

## HIGHER SECONDARY- SLIP TEST MATHEMATICS

| CHAPTER  | NO. OF<br>6 MARK<br>QUESTIONS | NO. OF<br>10 MARK<br>QUESTIONS | NO. OF<br>SLIP<br>TESTS<br>(6 MARK) | NO. OF<br>SLIP<br>TESTS<br>(10 MARK) | MINIMUM<br>MARK IN<br>PUBLIC<br>EXAM |
|--|-------------------------------|--------------------------------|-------------------------------------|--------------------------------------|--------------------------------------|
| 1. APPLICATIONS OF<br>MATRICES AND<br>DETERMINANTS | 32                            | -                              | 4                                   | 0                                    | 12                                   |
| 2. VECTOR ALGEBRA                                  | -                             | 20                             | 0                                   | 4                                    | 20                                   |
| 3. COMPLEX<br>NUMBERS                              | 32                            | 10                             | 4                                   | 2                                    | 16                                   |
| 4. ANALYTICAL<br>GEOMETRY                          | -                             | 17                             | 0                                   | 4                                    | 20                                   |
| 5. DIFFERENTIAL<br>CALCULUS –<br>APPLICATIONS I    | -                             | -                              | -                                   | -                                    | -                                    |
| 6. DIFFERENTIAL<br>CALCULUS –<br>APPLICATIONS II   | 16                            | 11                             | 2                                   | 2                                    | 16                                   |
| 7. INTEGRAL<br>CALCULAS AND ITS<br>APPLICATIONS    | 8                             | 9                              | 1                                   | 2                                    | 10                                   |
| 8. DIFFERENTIAL<br>EQUATIONS                       | -                             | 13                             | 0                                   | 2                                    | 10                                   |
| 9. DISCRETE<br>MATHEMATICS                         | 24                            | 15                             | 3                                   | 3                                    | 22                                   |
| 10. PROBABILITY<br>DISTRIBUTIONS                   | -                             | 12                             | -                                   | 2                                    | <u>10</u>                            |
| <b>TOTAL</b>                                       | <b>112</b>                    | <b>107</b>                     | <b>14</b>                           | <b>21</b>                            | <b>138</b>                           |

1. APPLICATIONS OF MATRICES AND DETERMINANTS (6 MARK QUESTIONS)

SLIP TEST -1

1. Find the adjoint of the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$  and verify the result

$$A(\text{adj } A) = (\text{adj } A)A = |A| \cdot I$$

2. If  $A = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$ , verify the result  $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$

3. Show that the adjoint of  $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$  is A itself.

4. For  $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$  show that  $A = A^{-1}$ .

5. Find the rank of the matrix  $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

6. Find the rank of the matrix  $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

7. Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 4 & 6 & -2 \\ 3 & 6 & 9 & -3 \end{bmatrix}$

8. Find the rank of the matrix  $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

SLIP TEST - 2

1. Find the rank of the matrix  $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

2. Find the rank of the matrix  $\begin{bmatrix} 3 & 1 & 2 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

3. Find the rank of the matrix  $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$

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4. Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$
5. Find the rank of the matrix  $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$
6. Find the rank of the matrix  $\begin{bmatrix} 1 & 1 & -1 \\ 3 & -2 & 3 \\ 2 & -3 & 4 \end{bmatrix}$

Examine the consistency of the following system of equations. If it is consistent then solve the same.

7.  $x + y + z = 7$  ;  $x + 2y + 3z = 18$  ;  $y + 2z = 6$
8.  $x - 4y + 7z = 14$  ;  $3x + 8y - 2z = 13$  ;  $7x - 8y + 26z = 5$

