IMPORTANT TEN MARKS

APPLICATIONS OF MATRICES AND DETERMINANTS

1. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of red chair is Rs.240, cost of a blue chair is Rs.260 and the cost of green chair is Rs.300. the total of chair is Rs.25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased.
2. Solve the following non – homogeneous system of linear equation by determinant method $\frac{1}{x}+\frac{2}{y}-\frac{1}{z}=1; \frac{2}{x}+\frac{4}{y}+\frac{1}{z}=5; \frac{3}{x}-\frac{2}{y}-\frac{2}{z}=0.$
3. Examine the consistency of the following system of equations. If it is consistent then solve the same

   (i). $4x+3y+6z=25$ $\quad x+5y+7z=13$ $\quad 2x+9yz=1$

   (ii) $x+y-z=1$ $\quad 2x+2y-2z=2$ $\quad -3x-3y+3z=-3$

4. Discuss the solutions of the system of equations for all the values of λ.

   $x+y+z=2,$ $\quad 2x+y-2z=2,$ $\quad \lambda x+y+4z=2$
5. For what values of k, the system of equations

   $kx+y+z=1,$ $\quad x+ky+z=1$ $\quad x+y+kz=1$ have

   (i) One Solution (ii) more than one solution (iii) no solution
6. Show that the equations $x+y+z=6,\ x+2y+3z=14,\ x+4y+7z=30$ are consistent and solve them.
7. Verify whether the given system of equations is consistent. If it is consistent, solve them.

   $2x+5y+7z=52,$ $\quad x+y+z=9,$ $\quad 2x+y-z=0$
8. For what value of μ the equations $x+y+3z=0,\ 4x+3y+\mu z=0,\ 2x+y+2z=0$ have a (i) trivial solution (ii) non – trivial solution.
9. A bag contains 3 types of coins namely Rs.1, Rs.2 and Rs.5. There are 30 coins amounting to Rs.100 in total. Find the number of coins in each category.
10. Verify whether the given system of equations is consistent. If it is consistent solve them: $x-2y+z=5, -x+y-z=-5,\ 2x-2y+2z=10.$

11. Investigate for what values of λ, μ the simultaneous equation $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution and (iii) infinite number of solutions.
12. Solve the following homogeneous linear equations $x+2y-5z=0, 3x+4y+6z=0, x+y+z=0$.

VECTOR ALGEBRA

1. If $\vec{a}=\overrightarrow{2i}+\overrightarrow{3j}-\vec{k},\ \vec{b}=-\overrightarrow{2i}+\overrightarrow{5k},\ \vec{c}=\vec{j}-\overrightarrow{3k}$ verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\,.\vec{b})\vec{c}$.
2. Verify $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\vec{b}\vec{d}]\vec{c}-[\vec{a}\vec{b}\vec{c}]\vec{d}$.
3. Show that the lines $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x-2}{1}=\frac{y-1}{2}=\frac{-z-1}{1}$ intersect and find their point of intersection.
4. Find the vector and Cartesian equation of the plane through the points (1, 2, 3) and (2, 3, 1) perpendicular to the plane $3x-2y+4z-5=0$.
5. Find the vector and Cartesian equation of the plane passing through the points with the position vectors $\overrightarrow{3i}+\overrightarrow{4j}+\overrightarrow{2k},\ \overrightarrow{2i}-\overrightarrow{2j}-\vec{k}$ and $\overrightarrow{7i}+\vec{k}$.
6. Derive the equation of the plane in intercept form.
7. Find the vector and Cartesian equation to the plane through the point (-1, 3, 2) and perpendicular to the planes $x+2y+2z=5$ and $3x+y+2z=8$.
8. Find the vector and Cartesian equation of the plane passing through the points A(1,-2,3) and B(-1, 2, -1) and is parallel to the line $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-1}{4}$.
9. Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{-2}$ and passing through the point (-1, 1, -1).
10. Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+2}{1}$.
11. Find the vector and Cartesian equation of the plane through the point (1, 3, 2) and parallel to the lines $\frac{x+1}{2}=\frac{y+2}{-1}=\frac{z+3}{3}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+2}{2}$.

