, 5\}$

$(A \cap B)' = U \backslash (A \cap B)$

$= \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \backslash \{3, 5\}$

$= \{-2, -1, 0, 1, 2, 4, 6, 7, 8, 9, 10\}$ ----- (I)

R.H.S. $= A' \cup B'$

$A' = U \backslash A$

$A' = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \backslash \{-2, 2, 3, 4, 5\}$

$= \{-1, 0, 1, 6, 7, 8, 9, 10\}$

$B' = U \backslash B$

$B' = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \backslash \{1, 3, 5, 8, 9\}$

$= \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$A' \cup B' = \{-1, 0, 1, 6, 7, 8, 9, 10\} \cup \{-2, -1, 0, 2, 4, 6, 7, 10\}$

$= \{-2, -1, 0, 1, 2, 4, 6, 7, 8, 9, 10\}$ ----- (II)

I = II

$(A \cap B)' = A' \cup B'$

8. Let $A = \{a, b, c, d, e, f, g, x, y, z\}$, $B = \{1, 2, c, d, e\}$ and $C = \{d, e, f, g, z, y\}$ P.T. $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$

Solution :

$B \cup C = \{1, 2, c, d, e\} \cup \{d, e, f, g, z, y\}$

$= \{1, 2, c, d, e, f, g, y\}$

$A \backslash B \cup C = \{a, b, c, d, e, f, g, x, y, z\} \backslash \{1, 2, c, d, e, f, g, y\}$

$= \{a, b, x, z\}$ ---- (I)

$A \backslash B = \{a, b, c, d, e, f, g, x, y, z\} \backslash \{1, 2, c, d, e\}$

$= \{a, b, f, g, x, y, z\}$

$A \backslash C = \{a, b, c, d, e, f, g, x, y, z\} \backslash \{d, e, f, g, z, y\}$

$= \{a, b, c, x, z\}$

$(A \backslash B) \cap (A \backslash C) = \{a, b, f, g, x, y, z\} \cap \{a, b, c, x, z\}$

$= \{a, b, x, z\}$ ---- (I)

I = II

$A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$

9. In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also 32% speak English and Tamil, 13% speak Tamil and Hindi 10% speak English and Hindi. Find the percentage of people who can speak all the three languages.

Tamil - T

English - E

Hindi - H

No. of people who can speak Tamil n(T) = 85%

No. of people who can speak English n(E) = 40%

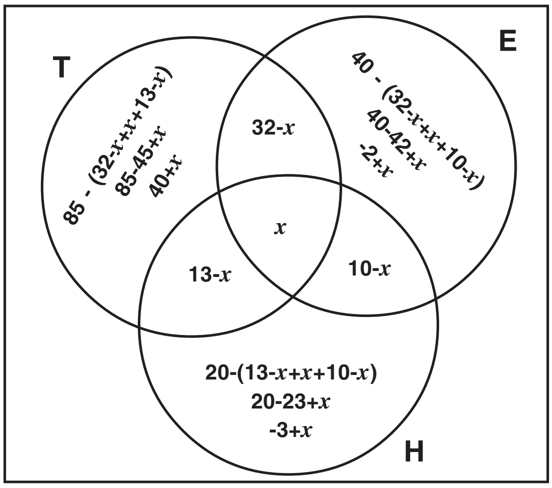
No. of people who can speak Hindi n(H) = 20%

$n(T \cap E) = 32\%$

$n(T \cap H) = 13\%$

$n(E \cap H) = 10\%$

$n(T \cap E \cap H) = x$



$40 + x + 32 - x + 13 - x + x - 2 + x - 3 + x + 10 - x = 100$

$95 - 5 + x = 100$

$90 + x = 100$

$x = 100 - 90$

$x = 10\%$

No. of people who can speak all the three languages = 10%

10. An advertising agency finds that, of its 170 clients, 115 use Television, 110 use Radio and 130 use Magazines. Also, 85 use Television and Magazines, 75 use Television and Radio, 95 use Radio and Magazines, 70 use all the three. Draw Venn diagram to represent these data. Find
(i) how many use only Radio? (ii) how many use only Television?
(iii) how many use Television and magazine but not radio?

Television = T

Radio = R

Magazine = M

$n(T) = 115$

$n(R) = 110$

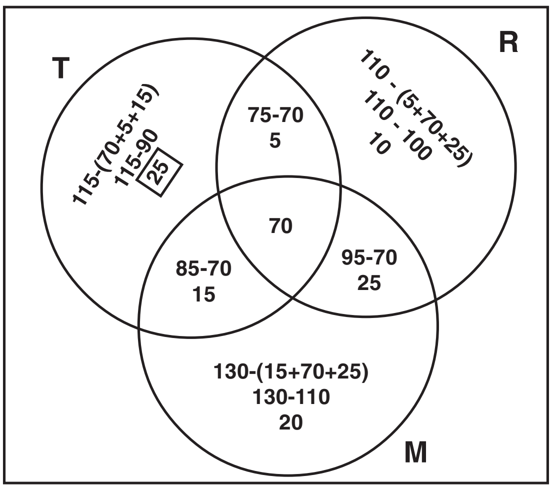
$n(M) = 130$

$n(T \cap M) = 85$

$n(T \cap R) = 75$

$n(R \cap M) = 95$

$n(T \cap R \cap M) = 70$



i) No. of clients using only Radio = 10

ii) No.of clients using only T.V. = 25

iii) No. of clients using T.V. and magazines but not radio = 15

11. A function f : [-3, 7) $\rightarrow$ R is defined by as follows

$$f(x) = \begin{cases} 4x^2 - 1 & : \quad -3 \le x < 2 \\ 3x - 2 & : \quad 2 \le x \le 4 \\ 2x - 3 & : \quad 4 < x < 7 \end{cases}$$

find (i) f(5) + f(6)

ii) f(1) - f(-3) (iii) f(-2) - f(4) (iv) $\frac{f(3)+f(-1)}{2f(6)-f(1)}$

Solution:

$$f(x) = \begin{cases} 4x^2 - 1 & : \quad -3 \le x < 2 \quad (-3,-2,-1,0,1) \\ 3x - 2 & : \quad 2 \le x \le 4 \quad (2,3,4) \\ 2x - 3 & : \quad 4 < x < 7 \quad (5,6) \end{cases}$$

i) f(5) + f(6) =?

