CLASS:XII

Time : 3.00 hrs. **MATHEMATICS** Max. Marks : 200

Section – A

Note: 1. ***Answer all the questions.***

2. ***Choose the most suitable answer from the given four alternatives.*** **40 x 1 = 40**

1. If A = $\begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$, then A^{12} is,

   a) $\begin{pmatrix} 0 & 0 \\ 0 & 60 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 0 \\ 0 & 5^{12} \end{pmatrix}$ c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2. If A and B are any two matrices such that AB = 0 and A is non-singular, then

   a) B = 0 b) B is singular c) B is non-singular d) B = A

3. If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(kI) is

   a) $k^n adj(I)$ b) $k\ adj(I)$ c) $k^2 adj(I)$ d) $k^{n-1} adj(I)$

4. If $\rho(A)=r$ then which of the following is correct ?

   a) all the minors of order r which does not vanish

   b) has atleast one minor of order r which does not vanish

   c) A has atleast one (r+1) order minor which vanishes

   d) all (r+1) and higher order minors should not vanish

5. The point of intersection of the line $\vec{r} = (\vec{i} - \vec{k}) + t(3\vec{i} + 2\vec{j} + 7\vec{k})$ and the plane $\vec{r} \cdot (\vec{i} + \vec{j} - \vec{k}) = 8$ is

   a) (8, 6, 22) b) (-8, - 6, -22) c) (4, 3, 11) d) (-4, -3, -11)

6. The shortest distance between the parallel lines $\frac{x-3}{4} = \frac{y-1}{2} = \frac{z-5}{-3}$ and $\frac{x-1}{4} = \frac{y-2}{2} = \frac{z-3}{-3}$

   a) 3 b) 2 c) 1 d) 0

7. If $\vec{a}$ is a non-zero vector and m is a non-zero scalar then $m\vec{a}$ is a unit vector if

   a) $m = \pm 1$ b) $a = |m|$ c) $a = \frac{1}{|m|}$ d) a = 1

8. The vectors $2\vec{i} + 3\vec{j} + 4\vec{k}$ and $a\vec{i} + b\vec{j} + c\vec{k}$ are perpendicular when

   a) a = 2, b = 3, c = -4 b) a = 4, b = 4, c = 5 c) a = 4, b = 4, c = -5 d) a = -2, b =3, c = 4

9. The value of $\vec{a} \cdot \vec{b}$ when $\vec{a} = \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} - 2\vec{k}$ is

   a) 7 b) -7 c) 5 d) 6

10. The work done in moving a particle from the point A, with position vector $2\vec{i} - 6\vec{j} + 7\vec{k}$, to the point B ,with position vector $3\vec{i} - \vec{j} - 5\vec{k}$, by a force $\vec{F} = \vec{i} + 3\vec{j} - \vec{k}$ is

    a) 25 b) 26 c) 27 d) 28

11. If $-i+2$ is one is one root equation $ax^2 - bx + c = 0$, then the other root is

    a) $-i-2$ b) $i-2$ c) $2+i$ d) $2i+i$

12. If ω is a cube root of unity then the value of $(1-\omega+\omega^2)^4 + (1+\omega-\omega^2)^4$ is

    a) 0 b) 32 c) -16 d) -32

13. If ω is a cube root of unity then the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is

    a) 9 b) -9 c) 16 d) 32

14. The complex conjugate of $\sqrt{5}$ is

    a) $\sqrt{5}$ b) $-\sqrt{5}$ c) $i\sqrt{5}$ d) $-i\sqrt{5}$

15. The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is.

    a) 4 b) 3 c) 2 d) 6

16. The asymptotes of the hyperbola $36y^2 - 25x^2 + 900 = 0$ are

a) $y = \pm\frac{6}{5}x$ b) $y = \pm\frac{5}{6}x$ c) $y = \pm\frac{36}{25}x$ d) $y = \pm\frac{25}{36}x$

17. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is.

a) $x^2 + y^2 = 25$ b) $x^2 + y^2 = 4$ c) $x^2 + y^2 = 3$ d) $x^2 + y^2 = 7$

18. The length of the L.R. of $y^2 = 4x$ is

a) 2 b) 3 c) 1 d) 4

19. For the curve $x = e^t \cos t$; $y = e^t \sin t$ the tangent line is parallel to the x axis when t is equal to

a) $-\pi/4$ b) $\pi/4$ c) 0 d) $\pi/2$

20. If the velocity of a particle moving along a straight line is directly proportional to the square of its distance from a fixed point on the line. Then its acceleration is proportional to.

a) s b) s^2 c) s^3 d) s^4.