12. Prove that $\cos(A-B) = Cos\ A\ Cos\ B + Sin\ A\ Sin\ B$.
13. Prove that $\cos(A+B) = Cos\ A\ Cos\ B - Sin\ A\ Sin\ B$.
14. Prove that $Sin(A+B) = Sin\ A\ Cos\ B + Cos\ A\ Sin\ B$.
15. Prove that $Sin(A-B) = Sin\ A\ Cos\ B - Cos\ A\ Sin\ B$.
16. Altitudes of a triangle are concurrent – prove by vector method.
17. Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$.
18. Find the vector and Cartesian equations of the plane through the points (2, -1, -3) and parallel to the lines $\frac{x-2}{3} = \frac{y-1}{-4} = \frac{z-3}{-4}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{2}$.
19. Find the vector and Cartesian equation of the plane passing through the points (-1, 1, 1) and (1, -1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.
20. Find the vector and Cartesian equations of the plane passing through the points (2, 2, -1), (3, 4, 2), (7, 0, 6).

COMPLEX NUMBERS

1. P represents the variable complex number z. find the locus P, if $Im\left[\frac{2z+1}{iz+1}\right] = -2$.
2. P represents the variable complex number z. find the locus P, if $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$.
3. If α and β are the roots of $x^2 - 2x + 4 = 0$, Prove that $\alpha^n - \beta^n = i2^{n+1}\sin\frac{n\pi}{3}$ and deduct $\alpha^9 - \beta^9$.
4. If α and β are the roots of the equation $x^2 - 2px + (p^2 + q^2) = 0$ and $\tan\theta = \frac{q}{y+p}$ show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha-\beta} = q^{n-1}\frac{\sin n\theta}{sin^n\ \theta}$.
5. Find all the values of $\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1.
6. Find all the values of $\left(-\sqrt{3} - i\right)^{\frac{2}{3}}$.
7. Solve: $x^4 - x^3 + x^2 - x + 1 = 0$.
8. P represents the variable complex number z, find the locus of P if (i) $Re\left(\frac{z+1}{z+i}\right) = 1$ (ii) $Re\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$.

9. If α and β are the roots of the equation $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$, show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}$.
10. Solve the equation $x^9 + x^5 - x^4 - 1 = 0$.
11. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$.
12. Find all the values of $\left(\sqrt{3} + i\right)^{\frac{2}{3}}$.

ANALYTICAL GEOMETRY

1. A cable of suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level, find the length of the support if the height of pillars are 55 mts.
2. Find the axis, vertex, focus, equation of the directrix, latus, rectum, length of the latus rectum for the following parabolas and hence sketch their graphs $y^2 + 8x - 6y + 1 = 0$.
3. Find the axis, vertex, focus, equation of the directrix, latus, rectum, length of the latus rectum for the following parabolas and hence sketch their graphs $x^2 - 6x - 12y - 3 = 0$.
4. Find the eccentricity, centre, foci, vertices, of the following ellipses and draw the diagram: $x^2 + 4y^2 - 8x - 16y - 68 = 0$.
5. Find the eccentricity, centre, foci, vertices, of the following ellipses and draw the diagram: $16x^2 + 9y^2 + 32x - 36y = 92$.
6. A kho-kho player in a practice session while running realizes that the sum of the distances from the two kho-kho poles from him is always 8m. Find the equations of the path traced by him if the distance between the poles is 6m.
7. A satellite is travelling around the earth in elliptical orbit having the earth at a focus and of eccentricity ½. The shortest distance that the satellite gets to the earth is 400 Kms. Find the longest distance that the satellite gets from the earth.
8. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi major axis is of the length 36 million miles and eccentricity of the orbit is 0.206. find (i) how close the mercury gets to sun? (ii) the greatest possible distance between mercury and sun.
9. Find the eccentricity, centre, foci, vertices, of the following hyperbolas and draw the diagram: $x^2 - 4y^2 + 6x + 16y - 11 = 0$.

