UNIT TEST – 1
APPLICATIONS OF MATRICES AND DETERMINANTS

Class : 12 **Tot. Marks: 50**

Subject : Mathematics **WWW.MATHSTIMES.COM**

Time: 45 Min

Part – A **2 x 1 = 2**

Answer all the Questions

1. If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2 , then λ is,

   a) 1 b) 2 c) 3 d) any real number

2. The system of equations ax+y+z = 0 ; x+by+z = 0; x+y+cz = 0 has a non – trivial solution then

   $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$

   a) 1 b) 2 c) –1 d) 0

Part – B **3 x 6 = 18**

Answer ANY 3 Questions.

3. Find the adjoint of the matrix $A=\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result $A(\text{adj } A) = (\text{adj } A)\ A = |A|\,.\,I$

4. Solve the following non-homogeneous equations of three unknowns.

   $2x+2y+z=5$; $x-y+z=1$; $3x+y+2z=4$

5. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

6. Solve the following non-homogeneous system of linear equations by determinant method: $4x+5y=9$; $8x+10y=18$

Part – C **3 x 6 = 18**

Answer ANY 3 Questions.

7. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chair is Rs. 25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased.

8. For what values of k, the system of equations $kx+y+z=1$, $x+ky+z=1$, $x+y+kz=1$ have

   (i) unique solution (ii) more than one solution (iii) no solution

9. Verify whether the given system of equations is consistent. If it is consistent, solve them

   $x-y+z=5$, $-x+y-z=-5$, $2x-2y+2z=10$

10. Solve by determinant method $\frac{1}{x}+\frac{2}{y}-\frac{1}{z}=1$; $\frac{2}{x}+\frac{4}{y}+\frac{1}{z}=5$; $\frac{3}{x}-\frac{2}{y}-\frac{2}{z}=0$

UNIT TEST – 2
VECTOR ALGEBRA

Class : 12 **Tot. Marks: 50**
Subject : Mathematics **Time: 45 Min**

Part – A **2 x 1 = 2**

Answer all the Questions

1. The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ is,
   a) $10\sqrt{3}$ b) $6\sqrt{30}$ c) $\frac{3}{2}\sqrt{30}$ d) $3\sqrt{30}$

2. The shortest distance between the parallel lines $\frac{x-3}{4}=\frac{y-1}{2}=\frac{z-5}{-3}$ and $\frac{x-1}{4}=\frac{y-2}{2}=\frac{z-3}{-3}$
   a) 3 b) 2 c) 1 d) 0

Part – B **3 x 6 = 18**

Answer ANY 3 Questions.

3. Show that diameter of a sphere subtends a right angle at a point on the surface by vector method.

4. Show that the two lines $\bar{r}=\left(\vec{i}-\vec{j}\right)+t\left(2\vec{i}+\vec{k}\right)$ and $\bar{r}=\left(2\vec{i}-\vec{j}\right)+s\left(\vec{i}+\vec{j}-\vec{k}\right)$ are skew lines and find the distance between them.

5. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a},\ \vec{b},\ \vec{c}\right]^2$

6. a) For any vector $\vec{r}$ prove that $\bar{r}=\left(\bar{r}\cdot\vec{i}\right)\vec{i}+\left(\bar{r}\cdot\vec{j}\right)\vec{j}+\left(\bar{r}\cdot\vec{k}\right)\vec{k}$

   b) A force given by $3\vec{i}+2\vec{j}-4\vec{k}$ is applied at the point (1,-1,2) . Find the moment of the force about the point (2,-1,3).

Part – C **3 x 6 = 18**

Answer ANY 3 Questions.

7. Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$.

8. Prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$

9. Show that the lines $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x-2}{1}=\frac{y-1}{2}=\frac{-z-1}{1}$ intersect and find their point of intersection.

10. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$, $\vec{c}=\vec{j}-3\vec{k}$ Verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$

UNIT TEST – 3
COMPLEX NUMBERS

Class : 12 **Tot. Marks: 50**

Subject : Mathematics **Time: 45 Min**

Part – A **2 x 1 = 2**

Answer all the Questions

1. If P represents the variable complex number z and if $|2z-1| = 2|z|$ then the locus of P is

   a) the straight line $x = \frac{1}{4}$ b) the straight line $y = \frac{1}{4}$

   c) the straight line $z = \frac{1}{2}$ d) the circle $x^2 + y^2 - 4x - 1 = 0$

2. If $a = \cos\alpha - i\sin\alpha$, $b = \cos\beta - i\sin\beta$ and $c = \cos\gamma - i\sin\gamma$ then $\frac{a^2c^2 - b^2}{abc}$ is

   a) $\cos 2(\alpha-\beta+\gamma) + i\sin(\alpha-\beta+\gamma)$ b) $-2\cos(\alpha-\beta+\gamma)$

   c) $-2i\sin(\alpha-\beta+\gamma)$ d) $2\cos(\alpha-\beta+\gamma)$

Part – B **3 x 6 = 18**

Answer ANY 3 Questions.

3. Find the square root of $(-8-6i)$

4. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$

5. For any two complex numbers Z_1, Z_2, show that

   (i) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$ (ii) $\arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2)$

6. If $\arg(z-1) = \frac{\pi}{6}$ and $\arg(z+1) = 2\frac{\pi}{3}$ then prove that $|z| = 1$

Part – C **3 x 6 = 18**

Answer ANY 3 Questions.

7. If α and β are the roots of the equation $x^2 - 2px + (p^2+q^2) = 0$ and $\tan\theta = \frac{q}{y+p}$ Show that

$$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha-\beta} = q^{n-1}\frac{\sin n\theta}{\sin^n\theta}\ ;\ n \in N$$

8. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$

9. Find all the values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1.

10. If P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$

UNIT TEST – 4

ANALYTICAL GEOMETRY

Class : 12 **Tot. Marks: 50**

Subject : Mathematics **Time: 45 Min**

Part – A **2 x 1 = 2**

Answer all the Questions

1. If the length of major and semi-minor axes of an ellipse are 8,2 and their corresponding equations y – 6 = 0 and x + 4 =0 then the equations of the ellipse is.

a) $\frac{(x+4)^2}{4}+\frac{(y-6)^2}{16}=1$ b) $\frac{(x+4)^2}{16}+\frac{(y-6)^2}{4}=1$

c) $\frac{(x+4)^2}{16}-\frac{(y-6)^2}{4}=1$ d) $\frac{(x+4)^2}{4}-\frac{(y-6)^2}{16}=1$

2. The normal to the rectangular hyperbola xy = 9 at $\left(6,\ \frac{3}{2}\right)$ meets the curve again at.

a) $\left(\frac{3}{8},24\right)$ b) $\left(-24,\ \frac{-3}{8}\right)$ c) $\left(\frac{-3}{8},-24\right)$ d) $\left(24,\ \frac{3}{8}\right)$

Part – B **3 x 6 = 18**

Answer ANY 3 Questions.

3. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

4. The headlight of a motor vehicle is a parabolic reflector of diameter 12cm and depth 4cm. Find the position of bulb on the axis of the reflector for effective functioning of the headlight.

5. The tangent at any point of the rectangular hyperbola $xy=c^2$ makes intercepts a, b and the normal at the point makes intercepts p,q on the axes. Prove that $ap+bq=0$

6. Find the angle between the asymptotes of the hyperbola $3x^2-y^2-12x-6y-9=0$

Part – C **3 x 6 = 18**

Answer ANY 3 Questions.

7. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x+2y-5=0$ and passes through the points (6,0) and (-3,0).

8. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.

9. The ceiling in a hallway 20ft wide is in the shape of a semi ellipse and 18ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12ft.

10. Find the eccentricity, centre, foci and vertices of the hyperbola $9x^2-16y^2-18x-64y-199=0$ and also trace the curve.

UNIT TEST – 5
DIFFERENTIAL CALCULAS APPLICATIONS - 1

Class : 12 **Tot. Marks: 50**

Subject : Mathematics **Time: 45 Min**

Part – A **2 x 1 = 2**

Answer all the Questions

1. $\lim_{x \to 0} \frac{a^x - b^x}{c^x - d^x}$

a) ∞ b) 0 c) $\log \frac{ab}{cd}$ d) $\frac{\log(a/b)}{\log(c/d)}$

2. The function y = tan x – x is.

a) an increasing function in $\left(0, \frac{\pi}{2}\right)$ b) a decreasing function in $\left(0, \frac{\pi}{2}\right)$

c) increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ d) decreasing in $\left(0, \frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Part – B **3 x 6 = 18**

Answer ANY 3 Questions.