21. The value of c in Rolle's Theorem for the function f(x) = cos x / 2 on $[\pi, 3\pi]$ is

a) 0 b) 2π c) $\pi/2$ d) $3\pi/2$

22. Food pockets were dropped from an helicopter during the flood and distance fallen in " t " seconds is given by $y = \frac{1}{2}gt^2$ $\left(g = 9.8\ m/s^2\right)$. Then the speed of the food pocket after it has falled for " 2 " seconds is

a) 19.6 m / sec b) 9.8 m / sec c) – 19.6 m / sec d) – 9.8 m / sec

23. The curve $ay^2 = x^2(3a - x)$ cuts the y axis at.

a) x = - 3a , x = 0 b) x =0, x = 3a. c) x = 0, x = a. d) x = 0

24. If $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to

a) yx^{y-1}. b) u log x c) u log y d) xy^{x-1}.

25. The value of $\int_0^{\pi/2} \frac{\cos^{5/3} x}{\cos^{5/3} x + \sin^{5/3} x}\,dx$ is

a) $\pi/2$ b) $\pi/4$ c) 0 d) π

26. The value $\int_0^1 x(1-x)^4\,dx$ is.

a) 1/ 12 b) 1/30 c) 1/ 24 d) 1/ 20

27. The value of $\int_0^{\pi/4} \cos^3 2x\,dx$ is

a) 2 / 3 b) 1/ 3 c) 0 d) $2\pi/3$

28. $\int_a^b f(x)\,dx$ is

a) $2\int_0^a f(x)\,dx$ b) $\int_a^b f(a-x)\,dx$ c) $\int_a^b f(b-x)\,dx$ d) $\int_a^b f(a+b-x)\,dx$

29. The P.I of $(3D^2+D-14)\,y = 13e^{2x}$ is..

a) $26x\,e^{2x}$. b) $13x\,e^{2x}$. c) $x\,e^{2x}$. d) $\frac{x^2}{2}e^{2x}$

30.If cos x is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then P =

a) – cot x. b) cot x c) tan x d) – tan x.

31. Solution of $\frac{dx}{dy} + mx = 0$, where m < 0 is.

a) $x = ce^{my}$. b) $x = ce^{-my}$. c) x = my + c. d) x = c.

32. The order and degree of the differential equation $y''+3y'^2+y^3=0$ are

a) 2,2 b) 2,1 c) 1,2 d) 3,1

33. Which of the following are statements ?

i) May God bless you ii) Rose is a flower iii) Milk is white. iv) 1 is a prime number.

a) (i), (ii), (iii) b) (i), (ii),(iv) c) (i), (iii), (iv) d) (ii), (iii), (iv)

34. If p is T and q is F, then which of the following have the truth value T ?

(i) p *v* q (ii) ~ p *v* q iii) p v ~q iv) p ^ ~ q.

a) (i), (ii), (iii) b) (i), (ii),(iv) c) (i), (iii), (iv) d) (ii), (iii), (iv)

35. The conditional statement $p \to q$ is equivalent to

a) p v q. b) p v ~ q c) ~p v q. d) p ^ q.

36. Which of the following are statements?

i. 7 + 2 < 10 ii. The set of rational numbers is finite

iii. How beautiful you are iv. Wish you all success.

(a) (iii) (iv) b) (i) , (ii) c) (i) , (iii) d) (ii) , (iv)

37. The marks secured by 400 students in a Mathematics test were normally distributed with mean 65. If 120 students got more marks above 85, the number of students securing marks between 45 and 65 is.

a) 120 b) 20 c) 80 d) 160

38. If $f(x) = \frac{A}{\pi}\frac{1}{16+x^2}, -\infty < x < \infty$ is a p.d.f of a continuous random variable X, then the value of A is.

a) 16 b) 8 c) 4 d) 1

39. A random variable X has the following p.d.f.

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

The value of k is

a) 1/ 8 b) 1/ 10 c) 0 d) -1 or 1/10

40. A continuous random variable X has p.d.f . f(x) then

a) $0 \le f(x) \le 1$ b) $f(x) \ge 0$ c) $f(x) \le 1$ d) $0 < f(x) < 1$

Section - B

Note: 1. ***Answer any 10 questions.***

2. ***Question No. 55 is compulsory and choose any nine questions from the remaining*** **10×6=60**

41. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

42. For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ prove that $\left[\vec{a}-\vec{b}\ ,\ \vec{b}-\vec{c}\ ,\ \vec{c}-\vec{a}\right]=0$

43. Find the vector and Cartesian equation of the sphere on the join of the points A and B having position vectors $2\vec{i}+6\ \vec{j}-7\ \vec{k}$ and $-2\vec{i}+4\ \vec{j}-3\ \vec{k}$ respectively as a diameter. Find also the centre and radius of the sphere.