### SLIP TEST - 3

1. Solve the following system of linear equations by determinant method  
 $2x + 3y = 8$  ;  $4x + 6y = 16$
2. Solve the following system of linear equations by determinant method  
 $4x + 5y = 9$  ;  $8x + 10y = 18$
3. Solve the following system of linear equations by determinant method  
 $2x + 2y + z = 5$  ;  $x - y + z = 1$  ;  $3x + y + 2z = 4$
4. Solve the following system of linear equations by determinant method  
 $x + y + 2z = 4$  ;  $2x + 2y + 4z = 8$  ;  $3x + 3y + 6z = 10$
5. Solve by matrix inversion method  $x + y = 3$ ,  $2x + 3y = 8$
6. Solve by matrix inversion method  $2x - y = 7$ ,  $3x - 2y = 11$
7. Solve by matrix inversion method  $7x + 3y = -1$ ,  $2x + y = 0$
8. State and prove reversal law for inverses of matrices

### SLIP TEST - 4

1. If  $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$  verify that  $(AB)^{-1} = B^{-1}A^{-1}$
2. If  $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$  verify that  $(AB)^T = B^T A^T$
3. If  $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$  verify that  $(AB)^{-1} = B^{-1}A^{-1}$ .

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4. Find the adjoint of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

5. Find the inverse of the following matrix:  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

6. Find the inverse of the following matrix:  $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

7. Find the inverse of the following matrix:  $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$

8. Find the inverse of the following matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

### 2. VECTOR ALGEBRA (10 MARK QUESTIONS)

#### SLIP TEST - 5

1. Altitudes of a triangle are concurrent – prove by vector method.
2. Prove that  $\cos(A-B) = \cos A \cos B + \sin A \sin B$
3. Prove that  $\sin(A+B) = \sin A \cos B + \cos A \sin B$
4. Prove that  $\cos(A+B) = \cos A \cos B - \sin A \sin B$
5. Prove that  $\sin(A-B) = \sin A \cos B - \cos A \sin B$ .

#### SLIP TEST – 6

1. If  $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$ ,  $\vec{b} = -2\vec{i} + 5\vec{k}$ ,  $\vec{c} = \vec{j} - 3\vec{k}$  Verify that  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
2. Verify  $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \ \vec{b} \ \vec{d}]\vec{c} - [\vec{a} \ \vec{b} \ \vec{c}]\vec{d}$  where  $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ ;  $\vec{b} = 2\vec{i} + \vec{k}$ ;  $\vec{c} = 2\vec{i} + \vec{j} + \vec{k}$ ;  $\vec{d} = \vec{i} + \vec{j} + 2\vec{k}$
3. Show that the lines  $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$  and  $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$  intersect and find their point of intersection.
4. Show that the lines  $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$  and  $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$  intersect and hence find the point of intersection.
5. Derive the equation of the plane in the intercept form. (both in vector and Cartesian form)

**SLIP TEST – 7**

1. Find the vector and Cartesian equations of the plane through the point (2,-1,-3) and parallel to the lines.  $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{-4}$  and  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{2}$ .
2. Find the vector and Cartesian equation of the plane through the point (1,3,2) and parallel to the lines  $\frac{x+1}{2} = \frac{y+2}{-1} = \frac{z+3}{3}$  and  $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+2}{2}$
3. Find the vector and Cartesian equation to the plane through the point (-1,3,2) and perpendicular to the planes  $x+2y+2z=5$  and  $3x+y+2z=8$ .
4. Find the vector and Cartesian equation of the plane containing the line  $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$  and parallel to the line  $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$ .
5. Find the vector and Cartesian equation of the plane containing the line  $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$  and passing through the point (-1,1,-1).

**SLIP TEST – 8**

1. Find the vector and Cartesian equations of the plane passing through the points (-1,1,1) and (1,-1,1) and perpendicular to the plane  $x+2y+2z=5$
2. Find the vector and Cartesian equation of the plane passing through the points A (1,-2,3) and B (-1,2,-1) and is parallel to the line  $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-1}{4}$
3. Find the vector and Cartesian equation of the plane through the points (1,2,3) and (2,3,1) perpendicular to the plane  $3x-2y+4z-5=0$ .
4. Find the vector and Cartesian equation of the plane passing through points with position vectors  $3\vec{i}+4\vec{j}+2\vec{k}$ ,  $2\vec{i}-2\vec{j}-\vec{k}$  and  $7\vec{i}+\vec{k}$ .
5. Find the vector and Cartesian equations of the plane passing through the points (2,2,-1), (3,4,2) and (7,0,6)

