

Class : 12  
Subject : Mathematics

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Maximum Marks: 200  
Time: 3 Hour

Part – A

40 x 1 = 40

Answer all the Questions

- The rank of the matrix  $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{pmatrix}$  is,  
a) 1    b) 2    c) 3    d) 4
- If the rank of the matrix  $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$  is 2, then  $\lambda$  is,  
a) 1    b) 2    c) 3    d) any real number
- If I is the unit matrix of order n, where  $k \neq 0$  is a constant, then  $\text{adj}(kI)$  is  
a)  $k^n \text{adj}(I)$     b)  $k \text{adj}(I)$     c)  $k^2 \text{adj}(I)$     d)  $k^{n-1} \text{adj}(I)$
- The rank of the matrix  $\begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$  is  
a) 1    b) 2    c) 0    d) 8
- The shortest distance of the point (2, 10, 1) from the plane  $\vec{r} \cdot (3\vec{i} - \vec{j} + 4\vec{k}) = 2\sqrt{26}$  is  
a)  $2\sqrt{26}$     b)  $\sqrt{26}$     c) 2    d)  $\frac{1}{\sqrt{26}}$
- The projection of  $\vec{OP}$  on a unit vector  $\vec{OQ}$  equals thrice the area of parallelogram OPRQ. Then  $\angle POQ$  is,  
a)  $\tan^{-1}\frac{1}{3}$     b)  $\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$     c)  $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$     d)  $\sin^{-1}\left(\frac{1}{3}\right)$
- If  $\vec{a}, \vec{b}, \vec{c}$  are a right handed triad of mutually perpendicular vectors of magnitude a, b, c then the value of  $[\vec{a}, \vec{b}, \vec{c}]$   
a)  $a^2b^2c^2$     b) 0    c)  $\frac{1}{2}abc$     d) abc
- If a line makes  $45^\circ, 60^\circ$  with positive direction of axes x and y then the angle it makes with the z axis is  
a)  $30^\circ$     b)  $90^\circ$     c)  $45^\circ$     d)  $60^\circ$
- The angle between the vectors  $\vec{i} - \vec{j}$  and  $\vec{j} - \vec{k}$  is  
a)  $\frac{\pi}{3}$     b)  $\frac{-2\pi}{3}$     c)  $\frac{-\pi}{3}$     d)  $\frac{2\pi}{3}$
- If the vectors  $\vec{a} = 3\vec{i} + 2\vec{j} + 9\vec{k}$  and  $\vec{b} = \vec{i} + m\vec{j} + 3\vec{k}$  are perpendicular then m is  
a) -15    b) 15    c) 30    d) -30
- The polar form of the complex number  $(i^{25})^3$  is  
a)  $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$     b)  $\cos\pi + i\sin\pi$     c)  $\cos\pi - i\sin\pi$     d)  $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
- If  $x = \cos\theta + i\sin\theta$  then the value of  $x^n + \frac{1}{x^n}$  is  
a)  $2\cos n\theta$     b)  $2i\sin n\theta$     c)  $2\sin n\theta$     d)  $2i\cos n\theta$
- $z_1 = 4 + 5i, z_2 = -3 + 2i$  then  $\frac{z_1}{z_2}$  is  
a)  $\frac{2}{13} - \frac{22}{13}i$     b)  $-\frac{2}{13} + \frac{22}{13}i$     c)  $-\frac{2}{13} - \frac{22}{13}i$     d)  $\frac{2}{13} + \frac{22}{13}i$
- The standard form  $(a + ib)$  of  $3 + 2i + (-7 - i)$  is  
a)  $4 - i$     b)  $-4 + i$     c)  $4 + i$     d)  $4 + 4i$
- The distance between the foci of the ellipse  $9x^2 + 5y^2 = 180$  is.  
a) 4    b) 6    c) 8    d) 2.
- The locus of foot of perpendicular from the focus to a tangent of the curve  $16x^2 + 25y^2 = 400$  is.  
a)  $x^2 + y^2 = 4$     b)  $x^2 + y^2 = 25$     c)  $x^2 + y^2 = 16$     d)  $x^2 + y^2 = 9$
- The equation of the chord of contact of tangents from (2,1) to the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  is  
a)  $9x - 8y - 72 = 0$     b)  $9x + 8y + 72 = 0$     c)  $8x - 9y - 72 = 0$     d)  $8x + 9y + 72 = 0$
- The vertex of the parabola  $x^2 = 20y$  is