10. Find the eccentricity, centre, foci, vertices, of the following hyperbolas and draw the diagram: $x^2 - 3y^2 + 6x + 6y + 18 = 0$.
11. Prove that the line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ and find its point of contact.
12. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse$x^2 + 3y^2 = 12$. Find the co-ordinates of the point of contract.
13. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, (2, 4) is the centre of the hyperbola and it passes through (2,0).
14. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line$x + 2y - 5 = 0$ and passes through the points (6,0) and (-3, 0).
15. Find the axis, vertex, focus, equation of the directrix, latus, rectum, length of the latus rectum for the following parabolas and hence sketch their graphs $y^2 - 8x + 6y + 9 = 0$.
16. Find the axis, vertex, focus, equation of the directrix, latus, rectum, length of the latus rectum for the following parabolas and hence sketch their graphs $x^2 - 2x + 8y + 17 = 0$.
17. The girder of a railway bridge is in the parabolic form with span 100 ft. and the highest point on the arch is 10ft. above the bridge. Find the height of the bridge at 10ft. to the left or right from the midpoint of the bridge.
18. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point find the angle of projection.
19. Assume that the water issuing from the end of the horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of the water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?
20. A comet is moving in a parabolic orbit around the sun is which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comets orbit (ii) how close does the comet come nearer to the sun? (take the orbit as open rightward).
21. A cable of suspension bridge hangs in the form of parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of

the cable on the towers are 200 ft above the roadway and the lowest point of the cable on the towers are 200 ft above the roadway and lowest point on the cable is 70 ft above the roadway. Find the vertical distance to the cable (Parallel to the roadway) from a pole whose height is 122 ft.

22. Find the eccentricity, centre, foci, vertices, of the following ellipses $36x^2 + 4y^2 - 72x + 32y - 44 = 0$.
23. An arch is in the form of semi-ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at the height of 10 feet above the base?
24. The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.
25. The orbit of the earth around the sun is elliptical in shape with sun at a focus. The semi major axis is of the length 92.9 million miles and eccentricity is 0.017. find how close the earth gets to sun and the greatest possible distance between the earth and sun.
26. A ladder length 15m moves with its end always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.
27. Find the eccentricity, centre, foci, vertices, of the hyperbola $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ and also trace the curve.
28. Find the eccentricity, centre, foci, vertices, of the hyperbola and draw the diagram: $9x^2 - 16y^2 + 36x + 32y + 164 = 0$

DIFFERENTIAL CALCULUS APPLICATIONS – I

1. At noon, Ship A is 100 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 Km/hr. how fast is the distance between the ships changing at 4.00 p.m.
2. Two sides of a triangle have length 12 m and 15 m. the angle between them is increasing at a rate $2°\ /min$. How fast is the length of third side increasing when than age between the sides of the fixed length is 60°?
3. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high.

4. Show that the equation of the normal to the curve $x = a\,cos^3\theta$; $y = a\,sin^3\theta$at θ is $x\cos\theta - y\sin\theta = a\cos 2\theta$.
5. Let P be a point on the curve $y = x^3$ and suppose that the tangent line at P intersects the curve again at Q. prove that the slope at $Q$is four times the slope at P.
6. If the curve $y^2 = x$ and$xy = k$ are orthogonal then prove that $8k^2 = 1$.
7. Obtain the Maclaurin's Series expansion for : $\tan x\,, -\frac{\pi}{2} < x < \frac{\pi}{2}$.
8. Evaluate the limit for the following if exists $\lim_{x\to\frac{\pi}{2}}(\tan x)^{\cos x}$.
9. Find the local maximum and minimum values for the following: (i) $x^4 - 6x^2$ (ii)$(x^2-1)^3$ (iii) $sin^2\theta,\ [0,\pi]$.
10. Find the intervals of coactivity and the points of inflection of the following functions: $f(x) = x^4 - 6x^2$.
11. Find the intervals of coactivity and the points of inflection of the following functions: $f(x) = \sin 2\theta\ \ in\ (0,\pi)$.
12. Find the intervals of coactivity and the points of inflection of the following functions: $y = 12x^2 - 2x^3 - x^4$.
13. A water tank has the shape of an inverted circular cone with base radius metres and height 4 metres. If water is being pumped into the tank at a rate of $2m^3/min$. Find the rate at which the water level rising when the water is 3m deep.
14. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (Volume of the sphere).
15. Find the angles between the curves $y = x^2$ and $y = (x-2)^2$ at the point of intersection.
16. A car A is travelling from west at 50 Km/hr. and car B is travelling towards north at 60 Km/hr. both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 Kilometers and car B is 0.4 Kilometers from the intersection?
17. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.
18. Evaluate:$\lim_{x\to 0}(\cot x)^{\sin x}$.
19. A man is at a point P on a bank of straight river, 3 km wide and wants to reach point Q, 8 km downstream on an opposite bank, as quickly as possible. He could row his boat directly across the river to point R and then run to Q, or he could row directly to Q, or he could row to

some point S between $Q$and R and then ruin to Q. If he Can row at 6 Km/hr and run at 8 Km/hr where should he land to reach Q as soon as possible?

20. Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a\,cos^4\theta, y = a\,sin^4\theta, 0 \leq \ \theta \leq \frac{\pi}{2}$ is equal to a.
21. A boy, who is standing on a pole of height 14.7 *m,*throws a stone vertically upwards. It moves a vertical line slightly away from the pole and falls on ground. Its equation of motion in metres and seconds is $x = 9.8t - 4.9t^2$ (i) find the time taken for upward and downward motions (ii) also find the maximum height reached by the stone from the ground.
22. The top and bottom margins of a poster are each 6 cms. If the area of the printed material on the poster is fixed at 384 cms^2, find the dimension of the poster with the smallest area.
23. Find the point on a parabola $y^2 = 2x$ that is closest to the point (1, 4).
24. Evaluate: $\lim_{x \to 0+} x^{\sin x}$.
25. Discuss the curve $y = x^4 - 4x^3$ with respect to concavity and points of inflection.
26. Find the points of inflection and determine the intervals of convexity and concavity of the Gaussian curve $y = e^{-x^2}$.
27. A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders of Straight River. He needs no fence along the river. What are the dimension of the field that has the largest area?
28. Find the local maximum and minimum values of $f(x) = x^4 - 3x^3 + 3x^2 - x$.
29. A closed (cuboid) box with a square base is to have a volume of 2000 c.c. The material for the top and bottom of the box is to cost Rs. 3 per square cm. and the material for the sides is to cost Rs. 1.50 per square cm. if the cost of the materials is to be the least, find the dimensions of the box.
30. A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall of rate of 1 *m/sec* how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 m from the wall.
31. Find the condition for the curves $ax^2 + by^2 = 1,\ a_1x_1^2 + b_1y_1^2$ to intersect orthogonally.

DIFFERENTIAL CALCULUS APPLICATIONS – II

1. Trace the curve $y = x^3$.
2. Trace the curve $y^2 = 2x^3$.

3. Trace the curve $y = x^3 + 1$.
4. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions: $u = \frac{x}{y^2} - \frac{y}{x^2}$.
5. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions: $u = \sin 3x \cos 4y$.
6. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions: $u = \tan^{-1}\left(\frac{x}{y}\right)$.
7. Using Euler's theorem prove the following: if $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.
8. If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y \log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.
9. Verify the Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2+y^2}}$.
10. Using the Euler's theorem, Prove that $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ if $u = \sin^{-1}\frac{x-y}{\sqrt{x}+\sqrt{y}}$.

INTEGRAL CALCULUS AND ITS APPLICATIONS

1. Find the common area enclosed by the parabolas $4y^2 = 9x$ and $13x^2 = 16y$.
2. Find the area of the region bounded by the curve $y = 3x^2 - x$ and the *x* – axis between the ordinates $x = 3 \; and \; x = 7$.
3. Find the area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$between the two latus rectum.
4. $y = 1 + x^2, x = 1, x = 2, y = 0$ is solved about the *x* – axis.
5. Find the area of the region bounded by the parabolas $y^2 = 4x$ and the line $2x - y = 4$.
6. Find the perimeter of the circle with radius *a*.
7. Find the surface area of the solid generated by resolving the arc of the parabola $y^2 = 4ax$, bounded by its latus rectum about *x*-axis.
8. Prove that the curved surface area of sphere of radius *r* intercepted between two parallel planes at a distance *a* and *b* from the centre of the sphere is $2\pi r(b - a)$ and hence deduct the surface area of the sphere (*b*>*a*).
9. Find the length of the curve $x = a(t - \sin t), y = (a(1 - \cos t) \; between \; t = 0 \; and \; \pi$.
10. Find the surface area of solid generated by resolving the arc of the parabola$y^2 = 4ax$, bounded by its latus rectum about *x* – axis.
11. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$