**3. COMPLEX NUMBERS (6 MARK QUESTIONS)**

**SLIP TEST – 9**

1. Find the square root of  $(-8-6i)$
2. Find the square root of  $(-7+24i)$
3. Solve the equation  $x^4 - 4x^2 + 8x + 35 = 0$ , if one of its roots is  $2 + \sqrt{3}i$
4. Solve the equation  $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$  if  $3+i$  is a root.
5. Solve the equation  $x^4 - 4x^3 + 11x^2 - 14x + 10 = 0$  if one root is  $1+2i$
6. Solve :  $6x^4 - 25x^3 + 32x^2 + 3x - 10 = 0$  given that one of the root is  $2-i$

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7. If  $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = A + iB$ , prove that

(i)  $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \dots (a_n^2 + b_n^2) = A^2 + B^2$

(ii)  $\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + \dots + \tan^{-1}\left(\frac{b_n}{a_n}\right) = k\pi + \tan^{-1}\left(\frac{B}{A}\right), k \in \mathbb{Z}$

8. Prove that the points representing the complex numbers  $(7+5i)$ ,  $(5+2i)$ ,  $(4+7i)$  and  $(2+4i)$  form a parallelogram.

### SLIP TEST – 10

1. Prove that the triangle formed by the points representing the complex numbers  $(10+8i)$ ,  $(-2+4i)$  and  $(-11+31i)$  on the Argand plane is right angled.

2. Prove that the complex numbers  $3+3i$ ,  $-3-3i$ ,  $-3\sqrt{3}+3\sqrt{3}i$  are the vertices of an equilateral triangle in the complex plane.

3. Prove that the points representing the complex numbers  $2i$ ,  $1+i$ ,  $4+4i$  and  $3+5i$  on the Argand plane are the vertices of a rectangle.

4. Show that the points representing the complex numbers  $7+9i$ ,  $-3+7i$ ,  $3+3i$  form a right angled triangle on the Argand diagram.

5. For any two complex numbers  $Z_1, Z_2$ , show that

(i)  $|Z_1 Z_2| = |Z_1| |Z_2|$       (ii)  $\arg(Z_1 Z_2) = \arg(Z_1) + \arg(Z_2)$

6. For any two complex numbers  $Z_1, Z_2$ , show that

(i)  $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$       (ii)  $\arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2)$

7. If  $x + \frac{1}{x} = 2 \cos \theta$  prove that (i)  $x^n + \frac{1}{x^n} = 2 \cos n\theta$       (ii)  $x^n - \frac{1}{x^n} = 2i \sin n\theta$

8. If  $x = \cos \alpha + i \sin \alpha$ ;  $y = \cos \beta + i \sin \beta$  prove that  $x^m y^n + \frac{1}{x^m y^n} = 2 \cos(m\alpha + n\beta)$

### SLIP TEST – 11

1. Prove that  $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$

2. Prove that  $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}$

3. Prove that  $(1+\cos \theta + i \sin \theta)^n + (1+\cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n(\theta/2) \cos \frac{n\theta}{2}$

4. Prove that  $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1} \cos \frac{n\pi}{6}$

5. Simplify :  $\frac{(\cos \theta + i \sin \theta)^4}{(\sin \theta + i \cos \theta)^5}$

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6. Simplify:  $\frac{(\cos \alpha + i \sin \alpha)^3}{(\sin \beta + i \cos \beta)^4}$

7. If  $n$  is a positive integer, prove that  $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos n\left(\frac{\pi}{2} - \theta\right) + i \sin n\left(\frac{\pi}{2} - \theta\right)$

8. State and prove the triangle inequality of complex numbers.

### SLIP TEST – 12

1. Find all the values of the following:  $(8i)^{1/3}$

2. Solve:  $x^4 + 4 = 0$

3.  $P$  represents the variable complex number  $z$ . Find the locus of  $P$ , if  $|z - 5i| = |z + 5i|$

4.  $P$  represents the variable complex number  $z$ . Find the locus of  $P$ , if  $|2z - 3| = 2$ .