f(x) = 2x - 3

f(5) = 2 x 5 - 3

= 10 - 3

f(5) = 7

f(6) = 2 $\times$ 6 - 3

= 12 - 3

f(6) = 9

f(5) + f(6) = 7 + 9

f(5) + f(6) = 16

ii) f(1) - f(-3) = ?

f(x) = $4x^2$ - 1

f(1) = 4 x 1^2 - 1

= 4 - 1

f(1) = 3

f(-3) = 4 x $(-3)^2$ - 1

= 4 x 9 - 1

= 36 - 1

f(-3) = 35

f(1) - f(-3) = 3 - 35

f(1) - f(-3) = -32

iii) f(-2) - f(4)

f(x) = $4x^2$ - 1

f(-2) = 4x $(-2)^2$ - 1

= 4 x 4 - 1

= 16 - 1

f(-2) = 15

f(x) = 3x - 2

f(4) = 3 x 4 - 2

= 12 - 2

f(4) = 10

f(-2) - f(4) = 15 - 10

f(-2) - f(4) = 5

iv) $\frac{f(3)+f(-1)}{2f(6)-f(1)}$ = ?

f(3) + f(-1)

f(x) = 3x - 2

f(3) = 3 x 3 - 2

= 9 - 2

f(3) = 7

f(x) = 4x $(-1)^2$ - 1

= 4 - 1

f(-1) = 3

f(3) + f(-1) = 7 + 3

f(3) + f(-1) = 10

2f(6) - f(-1) = 10

2f(6) - f(1) = ?

2(x) = 2x - 3

f(6) = 2 x 6 - 3

= 12 - 3

f(6) = 9

2f(6) = 18

f(1) = 4 x 1^2 - 1

= 4 - 1

f(1) = 3

2f(6) - f(1) = 18 - 3

= 15

$\frac{f(3)+f(-1)}{2f(6)-f(1)} = \frac{10}{15}$

Ans: $\frac{2}{3}$

12. Let A= { 0, 1, 2, 3 } and B = { 1, 3, 5, 7, 9 } be two sets. Let f : A $\rightarrow$ B be a function given by f(x)=2x+1. Represent this function as (i) a set of ordered pairs (ii) a table (iii) an arrow diagram and (iv) a graph.

Solution

f(x) = 2x + 1

f (0) = 2 x 0 + 1 = 0+1 = 1

f (1) = 2 x 1 +1 = 2 + 1 = 3

f (2) = 2 x 2 + 1 = 4 + 1 = 5

f (3) = 2 x 3 + 1 = 6 + 1 = 7

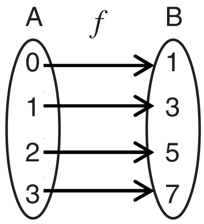
(i) Set of ordered pairs

{ (0, 1), (1, 3), (2, 5), (3, 7) }

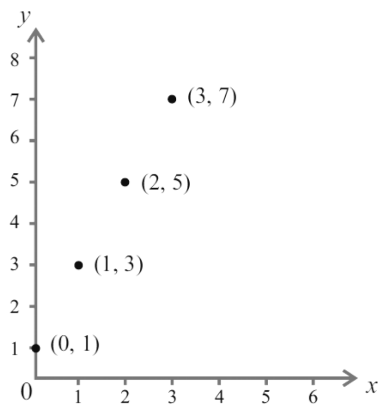
(ii) Table form

x	0	1	2	3
f (x)	1	3	5	7

(iii) Arrow Diagram



(iv) Graph



13. A function f : [1, 6) → R is defined as follows

$$f(x) = \begin{cases} 1+x & 1\le x<2 \\ 2x-1 & 2\le x<4 \\ 3x^2-10 & 4\le x<6 \end{cases}$$

Find the value of (i) f(5) (ii) f (3) (iii) f (1) (iv) f(2)- f(4) (v) 2 f(5) - 3f (1).

Solution:

$$f(x) = \begin{cases} 1+x & 1\le x<2 & (1) \\ 2x-1 & 2\le x<4 & (2, 3) \\ 3x^2-10 & 4\le x<6 & (4, 5) \end{cases}$$

i) f(x) $= 3x^2 - 10$

f(5) $= 3 \times 5^2 - 10$

$= 3 \times 25 - 10$

$= 75 - 10$

f(5) $= 65$

ii) f(x) $= 2x - 1$

f(3) $= 2 \times 3 - 1$

$= 6 - 1$

f(3) $= 5$

iii) f(x) $= 1 + x$

f(1) $= 1 + 1$

f(1) $= 2$

iv) f(2) - f(4)

$f(x) = 2x - 1$

$f(2) = 2 \times 2 - 1$

$= 4 - 1$

$f(2) = 3$

$f(x) = 3x^2 - 10$

$f(4) = 3 \times 4^2 - 10$

$= 3 \times 16 - 10$

$= 48 - 10$

$f(4) = 38$

$f(2) - f(4) = 3 - 38$

$f(2) - f(4) = -35$

v) 2f(5) - 3f(1)

$2f(5) = 2 \times 65$

$= 130$

$3f(1) = 3 \times 2$

$= 6$

$2f(5) - 3f(1) = 130 - 6$

$2f(5) - 3f(1) = 124$

14. Let A = {4, 6, 8, 10} and B = {3, 4, 5, 6, 7}. If f : A $\rightarrow$ B is defined by $f(x) = \frac{1}{2}x + 1$ then represent f by (i) an arrow diagram (ii) a set of ordered pair (iii) a table (iv) a graph.