12. Find the area between the curves $y = x^2 - x - 2$, x – axis and the lines $x = -2$ and $x = 4$.
13. Find the area bounded by the curve $y = x^3$ and the line $y = x$.
14. Find the area of the region enclosed by $y^2 = x$ and the line $y = x - 2$ are (1, -1) and (4, 2).
15. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.
16. Compute the area between the curve $y = \sin x$ and $y = \cos x$and the lines $x = 0, x = \pi$.
17. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
18. Find the area of the loop of the curve $3ay^2 = x(x-a)^2$.
19. Find the area bounded by x – axis and an arch of the cycloid $x = a(2t - \sin 2t)$, $y = a(1 - \cos 2t)$.
20. Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$.
21. Show that the surface area of the solid obtained by resolving the arc of the curve $y = \sin x$ from$x = 0$ to $x = \pi$ about x – axis is $2\pi[\sqrt{2} + \log(1 + \sqrt{2})]$.
22. Find the length of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$.
23. Find the surface area of the solid generated by resolving the cycloid $x = a(t + \sin t), y = a(1 + \cos t)$ about its base (x – axis).

DIFFERENTIAL EQUATIONS

1. Solve $(x + y)^2 \frac{dy}{dx} = 1$.
2. Solve $(x^2 + y^2)dx + 3xy\, dy = 0$.
3. Solve $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$.
4. Solve $\frac{dy}{dx} + \frac{y}{x} = \sin(x^2)$.
5. Solve $dx + xdy = e^{-y} \sec^2 y\, dy$.
6. Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y = 2(e^x - x - 1)$.
7. Solve the differential equation: $(D^2 + 2D + 3)y = \sin 2x$.
8. Solve the differential equation: $(D^2 - 6D + 9)y = x + e^{2x}$.
9. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$ when $x = \log 2$, $y = 0$ and when $x = 0, y = 0$.

10. Radium disappears at a rate proportional to the amount present. If *5%* of the original amount disappears in 50 years, how much will remain at the end of 100 years. [Take A_0 as the initial amount].
11. The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal? ($\log_e 2 = 0.6931$).
12. The rate at which the population of the city increases at many time is proportional to the population at the time. If there were 1,30,000 people in the city in 1960 and 1, 60, 000 in 1990 what population may be anticipated in 2020. [$\log_e \left(\frac{16}{13}\right) = .2070;\ e^{.42} = 1.52$.
13. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after further interval of 5 minutes.
14. A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from10 mgm to 5 mgm. [$\log_e 2 = 0.6931$].
15. Solve $(x+y)^2 \frac{dy}{dx} = a^2$.
16. A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. [Take $e^{.08} \approx 1.0833$.
17. Solve:$(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{(1-x^2)}$.
18. Solve: $(1+y^2)dx = (\tan^{-1} y - x)dx$.
19. In a certain chemical reaction the rate of conversion of a substance at time *t* is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the substance was there initially?
20. Find the cubic polynomial in *x* which attains its maximum value 4 and minimum value 0 at *x* = -1 and 1 respectively.
21. Solve: $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$.
22. Solve: $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ given that $y = 1$, where $x = 0$.

23. For a postmortem report, a doctor requires to know approximately the time of death of the deceased. He records the first temperature at 10.00 a.m. to be 93.4°F. After 2 hours he finds the temperature to be91.4°F. If the room temperature (which is constant) is 72°F, estimate the time of death. (Assume the normal temperature human body to be 98.6°F).
24. A number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria at the end of five hours will be 3^5 times of the population at initial time.

DISCRETE MATHEMATICS

1. Show that the set *G* of all positive rationals forms a group under the composition * defined by $a * b = \frac{ab}{3}\ for\ all\ a, b \in G$.
2. Show that $\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}\omega^2 & 0\\0 & \omega\end{pmatrix}, \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & \omega^2\\\omega & 0\end{pmatrix}, \begin{pmatrix}0 & \omega\\\omega^2 & 0\end{pmatrix}\right\}$ where$\omega^3 = 1, \omega \neq 1$ form a group with respect to matrix application.
3. Show that the set *M* of complex numbers *z* with the condition $|z| = 1$ forms a group with respect to the operation of multiplication of complex numbers.
4. Show that the set *G* of all rational numbers except –1 form an abelian group with respect to the operation * given by $a * b = a + b + ab\ for\ all\ a, b \in G$.
5. Show that the set $\{[1], [3], [4], [5], [9]\}$forms an abelian group under multiplication modulo 11.
6. Show that the set $G = \{2^n/n \ \in z\}$ is an abelian group under multiplication.
7. Show that $(z,\ *)$ is an infinite abelian group where * is defined as $a * b = a + b + 2$.
8. Show that the set *G* of all matrices of the form $\begin{pmatrix}x & x\\x & x\end{pmatrix}$, where $x \in R - \{0\}$, is a group under matrix multiplication.
9. Show that the set $G = \{a + b\sqrt{2}\ /a, b \ \in Q\}$ is an finite abelian group with respect to addition.
10. Let *G* be the set of all rational numbers except 1 and * be defined on *G* by $a * b = a + b - ab$ for all $a, b \ \in G$. Show that $(G,*)$ is an infinite abelian group.