5. Prove that if  $\omega^3 = 1$ , then  $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega} = 0$

If  $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$ , prove that

6. (i)  $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$

(ii)  $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$

7. (iii)  $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$

(iv)  $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

8. If  $\arg(z-1) = \frac{\pi}{6}$  and  $\arg(z+1) = 2\frac{\pi}{3}$  then prove that  $|z| = 1$

### 3. COMPLEX NUMBERS (10 MARK QUESTIONS)

#### SLIP TEST – 13

1. If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 2px + (p^2 + q^2) = 0$  and  $\tan \theta = \frac{q}{y+p}$  Show

that  $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = q^{n-1} \frac{\sin n\theta}{\sin^n \theta}; n \in N$

2. If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 2x + 2 = 0$  and  $\cot \theta = y + 1$ , Show that

$$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}; n \in N$$

3. If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 2x + 4 = 0$  Prove that  $\alpha^n - \beta^n = i 2^{n+1} \sin \frac{n\pi}{3}; n \in N$  and deduct  $\alpha^9 - \beta^9$

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4. If  $a = \cos 2\alpha + i \sin 2\alpha$ ,  $b = \cos 2\beta + i \sin 2\beta$  and  $c = \cos 2\gamma + i \sin 2\gamma$  Prove that

$$(i) \sqrt{abc} + \frac{1}{\sqrt{abc}} = 2 \cos(\alpha + \beta + \gamma) \quad (ii) \frac{a^2 b^2 + c^2}{abc} = 2 \cos 2(\alpha + \beta - \gamma)$$

5. If P represents the variable complex number z. Find the locus of P, if  $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$

### SLIP TEST – 14

- Find all the values of  $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$  and hence prove that the product of the values is 1.
- Solve:  $x^4 - x^3 + x^2 - x + 1 = 0$
- Solve the equation  $x^9 + x^5 - x^4 - 1 = 0$
- Solve the equation  $x^7 + x^4 + x^3 + 1 = 0$
- Find all the values of  $(-\sqrt{3} - i)^{\frac{2}{3}}$

### 4. ANALYTICAL GEOMETRY (10 MARK QUESTIONS)

#### SLIP TEST – 15

- Prove that the line  $5x + 12y = 9$  touches the hyperbola  $x^2 - 9y^2 = 9$  and find its point of contact.
- Show that the line  $x - y + 4 = 0$  is a tangent to the ellipse  $x^2 + 3y^2 = 12$ . Find the coordinates of the point of contact.
- Find the equation of the hyperbola if its asymptotes are parallel to  $x + 2y - 12 = 0$  and  $x - 2y + 8 = 0$ , (2,4) is the centre of the hyperbola and it passes through (2,0).
- Find the equation of the rectangular hyperbola which has for one of its asymptotes the line  $x + 2y - 5 = 0$  and passes through the points (6,0) and (-3,0).

#### SLIP TEST – 16

- A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level find the length of the support if the height of the pillars are 55 mts.
- A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500ft, the points of support of the cable on the towers are 200ft above the road way and the lowest point on the circle is 70ft above the roadway. Find the vertical distance to the cable from a pole whose height is 122 ft.

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- The girder of a railway bridge is in the parabolic form with span 100ft. and the highest point on the arch is 10ft, above the bridge. Find the height of the bridge at 10ft, to the left or right from the midpoint of the bridge.
- A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of  $\frac{\pi}{3}$  radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet nearer to the sun?( Take the orbit as open rightward ).