44. (I) Express the following complex number in polar form. $-1-i$

(II) Prove that if $\omega^3=1$, then $\left(\frac{-1+i\sqrt{3}}{2}\right)^5+\left(\frac{-1-i\sqrt{3}}{2}\right)^5=-1$

45. Find the square root of $(-7+24i)$

46. Prove that the product of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2+b^2}$

47. (i) Obtain the Maclaurin's Series for e^x

(ii) Evaluate: $\lim_{x\to\infty} \frac{x^2}{e^x}$

48. Determine for which values of x, the function $f(x)=2x^3-15x^2+36x+1$ is increasing and for which it is decreasing. Also determine the points where the tangents to the graph of the function are parallel to the x axis.

49. If $U=(x-y)(y-z)(z-x)$ then show that $U_x+U_y+U_z=0$

50. Find the equation of the curve passing through (1,0) and which has slope $1+\frac{y}{x}$ at (x, y).

51. Show that $p\leftrightarrow q \equiv ((\sim p)\vee q)\wedge((\sim q)\vee p)$

52. State and prove cancellation laws on groups.

53. Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x} & , 0<x<\infty \\ 0 & , \text{ elsewhere} \end{cases}$

54. The overall percentage of passes in a certain examination is 80. If 6 candidates appear in the examination what is the probability that at least 5 pass the examination.

55. (a) Examine the consistency of the following system of equation.If it is consistent then solve the same.

$x-4y+7z=14$; $3x+8y-2z=13$; $7x-8y+26z=5$ **(OR)**

(b) Evaluate $\int_0^{\pi/2} \log(\tan x)\, dx$

Section - c

Note: 1. ***Answer any 10 questions.***

2. ***Question No. 70 is compulsory and choose any nine questions from the remaining*** **10×10=100**

56. Discuss the solutions of the system of equations $x+y+z=2$, $2x+y-2z=2$, $\lambda x+y+4z=2$ for all values of λ

57. Derive the equation of the plane in the intercept form.

58. If α and β are the roots of $x^2-2x+4=0$ Prove that $\alpha^n-\beta^n=i\,2^{n+1}\sin\frac{n\pi}{3}; n\in N$ and deduct $\alpha^9-\beta^9$

59. Prove that the line $5x+12y=9$ touches the hyperbola $x^2-9y^2=9$ and find its point of contact.

60. Find the eccentricity, centre, foci, vertices of the following ellipse, $36x^2+4y^2+72x-32y-44=0$

61. The girder of a railway bridge is in the parabolic form with span 100ft. and the highest point on the arch is 10ft, above the bridge. Find the height of the bridge at 10ft, to the left or right from the midpoint of the bridge.

62. Find the angle between the curves $y=x^2$ and $y=(x-2)^2$ at the point of intersection.

63. A man is at a point P on a bank of a straight river, 3 km wide, and wants to reach point Q, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point R and then run to Q, or he could row directly to Q, or he could row to some point to between Q and R and then run to Q. If he can row at 6 km/h and run at 8 km/h where should he land to reach Q as soon as possible?

64. Trace the curve $y=x^3+1$

65. Find the area bounded by the curve $y=x^3$ and the line $y=x$.

66. Find the surface area of the solid generated by revolving the arc of the parabola $y^2=4ax$, bounded by its latus rectum about x - axis.

67. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1 hour. Show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

68. Show that the set $G=\left\{2^n \ / \ n \in Z\right\}$ an abelian group under multiplication.

69. The probability density function of a random variable X is

$$f(x)=\begin{cases} k\,x^{\alpha-1}\,e^{-\beta x^{\alpha}} & , \ x,\alpha,\beta>0 \\ 0 & , \ elsewhere \end{cases}$$ Find (i) k (ii) P (X > 10)

70. (a) If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$, $\vec{c}=\vec{j}-3\vec{k}$ Verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\,.\vec{c}\,\right)\vec{b}-\left(\vec{a}\,.\vec{b}\,\right)\vec{c}$

(OR)

(b) Show that the equation of the curve whose slope at any point is equal to $y+2x$ and which passes through the origin is $y=2\left(e^x-x-1\right)$