IMPORTANT SIX MARKS

APPLICATIONS OF MATRICES AND DETERMINANTS

1. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result $A(adj\ A) = (adj\ A)A = |A|.I$.
2. If $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ verify that (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) $(AB)^T = B^T A^T$.
3. Show that the adjoint of $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is $3A^T$.
4. For $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$, Show that $A = A^{-1}$.
5. Solve by matrix inversion method of each of the following system of linear equations :$2x - y = 7,\quad 3x - 2y = 11$.
6. Solve by matrix inversion method of each of the following system of linear equations :$7x + 3y = -1,\quad 2x + y = 0$.
7. Find the rank of the matrices: $\begin{bmatrix} 3 & 1 & 2 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 1 & 3 & 0 \end{bmatrix}$.
8. Find the rank of the matrices: $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$.
9. Find the rank of the matrices: $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$.
10. Find the rank of the matrices: $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$.
11. Solve the following non-homogeneous system of linear equation by determinant method: $4x + 5y = 9,\ 8x + 10y = 18$.
12. Examine the consistency of the following system of equations. If it is consistent solve the same. $x + y + z = 7, x + 2y + 3z = 18, y + 2z = 6$.
13. Examine the consistency of the following system of equations. If it is consistent solve the same. $x - 4y + 7z = 14, 3x + 8y - 2z = 13, 7x - 8y + 26z = 5$.

14. If $A = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$, verify that the result $A(adj\ A) = (adj\ A)A = |A|I_2$.

15. Find the inverses of the following matrices $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$.

16. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that$(AB)^{-1} = B^{-1}A^{-1}$.

17. Solve by matrix inversion method $x + y = 3, 2x + 3y = 8$.

18. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1\ 3 \\ 2 & -1 & 3\ 4 \\ 5 & -1 & 711 \end{bmatrix}$.

19. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

20. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 3-1 \\ 2 & 4 & 6-2 \\ 3 & 6 & 9-3 \end{bmatrix}$.

21. Find the rank of the matrix $\begin{bmatrix} 4 & 2 & 13 \\ 6 & 3 & 47 \\ 2 & 1 & 01 \end{bmatrix}$.

22. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5-1 \\ 1 & -2 & 1\ -5 \\ 1 & 5 & -7\ 2 \end{bmatrix}$.

23. Solve the following system of linear equations by determinant method.$2x + 3y = 8, 4x + 6y = 16$.

24. Solve the following non-homogeneous equation of three unknowns $2x + 2y + z = 5, x - y + z = 1, 3x + y + 2z = 4$.

25. Solve the following non-homogeneous equation of three unknowns $x + y + 2z = 4, 2x + 2y + 4z = 8, 3x + 3y + 6z = 10$.

VECTOR ALGEBRA

1. Prove by vector method if the diagonals of a parallelogram are equal then it is rectangle.
2. Forces and magnitudes 3 and 4 units acting in the directions $\overrightarrow{6i} + \overrightarrow{2j} + \overrightarrow{3k}$ and $\overrightarrow{3i} - \overrightarrow{2j} + \overrightarrow{6k}$ respectively act on a particle which is displaced from the point (2, 2, -1) to (4, 3, 1). Find the work done by the forces.

3. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.
4. Forces $\overrightarrow{2i} + \overrightarrow{7j},\ \overrightarrow{2i} - \overrightarrow{5j} + \overrightarrow{6k}, -\vec{i} + \overrightarrow{2j} - \vec{k}$. Find the moment of the resultant of three forces acting at P about the point Q whose position vector is $\overrightarrow{6i} + \vec{j} - \overrightarrow{3k}$.
5. Find the magnitude and direction cosines of the moment about point (1, -2, 3) of a force $\overrightarrow{2i} + \overrightarrow{3j} + \overrightarrow{6k}$.
6. Show that the points $(1,3,1), (1,1,-1), (-1,\ 1,1), (2,2,-1)$ are lying on the same plane. (Hint: it is enough to prove any three vectors formed by these four points are coplanar).
7. If$\vec{a} = \overrightarrow{2i} + \overrightarrow{3j} - \overrightarrow{5k}, \vec{b} = -\vec{i} + \vec{j} + \vec{k}\ and\ \vec{c} = \overrightarrow{4i} - \overrightarrow{2j} + \overrightarrow{3k}$, show that $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a}(\vec{b} \times \vec{c})$.
8. Prove that $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) + (\vec{b} \times \vec{c}).(\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}).(\vec{b} \times \vec{d}) = 0$.
9. Find the vector and Cartesian equation of the line through the point $(3,-4,-2)$ and parallel to the vector $\overrightarrow{9i} + \overrightarrow{6j} + \overrightarrow{2k}$.
10. Find the vector and Cartesian equation of the line joining the points$(1,-2,1)\, and\ (0,-2,3)$.
11. Show that the following lines are skew lines:$\vec{r} = (\overrightarrow{3i} + \overrightarrow{5j} + \overrightarrow{7k}) + t(\vec{i} - \overrightarrow{2j} + \vec{k})$and $\vec{r} = (\vec{i} + \vec{j} + \vec{k}) + s(\overrightarrow{7i} + \overrightarrow{6j} + \overrightarrow{7k})$.
12. Show that $(2,-1,3), (1,-1,0)\, and\ (3,\ -1,6)$ are collinear.
13. Can you draw a plane through the given two lines? Justify your answer. $\vec{r} = (\vec{i} + \overrightarrow{2j} - \overrightarrow{4k}) + t(\overrightarrow{2i} + \overrightarrow{3j} + \overrightarrow{6k})and\vec{r} = (\overrightarrow{3i} + \overrightarrow{3j} - \overrightarrow{5k}) + s(-\overrightarrow{2i} + \overrightarrow{3j} + \overrightarrow{8k})$.
14. Find the meeting point of the line $\vec{r} = (\overrightarrow{2i} + \vec{j} - \overrightarrow{3k}) + t(\overrightarrow{2i} - \vec{j} - \vec{k})$ and the plane$x - 2y + 3z + 7 = 0$.
15. Find the vector equation of the sphere with centre having position vector $\overrightarrow{2i} - \vec{j} + \overrightarrow{3k}$ and the radius 4 units. Also find the equation in Cartesian form.
16. Find the vector and Cartesian equation of the sphere on the join of the points A and B having position vectors$\overrightarrow{2i} + \overrightarrow{6j} - \overrightarrow{7k}$ and $-\overrightarrow{2i} + \overrightarrow{4j} - \overrightarrow{3k}$ respectively as a diameter. Find also the centre and radius of the sphere.
17. Obtain the vector and Cartesian equation of the sphere whose centre is $(1,-1,1)$ and radius is the same as that of the sphere$\left|\vec{r} - (\vec{i} + \vec{j} + \overrightarrow{2k})\right| = 5$.

18. Show that diameter of a sphere subtends a right angle at a point on the surface.
19. Angle in a semi-circle is right angle. Prove by vector method.
20. Diagonals of a rhombus are right angles. Prove by vector methods.
21. Prove that the area of quadrilateral $ABCD$ is $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$ where AC and BD are its diagonals.

If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of the triangle ABC, then prove that the area of the triangleABC is $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$ deduce the condition for the points $\vec{a}, \vec{b}, \vec{c}$ to be collinear.

