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PIMS EDUCATIONAL ACADEMY Regd. No. : 130/10

Register Number

+2 MODEL EXAMINATION

PART III – MATHEMATICS
[English Version]

Time allowed: 3 Hours] **[Maximum Marks: 200**

SECTION – A

Note: (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. The normal at 't_1' on the parabola $y^2 = 4ax$ meets the parabola at 't_2' then $\left(t_1 + \frac{2}{t_1}\right)$ is

(1) $-t_2$ (2) t_2 (3) $t_1 + t_2$ (4) $\frac{1}{t_2}$

2. If $u = y \sin x$ then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to

(1) $\cos x$ (2) $\cos y$ (3) $\sin x$ (4) 0

3. The curve $a^2 y^2 = x^2 (a^2 - x^2)$ has

(1) only one loop between $x = 0$ and $x = a$ (2) two loops between $x = 0$ and $x = a$
(3) two loops between $x = -a$ and $x = a$ (4) no loop

4. The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ is

(1) $\int P dx$ (2) $\int Q dx$ (3) $e^{\int Q dx}$ (4) $e^{\int P dx}$

5. The differential equation satisfied by all the straight lines in xy plane is

(1) $\frac{dy}{dx} =$ a constant (2) $\frac{d^2y}{dx^2} = 0$ (3) $y + \frac{dy}{dx} = 0$ (4) $\frac{d^2y}{dx^2} + y = 0$

6. If A $= \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then (adj A) A =

(1) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

7. The system of equations $ax + y + z = 0$; $x + by + z = 0$; $x + y + cz = 0$ has a non- trivial solution then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

(1) 1 (2) 2 (3) -1 (4) 0

8. If A is a matrix of order 3, then det (kA) is

(1) $k^3 \, det(A)$ (2) $k^2 \, det(A)$ (3) $k \, det(A)$ (4) $det(A)$

9. In the system of 3 linear equations with three unknowns $\rho(A) = \rho(A, B) = 1$ then the system
(1) has unique solution (2) reduces to 2 equations and has infinitely many solutions
(3) reduces to a single equation and has infinitely many solutions (4) is inconsistent

10. If $\vec{a}$ and $\vec{b}$ include an angle 120° and their magnitudes are 2 and $\sqrt{3}$ then $\vec{a} \cdot \vec{b}$ is equal to
(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) 2 (4) $-\frac{\sqrt{3}}{2}$

11. In the multiplicative group of *nth* roots of unity, the inverse of ω^k is $(k < n)$
(1) $\omega^{1/k}$ (2) ω^{-1} (3) ω^{n-k} (4) $\omega^{n/k}$

12. If a random variable X follows Poisson distribution such that E $(X^2) = 30$ then the variance of the distribution is
(1) 6 (2) 5 (3) 30 (4) 25

13. A random variable X has the following probability distribution

X	0	1	2	3	4	5
P(X = x)	1/4	2a	3a	4a	5a	1/4

Then $P(1 \leq x \leq 4)$ is
(1) $\frac{10}{21}$ (2) $\frac{2}{7}$ (3) $\frac{1}{14}$ (4) $\frac{1}{2}$

14. Var $(4X+3)$ is
(1) 7 (2) 16 Var (X) (3) 19 (4) 0

15. A continuous random variable takes
(1) only a finite number of values (2) all possible values between certain given limits
(3) infinite number of values (4) a finite or countable number of values.

16. In finding the differential equation corresponding to $y = e^{mx}$ where m is the arbitrary constant, then m is
(1) $\frac{y}{y'}$ (2) $\frac{y'}{y}$ (3) y' (4) y

17. '+' is not a binary operation on
(1) N (2) Z (3) C (4) $Q - \{0\}$

18. If p is T and q is F, then which of the following have the truth value T?
(i) $p \vee q$ (ii) $\sim p \vee q$ (iii) $p \vee \sim q$ (iv) $p \wedge \sim q$
(1) (i), (ii), (iii) (2) (i), (ii), (iv) (3) (i), (iii), (iv) (4) (ii), (iii), (iv)

19. Which of the following is not a group?
(1) $(Z_n , +_n)$ (2) $(Z , +)$ (3) $(Z , .)$ (4) $(R, +)$

20. The function $y = \tan x - x$ is
(1) an increasing function in $\left(0, \frac{\pi}{2}\right)$ (2) a decreasing function in $\left(0, \frac{\pi}{2}\right)$
(3) increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (4) decreasing in $\left(0, \frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

21. The curve $y = ax^3+bx^2+cx+d$ has a point of inflexion at $x = 1$ then a
(1) $a + b = 0$ (2) $a + 3b = 0$ (3) $3a + b = 0$ (4) $3a + b = 1$

22. If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero is

(1) $\frac{32}{3} m/\sec$ (2) $\frac{-16}{3} m/\sec$ (3) $\frac{16}{3} m/\sec$ (4) $\frac{-32}{3} m/\sec$

23. *l'Hopital's* rule cannot be applied to $\frac{x+1}{x+3}$ as $x \to 0$ because $f(x) = x+1$ and $g(x) = x+3$ are

(1) not continuous (2) not differentiable

(3) not in the indeterminate form as $x \to 0$ (4) in the indeterminate form as $x \to 0$