### SLIP TEST – 17

- On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.
- Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?
- An arch is in the form of a semi-ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base?
- The arch of a bridge is in the shape of a semi –ellipse having a horizontal span of 40ft and 16ft high at the centre. How high is the arch, 9ft from the right or left of the centre.

### SLIP TEST – 18

- The ceiling in a hallway 20ft wide is in the shape of a semi ellipse and 18ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12ft.
- A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.
- A kho-kho player in a practice session while running realizes that the sum of the distances from the two kho-kho poles from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

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- A satellite is traveling around the earth in an elliptical orbit having the earth at a focus and of eccentricity  $1/2$ . The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.
- The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun? (ii) The greatest possible distance between mercury and sun.

### 6. DIFFERENTIAL CALCULUS – APPLICATIONS – II

#### (6 MARK QUESTIONS)

#### SLIP TEST – 19

- If  $u = \log(\tan x + \tan y + \tan z)$ , prove that  $\sum \sin 2x \frac{\partial u}{\partial x} = 2$
- If  $U = (x-y)(y-z)(z-x)$  then show that  $U_x + U_y + U_z = 0$
- Suppose that  $z = ye^{x^2}$  where  $x = 2t$  and  $y = 1-t$  then find  $\frac{dz}{dt}$
- If  $w = x + 2y + z^2$  and  $x = \cos t$ ;  $y = \sin t$ ;  $z = t$ . Find  $\frac{dw}{dt}$ .
- If  $w = xy + z$  where  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$  Find  $\frac{dw}{dt}$ .
- If  $V = ze^{ax+by}$  and  $z$  is a homogenous function of degree  $n$  in  $x$  and  $y$  prove that  $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = (ax + by + n)V$ .
- The time of swing  $T$  of a pendulum is given by  $T = k\sqrt{\ell}$  where  $k$  is a constant. Determine the percentage error in the time of swing if the length of the pendulum / changes from 32.1 cm to 32.0 cm.
- If  $u$  is a homogenous function of  $x$  and  $y$  of degree  $n$ , prove that  $x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y^2} = (n-1) \frac{\partial u}{\partial y}$

#### SLIP TEST – 20

- Use differentials to find an approximate value for  $\sqrt[3]{65}$ .
- Use differentials to find an approximate value for  $\sqrt{36.1}$
- Use differentials to find an approximate value for  $\frac{1}{10.1}$
- Use differentials to find an approximate value for  $(1.97)^6$

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- Find  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial \theta}$  if  $w = \log(x^2 + y^2)$  where  $x = r \cos \theta$ ,  $y = r \sin \theta$
- Find  $\frac{\partial w}{\partial u}$  and  $\frac{\partial w}{\partial v}$  if  $w = x^2 + y^2$  where  $x = u^2 - v^2$ ,  $y = 2uv$
- Using Euler's theorem prove the following:  $u = xy^2 \sin\left(\frac{x}{y}\right)$  show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$ .
- If  $w = \frac{x}{(x^2 + y^2)}$  where  $x = \cos t$ ,  $y = \sin t$ , find  $\frac{dw}{dt}$

### 6. DIFFERENTIAL CALCULUS – APPLICATIONS - II (10 MARK QUESTIONS)

#### SLIP TEST – 21

- Trace the curve  $y = x^3 + 1$
- Trace the curve  $y^2 = 2x^3$
- Trace the curve :  $y = x^3$
- Verify Euler's theorem for  $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$
- Using Euler's theorem, prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$  if  $u = \sin^{-1}\left(\frac{x-y}{\sqrt{x} + \sqrt{y}}\right)$
- Using Euler's theorem prove that Prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ . If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$

#### SLIP TEST – 22

- Use differentials to find the an approximate value for the given number  
 $y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$
- If  $w = u^2 e^v$  where  $u = \frac{x}{y}$  and  $v = y \log x$ , find  $\frac{\partial w}{\partial x}$  and  $\frac{\partial w}{\partial y}$
- Verify  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$  if  $u = \frac{x}{y^2} - \frac{y}{x^2}$
3. Verify  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$  if  $u = \tan^{-1}\left(\frac{x}{y}\right)$
3. Verify  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$  if  $u = \sin 3x \cos 4y$