22. With usual notation prove that$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
23. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$.
24. Prove that $\left[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$.
25. Find the vector and Cartesian equations of the straight line passing through the points $(-5, 2, 3) and\ (4, -3, 6)$.
26. Find the shortest distance between the parallel lines$\vec{r} = (\vec{\imath} - \vec{\jmath}) + t\left(\overrightarrow{2\imath} - \vec{\jmath} + \vec{k}\right)$and $\vec{r} = \left(\overrightarrow{2\imath} + \vec{\jmath} + \vec{k}\right) + s\left(\overrightarrow{2\imath} - \vec{\jmath} + \vec{k}\right)$.
27. Show that the two lines $\vec{r} = (\vec{\imath} - \vec{\jmath}) + t\left(\overrightarrow{2\imath} + \vec{k}\right)$ and $\left(\overrightarrow{2\imath} - \vec{\jmath}\right) + s\left(\vec{\imath} + \vec{\jmath} - \vec{k}\right)$ are skew lines and find the distance between them.
28. Find the vector and Cartesian equation of the sphere whose center is $(1, 2, 3)$ and which passes through the point $(5, 5, 3)$.
29. Find the equation of the sphere on the join of the points A and B having the position vectors$\overrightarrow{2\imath} + \overrightarrow{6\jmath} - \overrightarrow{7k}$ and $\overrightarrow{2\imath} - \overrightarrow{4\jmath} + \overrightarrow{3k}$ respectively as a diameter.
30. Find the coordinates of the center and the radius of the sphere whose vector equation is$\vec{r}^2 - \vec{r}.\left(\overrightarrow{8\imath} - \overrightarrow{6\jmath} + \overrightarrow{10k}\right) - 50 = 0$.

COMPLEX NUMBERS

1. Find the real values x and y for which the following equations are satisfied. $\sqrt{x^2 + 3x + 8} + (x + 4)i = y(2 + i)$.

2. For what values of x and y, the numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are complex conjugate of each other?
3. Find the square root of $(-8 - 6i)$.
4. Prove that the triangle formed by the points representing the complex numbers $(7 + 5i), (5 + 2i), (4 + 7i) and\ (2 + 4i)$ form a parallelogram. (Plot points and use midpoint formula).
5. If $\arg(z - 1) = \frac{\pi}{6}$ and the $\arg(z + 1) = 2\ \frac{\pi}{3}$ then prove that $|z| = 1$.
6. P represents the variable complex number z. Find the locus of P, if $|z - 5i| = |z + 5i|$.
7. P represents the variable complex number z. Find the locus of P, if $|2z - 3| = 2$.
8. Solve the equation $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$ if $3 + i$ is a root.
9. Solve the equation $x^4 - 4x^3 + 11x^2 - 14x + 10 = 0$ if one root is $1 + 2i$
10. Solve : $6x^4 - 25x^3 + 32x^2 + 3x - 10 = 0$ given that one of the roots is $2 - i$
11. If $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = A + iB$, prove that $({a_1}^2 + {b_1}^2)({a_2}^2 + {b_2}^2) \ldots ({a_n}^2 + {b_n}^2) = A^2 + B^2$.
12. Find the square root of $(-7 + 24i)$.
13. Show that the point representing the complex numbers $7 + 9i, -3 + 7i, 3 + 3i$ form a right angled triangle on the Argand diagram.
14. Simplify $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$
15. If n is positive integer, prove that $\left(\frac{(1+\sin\theta + i\cos\theta)}{(1+\sin\theta - i\cos\theta)}\right)^n = \cos n\left(\frac{\pi}{2} - \theta\right) + i\sin n\left(\frac{\pi}{2} - \theta\right)$
16. If n is positive integer, prove that $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1}\cos\frac{n\pi}{6}$.
17. Prove that $\arg\left(\frac{z_1}{Z_2}\right) = \arg z_1 + \arg Z_2$.
18. Prove that $\arg\left(\frac{z_1}{Z_2}\right) = \arg z_1 - \arg Z_2$.
19. State and prove the triangle inequality.

ANALYTICAL GEOMETRY

1. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the distance of the focus from the centre of the reflector.