## CENTUM ASSURED MODEL PUBLIC EXAM-3

- a) (0,5)                      b) (0,0)                      c) (5,0)                      d) (0,-5)
19. If  $y = 6x - x^3$  and  $x$  increases at the rate of 5 units per second, the rate of change of slope when  $x = 3$   
 isa)  $-90$  units / sec.    b)  $90$  units / sec.                      c)  $180$  units / sec.                      d)  $-180$  units / sec.
20. The value of 'a' so that the curves  $y = 3e^x$  and  $y = \frac{a}{3}e^{-x}$  intersect orthogonally is.  
 a) -1                      b) 1                      c)  $1/3$                       d) 3.
21. The 'c' of Lagranges Mean Value Theorem for the function  $f(x) = x^2 + 2x - 1$ ;  $a = 0$ ,  $b = 1$  is.  
 a) -1                      b) 1                      c) 0                      d)  $1/2$
22. The curve  $y = f(x)$  and  $y = g(x)$  cut orthogonally if at the point of intersection  
 a) slope of  $f(x) =$  slope of  $g(x)$                       b) slope of  $f(x) +$  slope of  $g(x) = 0$   
 c) slope of  $f(x) /$  slope of  $g(x) = -1$                       d) [ slope of  $f(x)$  ] [ slope of  $g(x)$  ] = -1
23. If  $u = y \sin x$ , then  $\frac{\partial^2 u}{\partial x \partial y}$  is equal to  
 a)  $\cos x$ .                      b)  $\cos y$                       c)  $\sin x$ .                      d) 0.
24. If  $f(x, y)$  is a homogeneous functions of degree  $n$  then  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} =$   
 a)  $f$                       b)  $nf$                       c)  $n(n-1)f$                       d)  $n(n+1)f$
25. The surface area of the solid of revolution of the region bounded by  $y = 2x$ ,  $x = 0$  and  $x = 2$  about  $x$  axis is.  
 a)  $8\sqrt{5}\pi$                       b)  $2\sqrt{5}\pi$                       c)  $\sqrt{5}\pi$                       d)  $4\sqrt{5}\pi$
26. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is.  
 a)  $20\pi$                       b)  $40\pi$                       c)  $10\pi$                       d)  $30\pi$
27. The length of the arc of the curve  $x^{2/3} + y^{2/3} = 4$  is.  
 a) 48                      b) 24                      c) 12                      d) 96
28. If  $n$  is odd then  $\int_0^{\pi/2} \cos^n x dx$   
 a)  $\frac{n}{n-1} \cdot \frac{n-2}{n-3} \cdot \frac{n-4}{n-5} \dots \frac{\pi}{2}$                       b)  $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \frac{1}{2} \cdot \frac{\pi}{2}$   
 c)  $\frac{n}{n-1} \cdot \frac{n-2}{n-3} \cdot \frac{n-4}{n-5} \dots \frac{3}{2} \cdot 1$                       d)  $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \frac{2}{3} \cdot 1$
29. The amount present in a radio active element disintegrates at a rate proportional to its amount. The differential equation corresponding to the above statement is ( $k$  is negative)  
 a)  $\frac{dp}{dt} = \frac{k}{p}$                       b)  $\frac{dp}{dt} = kt$                       c)  $\frac{dp}{dt} = kp$                       d)  $\frac{dp}{dt} = -kt$
30. The differential equation formed by eliminating  $A$  and  $B$  from the relation  $y = e^x(A \cos x + B \sin x)$  is.  
 a)  $y_2 + y_1 = 0$                       b)  $y_2 - y_1 = 0$                       c)  $y_2 - 2y_1 + 2y = 0$                       d)  $y_2 - 2y_1 - 2y = 0$
32. The order and degree of the differential are  $\sin x (dx + dy) = \cos x (dx - dy)$   
 a) 1,1                      b) 0,0                      c) 1,2                      d) 2,1
33. The number of rows in the truth table of  $\sim [p \wedge (\sim q)]$  is.  
 a) 2                      b) 4                      c) 6                      d) 8.
34. A monoid becomes a group if it also satisfies the  
 a) closure axiom.                      b) associative axiom                      c) identity axiom                      d) inverse axiom.
35. The value of  $[3] + {}_{11}([5] + {}_{11}[6])$  is.  
 a) [0]                      b) [1]                      c) [2]                      d) [3]
36. Let  $p$  be "Kamala is going to school" and  $q$  be "There are twenty students in the class". "Kamala is not going to school or there are twenty students in the class" stands for  
 a)  $p \vee q$                       b)  $p \wedge q$                       c)  $\sim p$                       d)  $\sim p \vee q$
37. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively. Then the value of its parameter  $p$  is.

