1.If $tan^2\alpha = cos^2\beta - sin^2\beta, then\ prove\ that\ cos^2\alpha - sin^2\alpha = tan^2\beta$

Solution: *Given that* $tan^2\alpha = cos^2\beta - sin^2\beta$

Adding 1 on both sides , $1 + tan^2\alpha = 1 + cos^2\beta - sin^2\beta$

$$sec^2\alpha = 2cos^2\beta$$

$$cos^2\alpha = \frac{1}{2}sec^2\beta \quad ----(1)$$

$$sin^2\alpha = 1 - cos^2\alpha = 1 - \frac{1}{2}sec^2\beta \quad ----(2)$$

(1)-(2) $cos^2\alpha - sin^2\alpha = \frac{1}{2}sec^2\beta - (1 - \frac{1}{2}sec^2\beta)$

$$cos^2\alpha - sin^2\alpha = sec^2\beta - 1$$

$$cos^2\alpha - sin^2\alpha = tan^2\beta$$

2.Prove that $\frac{sec\theta - tan\theta}{sec\theta + tan\theta} = 1 - 2sec\theta\, tan\theta + 2tan^2\theta$

Solution:RHS $= 1 - 2sec\theta\, tan\theta + 2tan^2\theta$

$=1 - 2sec\theta\, tan\theta + tan^2\theta + tan^2\theta$

$=sec^2\theta + tan^2\theta - 2sec\theta\, tan\theta$

$=\frac{(sec\theta - tan\theta)^2}{1}$

$=\frac{(sec\theta - tan\theta)^2}{sec^2\theta - tan^2\theta}$

$=\frac{sec\theta - tan\theta}{sec\theta + tan\theta}$

RHS=LHS

3.Prove that $\frac{1+sec\theta}{sec\theta} = \frac{sin^2\theta}{1-cos\theta}$

Solution: RHS$=\frac{sin^2\theta}{1-cos\theta}$

$=\frac{sin^2\theta}{1-cos\theta} \times \frac{1+cos\theta}{1+cos\theta}$

$=\frac{sin^2\theta\,(1+cos\theta)}{sin^2\theta} = (1 + cos\theta)$

$=\frac{1+sec\theta}{sec\theta}$

RHS=LHS

4.Prove that $(cosec\theta - sin\theta)(sec\theta - cos\theta) = \frac{1}{tan\theta + cot\theta}$

$solution: (cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta)$

$$= \left(\frac{1}{sin\theta} - sin\theta\right)\left(\frac{1}{cos\theta} - cos\theta\right)\left(\frac{Sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)$$

$$= \left(\frac{cos^2\theta}{sin\theta}\right)\left(\frac{sin^2\theta}{cos\theta}\right)\left(\frac{1}{sin\theta cos\theta}\right)$$

=1

5.If $tan\theta + sin\theta = m, tan\theta - sin\theta = n \text{ and } m \neq n, \text{then show that } m^2 - n^2 = 4\sqrt{mn}$

$solution: \quad m^2 - n^2 = (tan\theta - sin\theta)^2 - (tan\theta - sin\theta)^2$

Using formula $(a+b)^2 - (a-b)^2 = 4ab$

$=4sin\theta tan\theta$

$$4\sqrt{mn} = 4\sqrt{(tan\theta + sin\theta)(tan\theta - sin\theta)} = 4\sqrt{tan^2\theta - sin^2\theta}$$

$=4sin\theta tan\theta$

Thus we get $m^2 - n^2 = 4\sqrt{mn}$

6.Prove that $\frac{1}{cosec\theta - cot\theta} - \frac{1}{sin\theta} = \frac{1}{sin\theta} - \frac{1}{cosec\theta + cot\theta}$

$$\frac{1}{cosec\theta - cot\theta} + \frac{1}{cosec\theta + cot\theta} = \frac{2}{sin\theta}$$

Using cross multiplication or l.c.m.   we get

$$\frac{1}{cosec\theta - cot\theta} + \frac{1}{cosec\theta + cot\theta} = \frac{2cosec\theta}{1} = \frac{2}{sin\theta}$$

7.Prove that $(sin\theta + cosec\theta)^2 + (cos\theta + sec\theta)^2 = 7 + tan^2\theta + cot^2\theta$

Solution: $LHS = (sin\theta + 1/sin\theta)^2 + (cos\theta + 1/cos\theta)^2$

$$= sin^2\theta + \frac{1}{sin^2\theta} + 2 + cos^2\theta + \frac{1}{cos^2\theta} + 2$$

$$= 5 + cosec^2\theta + cot^2\theta = 7 + tan^2\theta + cot^2\theta = RHS$$