3. Evaluate : $\lim_{x \to 1} x^{\frac{1}{x-1}}$

4. Find two positive numbers whose product is 100 and whose sum is minimum.

5. Find the intervals in which $f(x) = x^3 - 3x + 1$ is increasing and decreasing.

6. a) Obtain the Maclaurin's Series for e^x

b) Find the critical numbers of $x^{3/5}(4-x)$

Part – C **3 x 6 = 18**

Answer ANY 3 Questions.

7. Gravel is being dumped from a conveyor belt at a rate of 30 ft3 / min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

8. Show that the equation of the normal to the curve $x = a\cos^3\theta$; $y = a\sin^3\theta$ at ' θ ' is $x\cos\theta - y\sin\theta = a \cos 2\theta$.

9. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius 'a' is $\frac{8}{27}$ (volume of the sphere).

10. Find the intervals of concavity and the points of inflection of the function : $y = 12x^2 - 2x^3 - x^4$

UNIT TEST -6

DIFFERENTIAL CALCULUS – APPLICATION II

Time: 45 Minutes **Marks: 50**

MATHEMATICS –STD XII

Section-A

Choose the most suitable answer from the given four alternatives. **2X1=2**

1. If u = $\frac{1}{\sqrt{x^2+y^2}}$, then x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y}$ is equal to

a) $\frac{1}{2}u$ b) u c) $\frac{3}{2}u$ d) - u

2. An asymptote to the curve $y^2(a+2x) = x^2(3a-x)$ is..

a) x = 3a b) x = - a / 2 c) x = a / 2 d) x=0

Section-B

Answer any three questions **3X6=18**

3.If $u = \log(\tan x + \tan y + \tan z)$, prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$

4. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ if $w = \log(x^2 + y^2)$ where $x = r\cos\theta,\ y = r\sin\theta$

5. Use differentials to find an approximate value for the given number $\sqrt{36.1}$

6. If $w = x + 2y + z^2$ and $x = \cos t$; $y = \sin t$; $z = t$. Find $\frac{dw}{dt}$

Section-C

Answer any three questions **3X10=30**

7. Trace the curve $y = x^3 + 1$

8. If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y \log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

9. Verify $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}$ for the following function: $u = \frac{x}{y^2} - \frac{y}{x^2}$

10.If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. by using Euler's theorem.

UNIT TEST -7

INTEGRAL CALCULUS

MATHEMATICS – STD XII

Time: 45 Minutes **Marks: 50**

Section-A

Choose the most suitable answer from the given four alternatives. **2X1=2**

1. The value of $\int_0^{\pi} \sin^2 x \cos^3 x dx$ is

a) π b) $\pi/2$ c) $\pi/4$ d) 0.

2. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is.

a) 48 b) 24 c) 12 d) 96

Section-B

Answer any three questions **3X6=18**

3. Evaluate: $\int_0^3 \frac{\sqrt{x}\, dx}{\sqrt{x}+\sqrt{3-x}}$

4. Evaluate $\int_0^1 x e^{-4x}\, dx$

5. Evaluate $\int_0^{\pi/2} \log(\tan x)\, dx$

6. Find the volume of the solid that results when the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b>0)$ is revolved about the minor axis.

Section-C

Answer any three questions **3X10=30**

7. Find the area of the region bounded by the curve $y=3x^2-x$ and the x – axis between x = -1 and x = 1.

8. Find the length of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{a}\right)^{2/3}=1$

9. Find the area of the region enclosed by $y^2=x$ and $y=x-2$

10. Derive the formula for the volume of a right circular cone with radius ‘ r ‘ and height ‘ h ‘.

UNIT TEST –8

DIFFERENTIAL EQUATIONS

MATHEMATICS – STD XII

Time: 45 Minutes **Marks: 50**

Section-A

Choose the most suitable answer from the given four alternatives. **2X1=2**

1. The integrating factor of $\frac{dy}{dx}+2\frac{y}{x}=e^{4x}$ is

a) log x. b) x^2 c) e^x. d) x.