**7. INTEGRAL CALCULAS AND ITS APPLICATIONS  
(6 MARK QUESTIONS)**

**SLIP TEST – 23**

1. Evaluate  $\int_0^{\pi/2} \log(\tan x) dx$
2. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$
3. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$
4. Evaluate  $\int_0^3 \frac{\sqrt{x} dx}{\sqrt{x} + \sqrt{3-x}}$
5. Evaluate  $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$
6. Evaluate  $\int_0^1 x(1-x)^n dx$
7. Evaluate  $\int_0^{2\pi} \sin^9 \frac{x}{4} dx$
8. Evaluate  $\int_0^{\pi/2} \sin^4 x \cos^2 x dx$

**7. INTEGRAL CALCULAS AND ITS APPLICATIONS  
(10 MARK QUESTIONS)**

**SLIP TEST – 24**

1. Derive the formula for the volume of a right circular cone with radius 'r' and height 'h'.
2. Find the length of the curve  $4y^2 = x^3$  between  $x=0$  and  $x=1$
3. Find the length of the curve  $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$
4. Find the perimeter of the circle with radius a.
5. Find the length of the curve  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$  between  $t=0$  and  $\pi$ .

**SLIP TEST – 25**

1. Show that the surface area of the solid obtained by revolving the arc of the curve  $y = \sin x$  from  $x=0$  to  $x=\pi$  about x-axis is  $2\pi\left[\sqrt{2} + \log(1 + \sqrt{2})\right]$

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2. Find the surface area of the solid generated by revolving the cycloid  $x = a(t + \sin t)$ ,  $y = a(1 + \cos t)$  about its base (x-axis).
3. Find the surface area of the solid generated by revolving the arc of the parabola  $y^2 = 4ax$ , bounded by its latus Rectum about x-axis.
4. Prove that the curved surface area of a sphere of radius  $r$  intercepted between two parallel planes at a distance  $a$  and  $b$  from the centre of the sphere is  $2\pi r(b-a)$  and hence deduce the surface area of the sphere. ( $b > a$ ).

### 8. DIFFERENTIAL EQUATIONS

#### (10 MARK QUESTIONS)

#### SLIP TEST – 26

1. In a certain chemical reaction the rate of conversion of a substance at time  $t$  is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the first substance was there initially?
2. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1 hour. Show that the number of bacteria at the end of five hours will be  $3^5$  times of the population at initial time
3. The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020?  
[  $\log_e \left( \frac{16}{13} \right) = .2070$ ,  $e^{42} = 1.52$  ]
4. A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm. (  $\log_e 2 = 0.6931$  )
5. Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years. [ Take  $A_0$  as the initial amount ].

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- A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. [ Take  $e^{0.08} = 1.0833$  ].
- The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal ? (  $\log_e 2 = 0.6931$  )

### SLIP TEST – 27

- A cup of coffee at temperature  $100^\circ \text{C}$  is placed in a room whose temperature is  $15^\circ \text{C}$  and it cools to  $60^\circ \text{C}$  in 5 minutes. Find its temperature after a further interval of 5 minutes.
- For a postmortem report, a doctor requires to know approximately the time of death of the deceased. He records the first temperature at 10.00 a.m. to be  $93.4^\circ \text{F}$ . After 2 hours he finds the temperature to be  $91.4^\circ \text{F}$ . If the room temperature (which is constant) is  $72^\circ \text{F}$ , estimate the time of death. (Assume normal temperature of a human body to be  $98.6^\circ \text{F}$ ).  $\left[ \log_e \frac{19.4}{21.4} = -0.0426 \times 2.303 \text{ and } \log_e \frac{26.6}{21.4} = 0.00945 \times 2.303 \right]$
- A drug is excreted in a patient's urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of drug at time  $t = 0$ , which is excreted at a Rate of  $-3t^{1/2}$  mg/h.
  - What is the general equation for the amount of drug in the patient at time  $t > 0$  ?
  - When will the patient be drug free?
- solve  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$  when  $x = \log 2$ ,  $y = 0$  and  $x = 0$ ,  $y = 0$
- solve  $(D^2 - 6D + 9)y = x + e^{2x}$
- Solve the differential equation  $(D^2 - 1)y = \cos 2x - 2 \sin 2x$