Solution:

$$f(x) = \frac{1}{2}x + 1$$

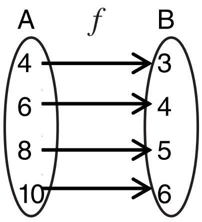
$$f(4) = \frac{1}{2} \times 4 + 1 = 2 + 1 = 3$$

$$f(6) = \frac{1}{2} \times 6 + 1 = 3 + 1 = 4$$

$$f(8) = \frac{1}{2} \times 8 + 1 = 4 + 1 = 5$$

$$f(10) = \frac{1}{2} \times 10 + 1 = 5 + 1 = 6$$

i) An arrow diagram



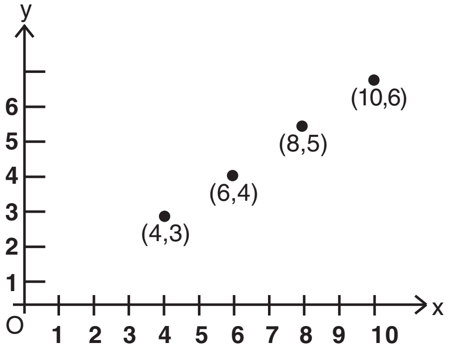
ii) Set of ordered pairs

f = {(4, 3) (6, 4) (8, 5) (10, 6)}

iii) Table

x	4	6	8	10
f(x)	3	4	5	6

iv) Graph



15. A function f : (-7, 6) $\rightarrow$ R is defined as following f(x) = $\begin{cases} x^2+2x+1 & -7 \le x < -5 \\ x+5 & -5 \le x \le 2 \\ x-1 & 2 < x < 6 \end{cases}$

find (i) 2f(-4) + 3f(2) ii) f(-7) -f(-3) iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$

$$f(x) = \begin{cases} x^2+2x+1 & -7 \le x < -5 & (-7,-6) \\ x+5 & -5 \le x \le 2 & (-5,-4,-3,-2,-1,0,1,2) \\ x-1 & 2 < x < 6 & (3,4,5) \end{cases}$$

i) 2f(-4) + 3f(2)

f(x) = x + 5

f(-4) = - 4 + 5 = 1

2xf(-4)= 1 x 2

2f(-4) = 2

f(2) = 2 + 5 = 7

3 x f(2) = 7 x 3

3f(2) = 21

2f(-4) + 3f(2) = 2 + 21

2f(-4) + 3f(2) = 23

ii) f(-7) - f(-3)

f(x) = x^2 + 2x + 1

f(-7) = $(-7)^2$ + 2x (- 7) + 1

= 49 - 14 + 1

= 50 - 14

f(-7) = 36

f(x) = x + 5

f(-3) = - 3 + 5

= 2

f(-3) = 2

f(-7) - f(-3) = 36 - 2

f(-7) - f (-3) = 34

iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$

f(x) = x + 5

f(-3) = - 3 + 5 = 2

4f(-3) = 2 x 4

4f(-3) = 8

f(x) = x - 1

f(4) = 4 - 1 = 3

2f(4) = 3 x 2

2f(4) = 6

4f(-3) + 2f(4) = 8 + 6

4f (-3) + 2f(4) = 14

f(x) = x^2 + 2x + 1

f(-6) = $(-6)^2$ + 2 x (- 6) + 1

= 36 - 12 + 1

= 37 - 12

f(-6) = 25

f(x) = x + 5

f(1) = 1 + 5 = 6

3f(1) = 6 x 3

3f(1) = 18

f(-6) - 3f(1) = 25 - 18

f(-6) - 3f(1) = 7

$$\frac{4f(-3)+2f(4)}{f(-6)-3f(1)} = \frac{14}{7}$$

Ans : 2

16. Let A = { 6, 9, 15, 18, 21 }; B = { 1, 2, 4, 5, 6 } and f : A $\rightarrow$ B be defined by f(x) = . Represent f by (i) an arrow diagram (ii) a set of ordered pairs (iii) a table (iv) a graph .