2. The focus of a parabolic mirror is at a distance of 8cm from its centre(vertex). If the mirror is 25cm deep, find the diameter of the mirror.
3. Find the equation of the two tangents that can be drawn (i) from the point (2, -3) to the parabola $y^2 = 4x$. (ii) from the point (1, 3) to the ellipse $4x^2 + 9y^2 = 36$. (iii) from the point (1, 2) to the hyperbola $2x^2 - 3y^2 = 6$.
4. Find the equation of the asymptotes to the hyperbola $8x^2 + 10xy - 3y^2 - 2x + 4y - 2 = 0$.
5. Find the equation of the hyperbola if the asymptotes are $2x + 3y - 8 = 0$ and $3x - 2y + 1 = 0$ and (5, 3) is a point of hyperbola.
6. Find the angle between the asymptotes of the hyperbola$24x^2 - 8y^2 = 27$.
7. A standard rectangular hyperbola has its vertices at (5, 7) and (-3, -1). Find its equation and asymptotes.
8. Find the equation of the rectangular hyperbola which has its centre at (2, 1), one of its asymptotes $3x - y - 5 = 0$ and which passes through the point (1, -1).
9. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of a constant.
10. The headlight of a motor vehicle is a parabolic reflector of diameter 12 cm and depth 4 cm. find the position of the bulb on the axis of the reflector for effective functioning of the headlight.
11. A reflecting telescope has a parabolic mirror for which the distance from the vertex to the focus is 9 mts. If the distance across (diameter) the top of the mirror is 160 cm, how deep is the mirror at the middle?
12. Find the equation of te ellipse whose one of the foci is (2, 0) and the corresponding directrix is $x = 8$ and the eccentricity is $\frac{1}{2}$.
13. Find the equation of the ellipse with focus (-1, -3), directrix$x - 2y = 0$ and eccentricity$\frac{4}{5}$.
14. Find the equation of hyperbola whose directrix is $2x + y = 1$ focus (1, 2) and eccentricity $\sqrt{3}$.
15. Find the equation of the hyperbola whose centre is (2, 1), one of the foci is (8, 1) and the corresponding directrixis$x = 4$.

16. Find te separate equations of the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
17. Find the equation of the hyperbola which passes through the point (2, 3) and has the asymptotes $4x + 3y - 7 = 0$and $x - 2y = 1$.
18. Find the angle between the asymptotes of the hyperbola$3x^2 - y^2 - 12x - 6y - 9 = 0$.
19. Find the angle between the asymptotes to the hyperbola$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
20. Prove that the product of the perpendiculars from any point on the hyperbola$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2+b^2}$.
21. The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts *a,b* and the normal at the point makes intercepts *p, q* on the axes. Prove that *ap+bq=0*.
22. Show that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact.

DIFFERENTIAL CALCULUS APPLICATIONS – I

1. A particle of unit mass moves so that displacement after *t* secs is given by $x = 3\cos(2t - 4)$. Find the acceleration and kinetic energy at the end of 2 secs. $\left[K.E = \frac{1}{2}mv^2, m\ is\ mass\right]$.
2. Newton's law cooling is given by $\theta_0^{\circ} e^{-kt}$, where the excess of temperature at zero time is $\theta_0^{\circ}C$ and at the time t seconds is $\theta^0 C$. Determine the rate of change of temperature after 40 *s*, given that $\theta_0 = 16°C$and $k = -0.03$. $[e^{1.2} = 3.3201]$.
3. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/sec. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is$\frac{\pi}{6}$.
4. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$ the tangent is parallel to (i) *x* – axis (ii) *y* – axis.
5. Prove that the curve$2x^2 + 4y^2 = 1$ and $6x^2 - 12y^2 = 1$ cut each other at right angles.
6. At what angle θ do the curves $y = a^x$ and $y = b^x$intersect $(a \neq b)$?
7. Using the Rolle's theorem find the points on the curve$y = x^2 + 1, -x \leq x \leq 2$.
8. Verify Lagrange's law of mean for the following functions$f(x) = 2x^3 + x^2 - x - 1, [0,2]$.
9. Verify Lagrange's law of mean for the following functions $f(x) = x^3 - 5x^2 - 3x, [1,3]$.