### 9. DISCRETE MATHEMATICS (6 MARK QUESTIONS)

#### SLIP TEST – 28

- Show that  $p \rightarrow q \equiv (\sim p) \vee q$
- Show that  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- Show that  $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$
- Show that  $\sim(p \wedge q) \equiv ((\sim p) \vee (\sim q))$

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5. Show that  $p \rightarrow q$  and  $q \rightarrow p$  are not equivalent.
6. Show that  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology
7. Construct the truth table for  $(p \wedge q) \vee (\sim r)$
8. Construct the truth table for  $(p \vee q) \wedge r$

### SLIP TEST – 29

1. Construct the truth table for  $(p \wedge q) \vee [\sim(p \wedge q)]$
2. Construct the truth table for  $(p \vee q) \vee r$
3. Construct the truth table for  $(p \wedge q) \vee r$
4. Show that  $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
5. Show that  $((\sim p) \vee (\sim q)) \vee p$  is a tautology.
6. Show that  $((\sim q) \wedge p) \wedge q$  is a contradiction
7. Use the truth table to determine whether the statement  $((\sim p) \vee q) \vee (p \wedge (\sim q))$  is a tautology.
8. Use the truth table to determine whether the statement  $(p \wedge (\sim p)) \wedge ((\sim q) \wedge p)$  is a tautology.

### SLIP TEST – 30

1. Prove that  $(\mathbb{Z}, +)$  is an infinite abelian group.
2.  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$  form an abelian group, under multiplication of matrices.
3. State and prove cancellation laws on groups.
4. State and prove reversal law on inverse of a group.
5. Show that the cube roots of unity forms a finite abelian group under multiplication.
6. Prove that the set of all 4<sup>th</sup> roots of unity forms an abelian group under multiplication.
7. Show that the set of all 2 X 2 non-singular matrices forms a non-abelian infinite group under matrix multiplication, (where the entries belong to  $\mathbb{R}$ ).
8. Show that the set of all non-zero complex numbers is an abelian group under the usual multiplication of complex numbers.

9. DISCRETE MATHEMATICS (10 MARK QUESTIONS)

SLIP TEST – 31

1. Show that  $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \right\}$  Where  $\omega^3 = 1, \omega \neq 1$  form a group with respect to matrix multiplication.
2. Show that the set  $\{[1], [3], [4], [5], [9]\}$  forms an abelian group under multiplication modulo 11.
3. Show that  $(\mathbb{Z}_7 - \{0\}, \cdot_7)$  forms a group.
4. Prove that the set of four functions  $f_1, f_2, f_3, f_4$  on the set of non-zero complex numbers  $C - \{0\}$  defined by  $f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$  and  $f_4(z) = -\frac{1}{z} \forall z \in C - \{0\}$  forms an abelian group with respect to the composition of functions.
5. Show the set G of all matrices of the form  $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$  where  $x \in R - \{0\}$ , is a group under matrix multiplication.

SLIP TEST – 32

1. Show that the set of all matrices of the form  $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}, a \in R - \{0\}$  forms an abelian group under matrix multiplication.
2. Show that  $(\mathbb{Z}, *)$  is an infinite abelian group where  $*$  is defined as  $a*b = a + b + 2$ .
3. Show that the set G of all positive rational forms a group under the composition  $*$  defined by  $a*b = \frac{ab}{3}$  for all  $a, b \in G$ .
4. Show that the set G of all rational numbers except -1 forms an abelian group with respect to the operation  $*$  given by  $a*b = a + b + ab$  for all  $a, b \in G$ .
5. Let G be the set of all rational numbers except 1 and  $*$  be defined on G by  $a*b = a + b - ab$  for all  $a, b \in G$ . Show that  $(G, *)$  is an infinite abelian group.