11. Prove that the set of four functions f_1, f_2, f_3, f_4 on the set of non – zero complex numbers $C-\{0\}$ defined by $f_1(z)=z, f_2(z)=-z, f_3(z)=\frac{1}{z}, f_4(z)=-\frac{1}{z} \forall z \in C-\{0\}$ forms an abelian group with respect to the composition of functions.
12. Show that $(z_n, +_n)$ forms group.
13. Show that $(z_7-\{[0]\}, \bullet_7)$ forms a group.
14. Show that the *n*th root of unity form an abelian group of finite order with usual multiplication.

PROBABILITY DISTRIBUTIONS

1. The probability density functions of a random variable *x* is $f(x)=\begin{cases} kx^{\alpha-1}e^{-\beta x^{\alpha}}, & x,\alpha,\beta>0 \\ 0, & elsewhere \end{cases}$. Find (i) k (ii) $P(X>10)$.
2. Four coins are tossed simultaneously. What is the probability of getting *(a)* exactly 2 heads *(b)* at least 2 heads *(c)* at most two heads.
3. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year. (ii) more than three accident in a year $[e^{-3}=0.0498]$.
4. The mean weight of 500 male students in a certain college in 151 pounds and the standard deviation is 15 pounds. Assuming the weight are normally distributed, find how many students weigh (i) between 120 and 155 Pounds (ii) more than 180 pounds,
5. Find $c, \mu, and\ \sigma^2$ of the normal distribution whose probability function is given by $f(x)=ce^{-x^2+3x}, -\infty<X<\infty$.
6. A random variable *X* has the following probability mass function.

X	0	1	2	3	4	5	6
P(X=x)	*k*	*3k*	*5k*	*7k*	*9k*	*11k*	*13k*

7. An urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn. (i) with replacement (ii) without replacement.
8. If X is normally distributed with mean 6 and standard deviation 5 find. (i) $P(0\le X\le 8)$ (ii) $P(|X-6|<10)$.

9. The air pressure in a randomly selected tyre put on a certain model new car is normally distributed with mean value 31 psi and standard deviation 0.2 psi. (i) what is the probability that the pressure for a randomly selected tyre (a) between 30.5 and 31.5 psi (b) between 30 and 32 psi. (ii) what is the probability that the pressure for a randomly selected tyre exceeds 30.5 psi?
10. The mean score of 1000 students for an examination is 34 and S.D is 16. (i) How many candidates can be expected to obtain marks between 30 and 60 assuming the normality of the distribution and (ii) determine the limit of the marks of the central *70%* of the candidates.
11. Find $k, \mu, and\ \sigma^2$ of the normal distribution whose probability function is given by $f(x) = ke^{-2x^2+4x}, -\infty < X < \infty$.
12. If the number of incoming buses per minute at a bus terminus is a random variable having a Poisson distribution with $\lambda = 0.9$, find the probability that there will be (i) exactly 9 incoming buses during a period of 5 minutes. (ii) Fewer than 10 incoming buses during a period of 8 minutes. (iii) Atleast 14 incoming buses during a period of 11 minutes.
13. The total time (in year) of 5 year old dog certain breed is a random variable whose distribution is given by $F(x) = \begin{cases} 0 & , \ for\ x \leq 5 \\ 1 - \frac{25}{x^2} & , for\ x > 5 \end{cases}$ find the probability such that a five year old dog will live (i) beyond 10 years (ii) less than 8 years (iii) anywhere between 12 to 15 years.