10. Obtain the Maclaurin's Series expansion for :$\frac{1}{1+x}$.
11. Evaluate the limit for the following if exists, $\lim_{x\to 1} x^{\frac{1}{x-1}}$.
12. Evaluate the limit for the following if exists, $\lim_{x\to 0+} x^{x}$.
13. Find the intervals on which *f* is decreasing or increasing: $f(x) = 20 - x - x^2$.
14. Find the intervals on which *f* is decreasing or increasing: $f(x) = 20 - x - x^2$.
15. Find the intervals on which *f* is decreasing or increasing: $f(x) = x^3 + x + 1$.
16. Find the intervals on which *f* is decreasing or increasing: $f(x) = sin\,^4x + cos^4x\ in\ \left[0, \frac{\pi}{2}\right]$.
17. Find the absolute maximum and absolute minimum values of *f* on the given interval: $f(x) = \sqrt{9 - x^2},\ [-1, 2]$.
18. Find the absolute maximum and absolute minimum values of *f* on the given interval: $f(x) = \frac{x}{x+1},\ [1, 2]$.
19. Find two numbers whose sum is 100 and whose product is maximum.
20. Find two positive numbers whose product is 100 and whose sum is minimum.
21. Find the intervals of concavity and the points of inflection of the following functions: $f(x) = 2x^3 + 5x^2 - 4x$.
22. Find the equations of the tangent and normal to the curve $y = x^3$.
23. Find the equations of the tangent and normal to the curve $y = x^2 - x - 2\ at\ the\ point\ (1, -2)$.
24. Show that $x^2 - y^2 = a^2$ and $xy = c^2$ cut orthogonally.
25. Verify Rolle's theorem for the following: $f(x) = e^x \sin x,\ 0 \le x \le \pi$.
26. Apply Rolle's theorem to find points on curve $y = -1 + \cos x$, where the tangent is parallel to *x* – axis in $[0, 2\pi]$.
27. Find the Maclaurin's series for: $\log_e(1 + x)$.
28. Find the Maclaurin's series for: $arc\tan x\ \ or\ \ \tan^{-1} x$.
29. Evaluate: $\lim_{x\to 0}\left(cosec\ x - \frac{1}{x}\right)$.
30. The current at time *t* in a coil with resistance *R*, inductance *L* and subjected to a constant electromotive force *E* is given by $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$. Obtain a suitable formula to be used when *R* is very small.

31. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing $[0, 1]$.
32. Find the intervals in which $f(x) = 2x^3 + x^2 - 20x$ is increasing or decreasing.
33. Discuss the monotonicity of the function$f(x) = \sin x, x \in [0,2\pi]$.
34. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 1, -\frac{1}{2} \leq x \leq 4$.
35. Determine the points of inflection if any, of the function $y = x^3 - 3x + 2$.
36. Test for points of inflection of the curve$y = \sin x, x \in (0, 2\pi)$.

DIFFERENTIAL CALCULUS APPLICATIONS – II

1. Use differentials to find an approximate value for the given number: $\sqrt{36.1}$.
2. Use differentials to find an approximate value for the given number: $\frac{1}{10.1}$.
3. Using chain rule find$\frac{dw}{dt}$ for each of the following$w = e^{xy}$ *where* $x = t^2, y = t^3$.
4. Using chain rule find $\frac{dw}{dt}$ for each of the following $w = \frac{x}{(x^2+y^2)}$ *where* $x = \cos t, y = \sin t$
5. Using chain rule find $\frac{dw}{dt}$ for each of the following $w = \log(x^2 + y^2)$ *where* $x = e^t, y = e^{-t}$.
6. Using chain rule find $\frac{dw}{dt}$ for each of the following $w = xy + z$ *where* $x = \cos t, y = \sin t, z = t$.
7. Find $\frac{\partial w}{\partial r}$ *and* $\frac{\partial w}{\partial \theta}$if $w = \log(x^2 + y^2)$ *where* $x = r\cos\theta, y \sin\theta$.
8. Find $\frac{\partial w}{\partial u}$ *and* $\frac{\partial w}{\partial v}$if $w = x^2 + y^2$ *where* $x = u^2 - v^2, y = 2uv$.
9. Find $\frac{\partial w}{\partial u}$ *and* $\frac{\partial w}{\partial v}$if $w = \sin^{-1} xy$ *where* $x = u + v, y = u - v$.
10. Using the Euler's theorem prove the following: $u = xy^2 \sin\left(\frac{x}{y}\right)$, *show that* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$.
11. Using the Euler's theorem prove the following: if *u* is a homogeneous function of *x* and *y* degree *n*, prove that $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$.
12. Using the Euler's theorem prove the following: If $V = ze^{ax+by}$ and *z* is a homogeneous function of degree *n* in *x* and *y* prove that $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = (ax + by + n)V$.