Solution:

$$f(x) = \frac{x-3}{3}$$

$$f(6) = \frac{6-3}{3} = \frac{3}{3} = 1$$

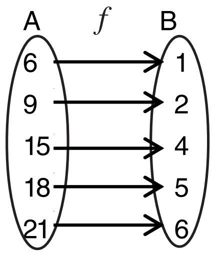
$$f(9) = \frac{9-3}{3} = \frac{6}{3} = 2$$

$$f(15) = \frac{15-3}{3} = \frac{12}{3} = 4$$

$$f(18) = \frac{18-3}{3} = \frac{15}{3} = 5$$

$$f(15) = \frac{21-3}{3} = \frac{18}{3} = 6$$

i) An arrow diagram



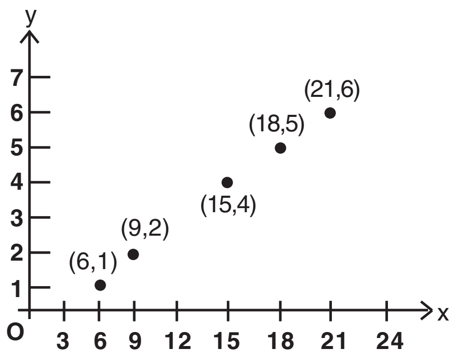
ii) a set of ordered pairs

f = {(6, 1), (9, 2), (15, 4), (18, 5), (21, 6)}

iii) a table

x	6	9	15	18	21
f(x)	1	2	4	5	6

iv) Graph



17. Let A = {5, 6, 7, 8}, B = {-11, - 4, 7, -1, -7, -9, -13} and f = {(x, y); y = 3-2x, x∈ A, y∈ B}

i) Write down the elements of f. ii) What is the co-domain iii) What is the range

iv) Identify the type of function.

Solution:

y = 3-2x

x = 5, y = 3-2 x 5 = 3 - 10 = -7

x = 6, y = 3 - 2 x 6 = 3 - 12 = -9

x = 7, y = 3 - 2 x 7 = 3 - 14 = -11

x = 8, y = 3 - 2 x 8 = 3 - 16 = -13

i) The elements of f

f = {(5, -7), (6, -9), (7, -11), (8, -13)}

ii) The co-domain = {-11, 4, 7, -10, -7, -9, -13}

iii) The range = {-7, -9, -11, -13}

iv) The type of function is

one - one function.

2. SEQUENCES AND SERIES OF REAL NUMBERS

1. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find the 27th term.

Solution : ★

Given that

$t_{10} = 41 \Rightarrow a + 9d = 41$ ---- (1)

$t_{18} = 73 \Rightarrow a + 17d = 73$ ---- (2)

(1) - (2) $\Rightarrow$ - 8d = 32

$$d = \frac{-32}{-8}$$

d = 4 sub in (1) we get

a + 9d = 41

a + 9 x 4 = 41

a + 36 = 41

a = 5

a = 41 - 36

a = 5

$t_{27} = a + 26d$

Sub a = 5 & d = 4, $t_{27} = 5 + 26\,(4)$

= 5 + 104

$t_{27} = 109$

2. If a, b, c are in A.P. then prove that $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

Solution:

If a, b, c are in A.P.

Divide each term by abc.

$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are also in A.P.

$\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

3. The 4th term of a geometric sequence of is $\frac{2}{3}$ and the seventh term is $\frac{16}{81}$. Find the geometric sequence.

$t_4 = \frac{2}{3} \Rightarrow ar^3 = \frac{2}{3}$ --- (1)

$t_7 = \frac{16}{81} \Rightarrow ar^6 = \frac{16}{81}$ --- (2)

$(2) \div (1) \Rightarrow \frac{ar^6}{ar^3} = \frac{16/81}{2/3}$

$r^{6-3} = \frac{16}{81} \times \frac{3}{2}$

$r^3 = \frac{8}{27}$

$r^3 = \left(\frac{2}{3}\right)^3$

$r = \frac{2}{3}$ sub in (1) we get

$ar^3 = \frac{2}{3}$

$a \times \left(\frac{2}{3}\right)^3 = 2/3$

$a = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$

$a = \frac{9}{4}$

G.P. is a, ar, ar^2 .....

$\frac{9}{4}, \left(\frac{9}{4}\right)\left(\frac{2}{3}\right), \left(\frac{9}{4}\right)\left(\frac{2}{3}\right)^2 .....$

4. In a geometric sequence , the first term is $\frac{1}{3}$ and the sixth term is $\frac{1}{729}$, find the G.P.

$a = \frac{1}{3}$

$t_6 = \frac{1}{729} \quad ar^5 = \frac{1}{729}$

$\left(\frac{1}{3}\right) r^5 = \frac{1}{729}$

$r^5 = \frac{1}{729} \times 3$

$= \frac{1}{243}$

$r^5 = \frac{1}{3^5}$

$r^5 = \left(\frac{1}{3}\right)^5$

$r = \frac{1}{3}$

G.P. is a, ar, ar^2 .....

$= \frac{1}{3}, \left(\frac{1}{3}\right) \frac{1}{3}, \frac{1}{3}\left(\frac{1}{3}\right)^2 ......$

$= \frac{1}{3}, \frac{1}{9}, \frac{1}{27} .....$

5. If the 4th and 7th terms of a G.P. are 54 and 1458 respectively, find the G.P. 54 and 1458 respectively, find the G.P.

$t_4 = 54 \Rightarrow ar^3 = 54$ ---- (1)
$t_7 = 1458 \Rightarrow ar^6 = 1458$ ---- (2)

$$(2) \div (1) \quad \frac{ar^6}{ar^3} = \frac{1458}{54}$$

$r^3 = 27$
$r^3 = (3)^3$
$r = 3$ sub in (1) we get
$ar^3 = 54$
$a(3)^3 = 54$

$$a = \frac{54}{3 \times 3 \times 3}$$

$a = 2$
G.P. is a, ar, ar^2
$= 2, (2)(3), (2)(3)^2 ....$
$= 2, 6, 18 .....$

6. Find the sum of all 3 digit natural numbers, which are divisible by 8.
Three digits natural numbers are 100, 101, ... 999.
Three digits natural numbers divisible by 8 are 104, 112, 120, ... 992.
$a = 104, d = 8, l = 992$
Step 1 :

$$n = \left(\frac{\ell - a}{d}\right) + 1$$
$$= \left(\frac{992 - 104}{8}\right) + 1$$
$$= \left(\frac{888}{8}\right) + 1$$
$= 111 + 1$
$n = 112$

```
     12                      125
8 | 100 + 4 = 104       8 | 999 -7 = 992
     8                       8
    ---                     ---
     20                      19
     16                      16
    ---                     ---
    4+4                      39
                             32
                            ---
                              7
```