2. The differential equation satisfied by all the straight lines in xy plane is

a) $\frac{dy}{dx}=\text{a constant}$ b) $\frac{d^2y}{dx^2}=0$ c) $y+\frac{dy}{dx}=0$ d) $\frac{d^2y}{dx^2}+y=0$

Section-B

Answer any three questions **3X6=18**

3. Solve: $\frac{dy}{dx}+xy=x$

4. Solve : $\left(3D^2+4D+1\right)y=3e^{-x/3}$

5. Solve: $3e^x \tan y\, dx+\left(1+e^x\right)\sec^2 y\, dy=0$

6 Solve: $\left(x^2+y^2\right)dy=xy\ dx$

Section-C

Answer any three questions **3X10=30**

7. Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years. [Take A_0 as the initial amount]

8. Solve $\left(D^2-6D+9\right)y=x+e^{2x}$

9. Show that the equation of the curve whose slope at any point is equal to $y+2x$ and which passes through the origin is $y=2\left(e^x-x-1\right)$

10. Solve : $(x+y)^2\ \frac{dy}{dx}=a^2$

UNIT TEST -9

DISCRETE MATHEMATICS

MATHEMATICS – STD XII

Time: 45 Minutes **Marks: 50**

Section-A

Choose the most suitable answer from the given four alternatives. **2X1=2**

1. monoid becomes a group if it also satisfies the

   a) closure axiom. b) associative axiom c) identity axiom d) inverse axiom.

2. The order of –i in the multiplicative group of 4^{th} roots of unity

   a)4 b) 3 c) 2 d) 1

Section-B

Answer any three questions **3X6=18**

3. State and prove cancellation laws on groups.
4. Show that $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$
5. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ form an abelian group, under multiplication of matrices.
6. Construct the truth tables for $(p \vee q) \vee r$

Section-C

Answer any three questions **3X10=30**

7. Let G be the set of all rational numbers except 1 and * be defined on G by $a * b = a + b - ab$ for all $a, b \in G$. Show that (G , *)is an infinite abelian group.
8. Show that the set $\{[1], [3], [4], [5], [9]\}$ forms an abelian group under multiplication modulo 11.
9. Show that the set of all matrices of the form $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}, a \in R - \{0\}$ forms an abelian group under matrix multiplication.
10. Show that the set $G = \{a + b\sqrt{2} \ / \ a, b \in Q\}$ is an infinite abelian group with respect to addition.

UNIT TEST-10---PROBABILITY DISTRIBUTIONS---MATHEMATICS – STD XII

Time: 45 Minutes **Section-A** **Marks: 50**

Choose the most suitable answer from the given four alternatives. **2X1=2**

1. A random variable X has the following probability mass function as follows

x	-2	3	1
P(X=x)	$\lambda/6$	$\lambda/4$	$\lambda/12$

Then the value of λ is…

a) 1 b) 2 c) 3 d) 4

2. The distribution function F(x) of a random variable X is.

a) a decreasing function b) a non decreasing

c) a constant function d) increasing first and then decreasing

Answer any three questions **Section-B** **3X6=18**

3. (I) Prove that the total probability is one.(ii) If the sum of mean and variance of a Binomial Distribution is 4.8 for 5 trials find the distribution.

4. Find the Mean and Variance for the following probability density function:

$$f(x) = \begin{cases} x e^{-x} & , if\ x > 0 \\ 0 & , \ otherwise \end{cases}$$

5. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

6. In a Poisson distribution if $P(X=2) = P(X=3)$ find $P(X=5)$ $\left[given \ \ e^{-3} = 0.050\right]$.

Answer any three questions **Section-C** **3X10=30**

7. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with(i) no accident in a year (ii) more than 3 accidents in a year $\left[e^{-3} = 0.0498\right]$.

8. Obtain k , μ and σ^2 of the normal distribution whose probability distribution function is given by

$$f(x) = ke^{-2x^2+4x} \qquad -\infty < X < \infty.$$

9. The mean weight of 500 male students in a certain college in 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh (i) between 120 and155pounds (ii) more than 185 pounds

10.An urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn.(i) With replacement (ii) without replacement.