13. If $u = \log(\tan x + \tan y + \tan z)$, prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$.
14. If $U = (x - y)(y - z)(z - x)$ then show that $U_x + U_y + U_z = 0$.
15. Suppose that ye^{x^2} where $x = 2t$ and $y = 1 - t$ then find $\frac{dz}{dt}$.
16. If $w = x + 2y + z^2$and $x = \cos t\,; y = \sin t\,; z = t\; Find\, \frac{dw}{dt}$.
17. If u is a homogeneous function of x and$y$ of degree n, prove that $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n - 1)\frac{\partial u}{\partial x}$.
18. Use the differentials to find an approximate value for $\sqrt[3]{65}$.
19. The time of swing T of a pendulum is given by $T = \sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of the pendulum l changes from 32.1 to 32cm.
20. Show that the percentage error in the n^{th}root of a number is approximately $\frac{1}{n}$ times the percentage error in thenumber.

INTEGRAL CALCULUS AND ITS APPLICATIONS

1. Evaluate the following problems using second fundamental theorem: $\int_0^1 \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$
2. Evaluate the following problems using second fundamental theorem: $\int_0^1 x^2 e^2 dx$.
3. Evaluate the following problems using properties of integration: $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$.
4. Evaluate the following problems using properties of integration: $\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx$.
5. Evaluate the following problems using properties of integration: $\int_0^1 x(1-x)^{10} dx$.
6. Evaluate the following problems using properties of integration: $\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{dx}{1+\sqrt{\tan x}}$.
7. Evaluate: $\int_0^{\frac{\pi}{4}} \cos^8 2x\, dx$.
8. Evaluate: $\int_0^{\frac{\pi}{4}} \sin^7 3x\, dx$.
9. Evaluate: $\int_0^1 x\, e^{-2x} dx$.
10. Evaluate: $\int_0^1 x^6\, e^{-\frac{x}{2}} dx$.
11. Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.

12. Find the area of the circle whose radius is *a*.

13. Find the volume of the solid that results when the region enclosed by the given curves: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is resolved about major axis $a>b>0$.

14. Evaluate $\int_0^1 x(1-x)^n dx$.

15. Evaluate $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$.

16. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$.

17. Evaluate: $\int_0^{\frac{\pi}{6}} \cos^7 3x\, dx$.

18. Evaluate: $\int_0^{2\pi} \sin^9 \frac{x}{4} dx$.

19. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x\, dx$.

20. Find the area bounded by the curve $y=\sin 2x$ between the ordinates $x=0, x=\pi$ *and the* $x-$*axis*.

21. Find the volume of the solid that results when ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b>0)$ is resolved about the minor axis.

22. Find the volume of the solid generated when the region enclosed by $y=\sqrt{x}, y=2$ and $x=0$ is resolved about the y –axis.

DIFFERENTIAL EQUATIONS

1. Find the order and degree of the following differential equations $\left(\frac{dy}{dx}\right)^2+x=\frac{dx}{dy}+x^2$.
2. Find the differential equation that will represent the family of all circles having centres on the x – axis and the radius is unity.
3. Solve the following $(x^2+5x+7)dy+\sqrt{9+8y-y^2}dx=0$.
4. Solve the following $\frac{dy}{dx}=\sin(x+y)$.
5. Solve the following $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$.
6. Solve the following $(x^2+y^2)dy=xy\, dx$.
7. Solve the following $x^2\frac{dy}{dx}= y^2+2xy$ *given that* $y=1,$ *when* $x=1$.
8. Solve the following $\frac{dy}{dx}+y=x$.