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$
$$S_{112} = \frac{112}{2}[104 + 992]$$
$= 56 \times 1096$
$S_{112} = 61376$

7. Find the sume of all 3 digit natural numbers, which are divisible by 9.
Three digit natural numbers are 100, 101, ... 999.
Three digit natural numbers divisible by 9 are 108, 117, ... 999
$a = 108, d = 9, \ell = 999$
Step 1 :

$$n = \left(\frac{\ell - a}{d}\right) + 1$$
$$= \left(\frac{999 - 108}{9}\right) + 1$$
$$= \left(\frac{891}{9}\right) + 1$$

```
     11                      111
8 | 100 + 8             8 | 999 - 0
     9                       9
    ---                     ---
     10                      9
     9                       9
    ---                     ---
    1+8                      9
                             9
                            ---
                             0
```

$= 99 + 1$

$n = 100$

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{100} = \frac{100}{2}[108 + 999]$$

$= 56 \times 1107$

$S_{100} = 55350$

8. Find the sum of all natural numbers between 300 and 500 which are divisible by 11.
The natural numbers between 300 and 500, which are divisible by 11 are 308, 319, ... 495.
$a = 308$, $d = 11$, $l = 495$
Step 1 :

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{495 - 308}{11}\right) + 1$$

$$= \left(\frac{187}{11}\right) + 1$$

$= 17 + 1$

$n = 18$

```
      27                         45
11 | 300 + 8 = 308         11 | 499 - 4 = 495
     22                          44
     --                          --
      80                          59
      77                          55
     --                          --
      3+8                          4
```

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{18} = \frac{18}{2}[308 + 495]$$

$= 9 \times 803$

$S_{18} = 7227$

9. Find the sum of all numbers between 100 and 200 which are not divisible by 5.
Numbers which are divisible by 5 are 105, 110, ... 195, $a = 105$, $d = 5$, $\ell = 195$
Step 1 :

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{195 - 105}{5}\right) + 1$$

$$= \left(\frac{90}{5}\right) + 1$$

$= 18 + 1$

$n = 19$

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{19} = \frac{19}{2}[105 + 195]$$

$= 19 \times 150$

$S_{19} = 2850$

Step 2 :

The sum of natural nos are 101 + 102 + ... 199

$$\sum n = \frac{n(n+1)}{2}$$

$101 + 102 + ... + 199 = (1 + 2 + ... + 199) - (1 + 2 + ... + 100)$

$= \frac{199 \times 200}{2} - \frac{100 \times 101}{2}$

$= 19900 - 5050$

$= 14850$

Step 3:

Sum of numbers which are not divisible $= 14850 - 2850$

$= 12000$

10. Find the sum of first n terms of the series 6 + 66+ 666 +...

$S_n = 6 + 66 + 666 + ...$ to n times

$= 6\ (1 + 11 + 111 + ...$ to n times)

$= \frac{6}{9}(9 + 99 + 999 + ...$ to n times)

$= \frac{2}{3}[(10 - 1) + (100 - 1) + (1000 - 1) .....$ to n times]

$= \frac{2}{3}\mathbf{[}(10 + 100 + 1000 + ...$ to n times) - n$\mathbf{]}$ Here a = 10, r = 10>1

$= \frac{2}{3}\left[\frac{10(10^n-1)}{10-1} - n\right]$ $S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1}$

$S_n = \frac{2}{3}\left[\frac{10(10^n-1)}{9} - n\right]$

11. Find the sum of first n terms of the series 7 + 77 + 777 +...

$S_n = 7 + 77 + 777 + ...$ to n times

$= 7\ (1 + 11 + 111 + ...$ to n times)

$= \frac{7}{9}(9 + 99 + 999 + ...$ to n times)

$= \frac{7}{9}[(10 - 1) + (100 - 1) + (1000 - 1) .....$ to n times]

$= \frac{7}{9}\mathbf{[}(10 + 100 + 1000 + ...$ to n times) - n$\mathbf{]}$ Here a = 10, r = 10>1

$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$ $S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1}$

$S_n = \frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right]$

12. Find the sum of first n terms of the series 1 + 11 + 111 +... to 20 terms.

$S_n = 1 + 11 + 111 + ...$ to n times

$= \frac{1}{9}(9 + 99 + 999 + ...$ to n times)

$= \frac{1}{9}[(10 - 1) + (100 - 1) + (1000 - 1) .....$ to n times]

$= \frac{1}{9}\mathbf{[}(10 + 100 + 1000 + ...$ to n times) - n$\mathbf{]}$

$S_n = \frac{1}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$ a = 10, r = 10>1

If r > 1

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{20} = \frac{1}{9}\left[\frac{10(10^{20}-1)}{10-1} - 20\right]$

$= \frac{1}{9}\left[\frac{10(10^{20}-1)}{9} - 20\right]$

$S_{20} = \left[\frac{10}{81}(10^{20}-1) - \frac{20}{9}\right]$

13. Find the sum of the series $16^2 + 17^2 + 18^2 .... + 25^2$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$16^2 + 17^2 + 18^2 .... + 25^2 = (1^2 + 2^2 + ... + 25^2) - (1^2 + 2^2 .... + 15^2)$