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- a) 1/2                                      b) 1/3                                      c) 2/3                                      d) 1/4.
38. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls is.
- a) 1/20                                      b) 18/125                                      c) 4/25                                      d) 3/10
39. For a Poisson distribution with parameter  $\lambda = 0.25$  the value of the 2<sup>nd</sup> moment about the origin is.
- a) 0.25                                      b) 0.3125                                      c) 0.0625                                      d) 0.025
40. For a standard normal distribution the mean and variance are
- a)  $\mu, \sigma^2$                                       b)  $\mu, \sigma$                                       c) 0,1                                      d) 1,1

### Part – B

**10x 6 = 60**

**1. Answer ANY 10 Questions.**

**2. Question No 55 is Compulsory and Choose any Nine from the remaining**

41. For  $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$  show that  $A = A^{-1}$ .

42. a) Solve the following system of linear equations by determinant method.       $x - y = 2$  ;  $3y = 3x - 7$

b) **Solve:**  $x + y + 2z = 0$  ;  $2x + y - z = 0$  ;  $2x + 2y + z = 0$

43. a) The volume of a parallelepiped whose edges are represented by  $-12\vec{i} + \lambda\vec{k}$ ,  $3\vec{j} - \vec{k}$ ,  $2\vec{i} + \vec{j} - 15\vec{k}$  is 546. Find the value of  $\lambda$ .

b) Find the angle between the line  $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$  and the plane  $3x + 4y + z + 5 = 0$

44. Angle in a semi-circle is a right angle. Prove by vector method.

45. If  $x = \cos\alpha + i\sin\alpha$ ;  $y = \cos\beta + i\sin\beta$  prove that  $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

46. If n is a positive integer, prove that  $\left(\frac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta}\right)^n = \cos n\left(\frac{\pi}{2} - \theta\right) + i\sin n\left(\frac{\pi}{2} - \theta\right)$

47. The tangent at any point of the rectangular hyperbola  $xy = c^2$  makes intercepts a, b and the normal at the point makes intercepts p, q on the axes. Prove that  $ap + bq = 0$

48. A Particle of unit mass moves so that displacement after t secs is given by  $x = 3 \cos(2t - 4)$ . Find the acceleration and kinetic energy at the end of 2 secs.

49. Obtain the Maclaurin's Series for  $\log_e(1+x)$

50. Find the area included between the parabola  $y^2 = 4ax$  and its latus rectum.

51. Solve:  $(D^2 + 5)y = \sin^2 x$

52. Use the truth table to determine whether the statement  $((\sim P) \vee q) \vee (p \wedge (\sim q))$  is a tautology.

53. State and prove reversal law of a group.

54. Alpha particles are emitted by a radio active source at an average rate of 5 in a 20 minutes interval.

Using Poisson distribution find the probability that there will be (i) 2 emission (ii) at least 2 emission in a particular 20 minutes interval.  $[e^{-5} = 0.0067]$ .

55. a) Find the mean and variance of the distribution  $f(x) = \begin{cases} \alpha e^{-\alpha x} & , \text{if } x > 0 \\ 0 & , \text{otherwise} \end{cases}$

**(OR)**

b) If  $u = \log(\tan x + \tan y + \tan z)$ , prove that  $\sum \sin 2x \frac{\partial u}{\partial x} = 2$

**Part – C**

**10x 10 = 100**

**1. Answer ANY 10 Questions.**

**2. Question No 70 is Compulsory and Choose any Nine from the remaining**

56. For what value of  $\mu$  the equations  $x + y + 3z = 0$ ,  $4x + 3y + \mu z = 0$ ,  $2x + y + 2z = 0$  have a (i) trivial solution, (ii) non-trivial solution

57. Show that the lines  $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$  and  $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$  intersect and hence find the point of intersection.

58. Find the vector and Cartesian equation of the plane through the points (1,2,3) and (2,3,1) perpendicular to the plane  $3x - 2y + 4z - 5 = 0$ .

59. If  $a = \cos 2\alpha + i \sin 2\alpha$ ,  $b = \cos 2\beta + i \sin 2\beta$  and  $c = \cos 2\gamma + i \sin 2\gamma$  Prove that

(i)  $\sqrt{abc} + \frac{1}{\sqrt{abc}} = 2 \cos(\alpha + \beta + \gamma)$       (ii)  $\frac{a^2b^2 + c^2}{abc} = 2 \cos 2(\alpha + \beta - \gamma)$

60. A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level find the length of the support if the height of the pillars are 55 mts.

61. A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.

62. Find the local maximum and minimum values of the following functions:  $x^4 - 6x^2$

63. Using Euler's theorem, prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$  if  $u = \sin^{-1} \left( \frac{x-y}{\sqrt{x} + \sqrt{y}} \right)$

64. Find the common area enclosed by the parabolas  $y^2 = x$  and  $x^2 = y$

65. Find the surface area of the solid generated by revolving the arc of the parabola  $y^2 = 4ax$ , bounded by its latus rectum about x - axis.

66. In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the first substance was there initially?

68. Show that  $(Z_7 - \{0\}, \cdot_7)$  forms a group.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with  
(i) no accident in a year  
(ii) more than 3 accidents in a year  $[e^{-3} = 0.0498]$ .

70. a) If the curve  $y^2 = x$  and  $xy = k$  are orthogonal then prove that  $8k^2 = 1$ .

**(OR)**

b) the eccentricity centre, foci and vertices of the following hyperbolas and draw their diagrams.  
 $x^2 - 3y^2 + 6x + 6y + 18 = 0$