9. Solve the following$\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{(x^2+1)^2}$.

10. Solve the following $(1+x^2)\frac{dy}{dx}+2xy=\cos x$.

11. Solve the following $\frac{dy}{dx}+xy=x$.

12. Solve the following differential equations: $(D^2+14D+49)y=x^2$.

13. Solve the following differential equations: $(D^2-13D+12)y=e^{-2x}+5e^x$.

14. Solve the following differential equations:$(D^2-2D-3)y=\sin\ x\cos\ x$.

15. Solve the following differential equations:$(D^2+5)y=cos^2x$.

16. Solve :$\frac{dy}{dx}+y\cot x=2\cos x$.

17. Solve :$\frac{dy}{dx}+2y\tan x=\sin x$.

18. Solve $xdy-ydx=\sqrt{x^2+y^2}dx$.

19. Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ given that $y=1$, where $x=0$.

20. Solve: $(2D^2+5D+2)y=e^{-\frac{1}{2}x}$.

21. Solve: $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$.

22. Solve: $(D^2-6D+9)y=e^x$.

23. Solve: $(D^2+6D+8)y=e^{-2x}$.

24. Solve:$(D^2-4)y=\sin 2x$.

25. Solve: $(D^2+4D+13)y=\cos 3x$.

26. Solve: $(D^2-3D+2)y=x$.

27. A bank pays interest by continuous compounding that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. $[Take\ e^{.08}\approx 1.0833]$.

28. Solve $3e^x\tan y\,dx+(1+e^x)Sec^2\,ydy=0$.

29. Solve $\frac{dy}{dx}+\left(\frac{1-y^2}{1-x^2}\right)^{\frac{1}{2}}=0$.

DISCRETE MATHEMATICS

Construct the truth tables for the following statements:

1. $p \vee (\sim q)$
2. $(\sim p) \wedge (\sim q)$
3. $\sim(p \vee q)$
4. $(p \vee q) \vee (\sim p)$
5. $(p \wedge q) \vee (\sim q)$
6. $\sim(p \vee (\sim q))$
7. $(p \wedge q) \vee [\sim(p \wedge q)]$
8. $(p \wedge q) \vee (\sim q)$
9. $(p \vee q) \vee r$ (10) $(p \wedge q) \vee r$

Use the truth table to establish which of the following statements are tautologies and which are contradictions.

10. $((\sim p) \wedge q) \wedge p$
11. $(p \vee q) \vee (\sim(p \vee q))$
12. $(p \wedge (\sim q)) \vee ((\sim p) \vee q)$
13. $q \vee (p \vee (\sim q))$
14. $(p \wedge (\sim p)) \wedge ((\sim q) \wedge p)$
15. State and prove Cancellation Law.
16. Prove that (Z, +) is an infinite abelian group.
17. Prove that the set of all 4^{th} roots of unity forms an abelian group under multiplication.
18. Show that the set of all non – zero complex numbers is an abelian group under the usual multiplication of complex numbers.
19. Show that the set of four matrices $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ form an abelian group, under multiplication of matrices.
20. State and prove reversal law.

PROBABILITY DISTRIBUTIONS

1. Find the probability distribution of the number of sixes in throwing three dice once.
2. Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of queens.
3. A discrete random variable X has the following probability distributions.

X	0	1	2	3	4	5	6	7	8
P(x)	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) find the value of a (ii) Find P(x < 3) (iii) Find P(3<x < 7).

4. In an entrance examination a student has to answer all the 120 questions. Each question has four options and only one option is correct. A student gets 1 mark for a correct answer and loses half mark for a wrong answer. What is the expectation of the mark scored by a student if he chooses the answer to each question at random?
5. Two cards are drawn with replacement from a well shuffled desk of 52 cards. Find te mean and variance for the number of aces.
6. In a gambling game a man win Rs. 10 if he gets all heads or all tails and loses Rs. 5 if he gets 1 or 2 heads when 3 coins are tossed once. Find his expectation gain.
7. The probability distribution of a random variable X is given below

X)	*0*	*1*	*2*	*3*
P(X=x)	*0.1*	*0.3*	*0.5*	*0.1*

If $Y = X^2 + 2X$ find the mean and variance of Y.