$= \left(\frac{25\times26\times51}{6}\right) - \left(\frac{15\times16\times31}{6}\right)$

= (25 x 13 x 17) - (5 x 8 x 31)

= 5525 - 1240

= 4285

14. Find the sum of series $16^2 + 17^2 + ... 35^2$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$16^2 + 17^2 + ... + 35^2 = (1^2 + 2^2 + ... + 35^2) - (1^2 + 2^2 .... + 15^2)$

$= \left(\frac{35\times36\times71}{6}\right) - \left(\frac{15\times16\times31}{6}\right)$

= (35 x 6 x 17) - (5 x 8 x 31)

= 14910 - 1240

= 13670

15. Find the total area of 14 squares whose sides are 11 cm, 12 cm, ... 24 cm.

Area = $11^2 + 12^2 + 13^2 + ... + 24^2$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$11^2 + 12^2 + 13^2 .... + 24^2 = (1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 .... + 10^2)$

$= \left(\frac{24\times25\times49}{6}\right) - \left(\frac{10\times11\times21}{6}\right)$

= (4 x 25 x 49) - (5 x 11 x 7)

= 4900 - 385

= 4515

Total Area = 4515 cm^2

16. Find the total area of 12 squares whose sides are 12 cm, 13cm, ... 23cm. respectively. **(June 12)** ★

Solution :

Given that the side length of 12 squares are 12cm, 13 cm, 14 cm ...... 23 cm.

Total area of the 12 squares is

Area = $12^2 + 13^2 + 14^2 + ... + 23^2$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$12^2 + 13^2 + ... + 23^2 = (1^2 + 2^2 + ... + 23^2) - (1^2 + 2^2 + ... + 11^2)$

$= \frac{23\times24\times47}{6} - \frac{11\times 12\times 23}{6}$

= 23 x 4 x 47 - 22 x 23

= 4324 - 506

= 3818

Total Area = 3818 cm^2.

17. Find the total volume of 15 cubes whose edges are 16 cm, 17 cm, 18 cm, ..., 30 cm respectively.

Solution :

Given that the sides of the cubes are 16cm, 17cm, 18cm,.... 30 cm respectively.

Volume $= 16^3 + 17^3 + 18^3 + ... + 30^3$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$16^3 + 17^3 + 18^3 + ... + 30^3 = (1^3 + 2^3 + ... + 30^3) - (1^3 + 2^3 + ... + 15^3)$

$= \left(\frac{30\times31}{2}\right)^2 - \left(\frac{15\times16}{2}\right)^2$

$= (15 \times 31)^2 - (15 \times 8)^2$

$= (465)^2 - (120)^2$

= (465 + 120) (465 - 120)

= 585 x 345

Total Volume = 201825 cm^3

18. The sum of three consecutive terms in an A.P. is 6 and their product is -120. Find the three numbers.

Let three terms of an A.P. are a - d, a, a+d

Their sum = 6

a - d + a + a + d = 6

3a = 6

a = 6/3

a = 2

Their product = - 120

(a-d) (a) (a+d) = -120

$(a^2 - d^2)$ a = -120

sub a = 2

$(2^2 - d^2)2 = -120$

$4 - d^2 = \frac{-120}{2}$

$-d^2 = -60 - 4$

$d^2 = 64$

$d = \sqrt{8\times8}$

$d = \pm 8$

Three numbers are,

If a = 2; d = 8 $\Rightarrow$ 2- 8, 2, 2+8 = -6, 2 10 (or)

(or) If a = 2, d = -8 $\Rightarrow$ 2 - (-8), 2, 2 - 8 = 10, 2, -6

19. Find the sum of series 5 + 11 + 17 + ... + 95

$a = 5$, $d = 11-5 = 6$; $\ell = 95$

Step 1:

$$n = \left(\frac{\ell - a}{d}\right) + 1$$

$$= \left(\frac{95 - 5}{6}\right) + 1$$

$$= \left(\frac{90}{6}\right) + 1$$

$$= 15 + 1$$

$$n = 16$$

Step 2:

$$S_n = \frac{n}{2}[a + \ell]$$

$$S_{16} = \frac{16}{2}[5+95]$$

$$= 7 \times 100$$

$$S_{16} = 800$$

3. ALGEBRA

1. Using elimination method, solve $101x + 99y = 499$, $99x + 101y = 501$

$101x + 99y = 499$ --- (1)

$99x + 101y = 501$ --- (2)

(1)+(2) $200x + 200y = 1000$

÷ 200

$x + y = 5$ --- (3)

(1)-(2) $2x - 2y = -2$

÷ 2

$x - y = -1$ ---- (4)

(3)+(4) $2x = 4$

$x = 4/2 = 2$

Substitute in (1)

$2 + y = 5$

$y = 5 - 2$

$y = 3$

$x = 2$

$y = 3$

2. Factorise : $x^3 - 2x^2 - 5x + 6$

1	1	-2	-5	6	
	0	1	-1	-6	
3	1	-1	-6	0	$(x-1)$ is a factor
	0	3	6		
	1	2	0		$(x-3)$ is a factor

$(x+2)$ is a factor

$(x - 1)$, $(x - 3)$, $(x+2)$ are factors.

3. Factorize $4x^3 - 7x + 3$

1	4	0	-7	3
	0	4	4	-3
	4	4	-3	0

$(x - 1)$ is a factor

$4x^2 + 4x - 3 = (2x + 3)(2x - 1)$

$\therefore$ $(x-1), (2x-1), (2x+3)$ are factors.