SLIP TEST – 33

1. Show that the  $n$ th roots of unity form an abelian group of finite order with usual multiplication.
2. Show that  $(Z_n, +_n)$  forms group.
3. Show that the set  $G = \{2^n / n \in Z\}$  an abelian group under multiplication.
4. Show that the set  $M$  of complex numbers  $z$  with the condition  $|z| = 1$  forms a group with respect to the operation of multiplication of complex numbers.
5. Show that the set  $G = \{a + b\sqrt{2} / a, b \in Q\}$  is an infinite abelian group with respect to addition.

10. PROBABILITY DISTRIBUTIONS

(10 MARK QUESTIONS)

SLIP TEST – 34

1. A random variable  $X$  has the following probability mass function

|            |     |      |      |      |      |       |       |
|------------|-----|------|------|------|------|-------|-------|
| $X$        | 0   | 1    | 2    | 3    | 4    | 5     | 6     |
| $P(X = x)$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

- (1) Find  $k$ .
  - (2) Evaluate  $P(X < 4)$ ,  $P(X \geq 5)$  and  $P(3 < X \leq 6)$
  - (3) What is the smallest value of  $x$  for which  $P(X \leq x) > \frac{1}{2}$ ?
2. An urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn. (i) With replacement (ii) without replacement

3. The total life time (in year) of 5 year old dog of a certain breed is a Random Variable

whose distribution function is given by  $F(x) = \begin{cases} 0 & , \text{ for } x \leq 5 \\ 1 - \frac{25}{x^2} & , \text{ for } x > 5 \end{cases}$  Find the probability that

such a five year old dog will live (i) beyond 10 years (ii) less than 8 years (iii) anywhere between 12 to 15 years.

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4. The probability density function of a random variable X is

$$f(x) = \begin{cases} kx^{\alpha-1}e^{-\beta x^\alpha} & , x, \alpha, \beta > 0 \\ 0 & , \text{elsewhere} \end{cases}$$

Find (i) k (ii)  $P(X > 10)$

5. If the number of incoming buses per minute at a bus terminus is a random variable having a Poisson distribution with  $\lambda = 0.9$ , find the probability that there will be

- Exactly 9 incoming buses during a period of 5 minutes.
  - Fewer than 10 incoming buses during a period of 8 minutes.
  - At least 14 incoming buses during a period of 11 minutes.
6. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year

$$[e^{-3} = 0.0498]. \quad \text{SLIP TEST - 35}$$

1. Find  $c$ ,  $\mu$  and  $\sigma^2$  of the normal distribution whose probability function is given by

$$f(x) = ce^{-x^2+3x} \quad -\infty < X < \infty.$$

2. Obtain  $k$ ,  $\mu$  and  $\sigma^2$  of the normal distribution whose probability distribution

function is given by  $f(x) = ke^{-2x^2+4x} \quad -\infty < X < \infty.$

3. If X is normally distributed with mean 6 and standard deviation 5 find.

(i)  $P(0 \leq X \leq 8)$  (ii)  $P(|X - 6| < 10)$

4. The mean score of 1000 students for an examination is 34 and S.D. is 16. (i) How many candidates can be expected to obtain marks between 30 and 60 assuming the normality of the distribution and (ii) determine the limit of the marks of the central 70% of the candidates.

5. The air pressure in a randomly selected tyre put on a certain model new car is normally distributed with mean value 31 psi and standard deviation 0.2 psi.

(i) What is the probability that the pressure for a randomly selected tyre (a) between 30.5 and 31.5 psi (b) between 30 and 32 psi

(ii) What is the probability that the pressure for a randomly selected tyre exceeds 30.5 psi?

6. The mean weight of 500 male students in a certain college is 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh (i) between 120 and 155 pounds (ii) more than 185 pounds.