8. Find the Mean and Variance for the following probability density functions (i)$f(x) = \begin{cases} \frac{1}{24}, & -12 \leq x \leq 12 \\ 0, & Otherwise \end{cases}$.
9. Find the Mean and Variance for the following probability density functions (i)$f(x) = \begin{cases} \alpha e^{-\alpha x}, & if\ x > 0 \\ 0, & Otherwise \end{cases}$.
10. Find the Mean and Variance for the following probability density functions (i)$f(x) = \begin{cases} xe^{-x}, & if\ x > 0 \\ 0, & Otherwise \end{cases}$.
11. Four coins are tossed simultaneously. What is the probability of getting (a) exactly two heads (b) at least heads (c) at most two heads.

12. The overall percentage of passes in a certain examination is 80. If 6 candidates appear in the examination what is the probability that atleast 5 passes the examination.
13. 20% of bolts produced in a factory are found to be defective. Find the probability that in a sample of 10 bolts chosen at random exactly 2 will be defective using (i) Binomial distribution (ii) Poisson distribution. [$e^{-2} = 0.1353$].
14. Alpha particles are emitted by a radioactive source at an average rate of 5in a 20 minutes interval. Using a Poisson distribution find the probability that there will be (i) 2 emission (ii) at least 2 emission in a particular 20 minutes interval.[$e^{-5} = 0.0067$].
15. Suppose that the amount of cosmic radiation to which a person is exposed when flying by jet across the United States is a random variable having a normal distribution with a mean of 4.35 m rem and a standard deviation of 0.59 m rem. What is the probability that a person will be exposed to more than 5.20 m rem of cosmic radiation of such a flight.
16. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.
17. If the height of 300 students are normally distributed with mean 64.5 inches and standard deviation 3.3 inches, find the height below which 99% of the student lie.
18. Marks in an aptitude test given to 800 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90.
19. An urn contains 4 white and 3 Red balls. Find the probability distribution of the number of red balls in three draws when a ball is drawn at random with replacement. Also find its mean and variance.
20. A game is played with a single fair die, A player wins Rs. 20 if a 2 turns up, Rs. 40 if a 4 turns up, loses Rs. 30 if a 6 turns up. While he neither wins nor loses if any other face turns up. Find the expected sum of money he can win.
21. In a continuous distribution the p.d.f of X is $f(x) = \begin{cases} \frac{3}{4}x(2-x), & 0 < x < 2 \\ 0, & otherwise \end{cases}$.
22. Find the mean and variance of the distribution$f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & Otherwise \end{cases}$.

23. Let Xbe a binomially distributed variable with mean 2 and standard deviation$\frac{2}{\sqrt{3}}$. Find the corresponding probability function.
24. A pair of dice is thrown 10 times. If getting a doublet is considered a success find the probability of (i) 4 success (ii) No success.
25. If a publisher of non-technical books takes a great pain to ensure that his books are free of typological errors, so that the probability of any given page containing atleast one such error is 0.005 and errors are independent from page to page (i) what is the probability that one of its 400 page novels will contain exactly one page with error. (ii) atmost three pages with errors.$[e^{-2} = 0.1353;\ e^{-0.2} = 0.819]$.
26. Suppose that the probability of suffering a side effect from a certain vaccine is 0.005. If 1000 persons are inoculated, find approximately the probability that (i) atmost 1 person suffer. (ii) 4, 5 or 6 persons suffer.$[e^{-5} = 0.0067]$.
27. In a Poisson distribution if$P(X = 2) = P(X = 3) find\ P(X = 5)$. $[GIven\ e^{-3} = 0.050]$.