4. Factorize $x^3 - 7x + 6$

1	1	0	-7	6
	0	1	1	-6
2	1	1	-6	0
	0	2	6	
	1	3	0	

$(x-1)$ is a factor

$(x-2)$ is a factor

$(x+3)$ is a factor

$\therefore$ $(x-1), (x-2), (x+3)$ are factors.

5. Factorise: $x^3 - 3x^2 - 10x + 24$

2	1	-3	-10	24
	0	2	-2	-24
-3	1	-1	-12	0
	0	-3	12	
	1	-4	0	

$(x - 2)$ is a factor

$(x+3)$ is a factor

$(x-4)$ is a factor.

$(x-2)(x + 3)(x - 4)$ are factors.

(Note : If it is not possible to find all the factors leave as it is. In Ex. 3.5 problems IV, VIII and XI are all of the same type)

6. If $P = \frac{x}{x+y}$, $Q = \frac{y}{x+y}$, then find $\frac{1}{P-Q} - \frac{2Q}{P^2-Q^2}$

$$\frac{1}{P-Q} - \frac{2Q}{P^2-Q^2} = \frac{1}{P-Q} - \frac{2Q}{(P+Q)(P-Q)}$$

$$= \frac{P+Q-2Q}{(P+Q)(P-Q)}$$

$$= \frac{P-Q}{(P+Q)(P-Q)}$$

$$= \frac{1}{P+Q}$$

$$= \frac{1}{\frac{x}{x+y} + \frac{y}{x+y}}$$

$$= \frac{1}{\frac{x+y}{x+y}}$$

$= 1$

7. Find the square root of $(x^2 - 25)\ (x^2 + 8x + 15)\ (x^2 - 2x - 15)$

$= (x + 5)\ (x - 5)\ (x + 3)\ (x + 5)\ (x - 5)\ (x + 3)$

$= (x+5)^2\ (x - 5)^2\ (x + 3)^2$

Square root $= |(x + 5)\ (x - 5)\ (x + 3)|$

8. Find the square root of $9x^4 + 12x^3 + 10x^2 + 4x + 1$

			3	2	1		
		3	9	12	10	4	1
			9				
	6	2		12	10		
				12	4		
6	4	1			6	4	1
					6	4	1
						0	

Square root $= |3x^2 + 2x + 1|$

9. Find the square root of $x^4 - 10x^3 + 37x^2 - 60x + 36$

			1	-5	6		
	1		1	-10	37	-60	36
			1				
	2	-5		-10	37		
				-10	25		
2	-10	6			12	-60	36
					12	-60	36
						0	

Square root $= |x^2 - 5x + 6|$

10. Find the square root of $4+25x^2-12x-24x^3+16x^4$

Write in descending powers

$16x^4 - 24x^3 + 25x^2 - 12x + 4$

			4	– 3	2		
		4	16	– 24	25	– 12	4
			16				
	8	–3		–24	25		
				–24	9		
8	–6	2			16	– 12	4
					16	– 12	4
						0	

Square root $= |4x^2 - 3x + 2|$

11. If $m - nx + 28x^2 + 12x^3 + 9x^4$ is a perfect square, then find the values of m and n.

Write in descending order

$9x^4 + 12x^3 + 28x^2 - nx + m$

			3	2	4		
		3	9	12	28	-n	m
			9				
	6	2		12	28		
				12	4		
6	4	4			24	-n	m
					24	16	16
						0	

$m = 16,\ \ n = -16$

12. Find a and b if $ax^4 - bx^3 + 40x^2 + 24x + 36$ is a perfect square.

Write in ascending order

$36 + 24x + 40x^2 - bx^3 + ax^4$

	6	2	3		
6	36	24	40	-b	a
	36				
12 2		24	40		
		24	4		
12 4 3			36	-b	a
			36	12	9
				0	

a = 9

b = -12

13. The sum of a number and its reciprocal is $5\frac{1}{5}$, find the number.

Solution :

Let x be the number

$1/x$ be its reciprocal

$$\text{Sum} = 5\frac{1}{5}$$

$$x + \frac{1}{x} = \frac{26}{5}$$

$$\frac{x^2+1}{x} = \frac{26}{5}$$

$5(x^2+1) = 26x$

$5x^2 + 5 - 26x = 0$

$5x^2 - 26x + 5 = 0$

$(5x - 1)(x - 5) = 0$

$5x - 1 = 0$ or $x = 5$

$x = 1/5$ or $x = 5$

The number = {1/5, 5}

14. If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then prove that $c^2 = a^2(1+m^2)$

Given equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

$a = 1 + m^2$, $b = 2mc$, $c = c^2 - a^2$

equal roots $= b^2 - 4AC = 0$

$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$-4c^2 = -4a^2 - 4m^2a^2 = 0$

$\div -4$

$c^2 = a^2 + m^2a^2$

$c^2 = a^2(1+m^2)$

$\therefore c^2 = a^2(1+m^2)$

Thus proved.

4. MATRICES

1. Prove that $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ are multiplicative inverse to each other.

Solution:

$$\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 6-5 & -15+15 \\ 2-2 & -5+6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Also $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 6-5 & 10-10 \\ -3+3 & -5+6 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

The given matrices are inverses to each other under matrix multiplication.

2. Prove that $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$ are inverses to each other under matrix multiplication.

Solution :

$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 15-14 & -10+10 \\ 21-21 & -14+15 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$BA = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 15-14 & 6-6 \\ -35+35 & -14+15 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

The given matrices are inverses to each other under matrix multiplication.

3. If $A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$ then find AB and BA. Are they equal?

Solution:

$$AB = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9+6 & 0+4 \\ 12+0 & 0+0 \end{pmatrix} = \begin{pmatrix} 15 & 4 \\ 12 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 9+0 & 6+0 \\ 9+8 & 6+0 \end{pmatrix} =$$

$$BA = \begin{pmatrix} 9 & 6 \\ 18 & 6 \end{pmatrix}$$

$AB \neq BA$

4. If $A = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$ and B = (1 3 -6) then verify that $(AB)^T = B^T A^T$.

Solution:

$$AB = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} (1 \quad 3 \quad -6)$$

$$= \begin{pmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{pmatrix} \quad \text{----- (1)}$$

$$B^T = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix}$$

$$A^T = (-2 \quad 4 \quad 5)$$

$$B^T A^T = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix} (-2 \quad 4 \quad 5)$$

$$= \begin{pmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{pmatrix} \quad \text{--- (2)}$$

From (1) and (2) we get $(AB)^T = B^T A^T$

5. If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$.

Solution :

$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10-2 & -5+2 \\ 14-3 & -7+3 \end{pmatrix}$$

$$AB = \begin{pmatrix} 8 & -3 \\ 11 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \quad \text{------ (1)}$$

$$B^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$B^TA^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10-2 & 14-3 \\ -5+2 & -7+3 \end{pmatrix}$$

$$B^TA^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \qquad \text{----- (2)}$$

From (1) and (2), we get

$$(AB)^T = B^T A^T.$$

6. If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then show that $A^2 - 4A + 5I_2 = 0$

Solution

$$A^2 = A \times A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$$

$$4A = 4\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix}$$

$$5I_2 = 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$A^2 - 4A = 5I_2 = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} - \begin{pmatrix} 4 & -4 \\ 8 & 12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A^2 - 4A + 5I_2 = 0$$

7. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$. Verify that A(B+C) = AB + AC.

Solution:

$$B + C = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -3+2 & 18+20 \\ 1+4 & -6+40 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix} \qquad \text{----- (1)}$$

$$AB = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} -6+12 & 15+14 \\ 2+24 & -5+28 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix}$$

$$AC = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3-10 & 3+6 \\ -1-20 & -1+12 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix} + \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

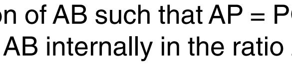
$$= \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix} \qquad \text{----- (2)}$$

From (1) and (2) we get

A (B+C) = AB + AC

5. COORDINATE GEOMETRY

1. Find the points of trisection of the line segment joining the points A(2,-2) and B(-7, 4).



Let P and Q be the points of trisection of AB such that AP = PQ = QB. Then the point P divides AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1

$$P = \left(\frac{1\times(-7)+(2\times2)}{1+2}, \frac{(1\times4)+2(-2)}{1+2} \right)$$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3} \right) = (-1, 0)$$

Thus, the point P is (-1, 0)

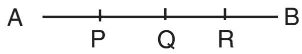
Again by section formula

$$Q = \left(\frac{2\times(-7)+(1\times2)}{2+1}, \frac{(2\times4)+1\times(-2)}{2+1}\right)$$

$= (-4, 2)$

Thus the point Q is (-4, 2)

2. Find the points which divide the line segment joining A (-4, 0) and (0,6) into four equal parts.
Let P,Q,R be the points which divide AB into four equal parts.



The point Q is the midpoint of AB

$$Q = \left(\frac{-4+0}{2}, \frac{0+6}{2}\right) = (-2, 3)$$

now P is the midpoint of AQ.

$$P = \left(\frac{-4-2}{2}, \frac{0+3}{2}\right)$$

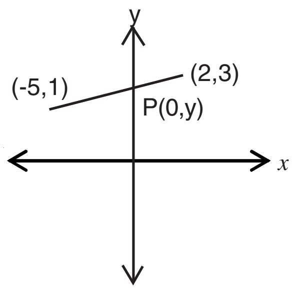
$$P = \left(\frac{-6}{2}, \frac{3}{2}\right) = \left(-3, \frac{3}{2}\right)$$

R is the midpoint of QB.

$$R = \left(\frac{-2+0}{2}, \frac{3+6}{2}\right) = \left(-1, \frac{9}{2}\right)$$

Hence the required points are $P = \left(-3, \frac{3}{2}\right)$ Q = (-2, 3) $R = \left(-1, \frac{9}{2}\right)$.

3. In what ratio is the line joining the points (-5, 1) and (2, 3) divided by the y-axis. Also find the point of intersection.



Let A (-5, 1) and B(2, 3) be the given points.
Let P (0, y) divide AB internally in the ratio ℓ: m. By section formula.

$$P(0, y) = P\left(\frac{(\ell\times2)+(m\times(-5))}{\ell+m}, \frac{(\ell\times3)+m\times1)}{\ell+m}\right) \quad \text{---- (1)}$$

$$P(0, y) = P\left(\frac{2\ell-5m}{\ell+m}, \frac{3\ell+m}{\ell+m}\right) \quad \text{---- (1)}$$

Equating x coordinates to zero.

$$\frac{2\ell-5m}{\ell+m} = 0 \Rightarrow 2l - 5m = 0 \Rightarrow \frac{\ell}{m} = \frac{5}{2}$$

The required ratio is 5 : 2

Also from (1) we have $P(0,y) = P\left(0, \frac{(5\times3)+(2\times1)}{5+2}\right)$