24. The surface area of the solid of revolution of the region bounded by $y=2x$, $x=0$ *and* $x=2$ about x-axis is

(1) $8\sqrt{5}\pi$ (2) $2\sqrt{5}\pi$ (3) $\sqrt{5}\pi$ (4) $4\sqrt{5}\pi$

25. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

(1) 48 (2) 24 (3) 12 (4) 96

26. The non-parametric vector equation of a plane passing through three non-collinear points whose P.Vs are $\vec{a}, \vec{b}, \vec{c}$ is

(1) $[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}] = 0$ (2) $[\vec{r} \quad \vec{a} \quad \vec{b}] = 0$ (3) $[\vec{r} \quad \vec{b} \quad \vec{c}] = 0$ (4) $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$

27. The polar form of the complex number $(i^{25})^3$ is

(1) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (2) $\cos\pi + i\sin\pi$ (3) $\cos\pi - i\sin\pi$ (4) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

28. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

(1) $z_1 + z_4 = z_2 + z_3$ (2) $z_1 + z_3 = z_2 + z_4$ (3) $z_1 + z_2 = z_3 + z_4$ (4) $z_1 - z_2 = z_3 - z_4$

29. If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

(1) 0 (2) 32 (3) –16 (4) –32

30. If z_1, and z_2, are any two complex numbers then which one of the following is false?

(1) $Re(z_1+z_2) = Re(z_1) + Re(z_2)$ (2) $Im(z_1+z_2) = I_m(z_1) + I_m(z_2)$

(3) $arg(z_1+z_2) = arg\, z_1 + arg\, z_2$ (4) $|z_1 z_2| = |z_1|\,|z_2|$

31. The value of $\int_0^1 x(1-x)^4 dx$ is

(1) $\frac{1}{12}$ (2) $\frac{1}{30}$ (3) $\frac{1}{24}$ (4) $\frac{1}{20}$

32. $\int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx =$

(1) $\int_0^a f(x)\,dx$ (2) $2\int_0^a f(x)\,dx$ (3) $\int_0^{2a} f(x)\,dx$ (4) $\int_0^{2a} f(a-x)\,dx$

33. The P.I of $(3D^2 + D - 14)\,y = 13e^{2x}$ is

(1) $26xe^{2x}$ (2) $13x\,e^{2x}$ (3) xe^{2x} (4) $x^2/2e^{2x}$

34. The equation of the chord of contact of tangents from (2, 1) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is

(1) $9x - 8y - 72 = 0$ (2) $9x + 8y + 72 = 0$ (3) $8x - 9y - 72 = 0$ (4) $8x + 9y + 72 = 0$

35. If the lengths of major and semi-minor axes of an ellipse are 8, 2 and their corresponding equations are $y - 6 = 0$ and $x + 4 = 0$ then the equation of the ellipse is

(1) $\frac{(x+4)^2}{4} + \frac{(y-6)^2}{16} = 1$ (2) $\frac{(x+4)^2}{16} + \frac{(y-6)^2}{4} = 1$

(3) $\frac{(x+4)^2}{16} - \frac{(y-6)^2}{4} = 1$ (4) $\frac{(x+4)^2}{4} - \frac{(y-6)^2}{16} = 1$

36. If $\overrightarrow{PR} = 2\vec{i} + \vec{j} + \vec{k}$, $\overrightarrow{QS} = -\vec{i} + 3\vec{j} + 2\vec{k}$ then the area of the quadrilateral PQRS is

(1) $5\sqrt{3}$ (2) $10\sqrt{3}$ (3) $\frac{5\sqrt{3}}{2}$ (4) $\frac{3}{2}$

37. If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{i} + a\vec{j} - \vec{k}$ acting through the point $\vec{i} + \vec{j}$ is $\sqrt{8}$ then the value of a is

(1) 1 (2) 2 (3) 3 (4) 4

38. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

(1) (0, 0, –4) (2) (1, 0, 0) (3) (0, 2, 0) (4) (1, 2, 0)

39. The work done in moving a particle from the point A with position vector $2\vec{i} - 6\vec{j} + 7\vec{k}$ to the point B, with position vector $3\vec{i} - \vec{j} - 5\vec{k}$ by a force $\vec{F} = \vec{i} + 3\vec{j} - \vec{k}$ is

(1) 25 (2) 26 (3) 27 (4) 28

40. The axis of the parabola $y^2 - 2y + 8x - 23 = 0$ is

(1) $y = -1$ (2) $x = -3$ (3) $x = 3$ (4) $y = 1$

SECTION - B

Note: **(i) Answer any *ten* questions.**
(ii) Question No.55 is compulsory and choose any nine questions from the remaining.
(iii) Each question carries six marks. **10 x 6 = 60**

41. Find the inverse of the matrix $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$

42. Solve the following non-homogeneous equations of three unknowns.
$2x + 2y + z = 5;$ $x - y + z = 1;$ $3x + y + 2z = 4$

43. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

44. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B ,C of a triangle ABC, then prove that the area of triangle ABC is $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$. Deduce the condition for points $\vec{a}, \vec{b}, \vec{c}$ to be collinear.

45. (a) If (1+i) (1+2i) (1+3i) ..... (1+ni) = x+iy , then show that 2.5.10......... $(1+n^2) = x^2+y^2$

(b) Simplify: $\dfrac{(\cos 2\theta - i\sin 2\theta)^7(\cos 3\theta + i\sin 3\theta)^{-5}}{(\cos 4\theta + i\sin 4\theta)^{12}(\cos 5\theta - i\sin 5\theta)^{-6}}$

46. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\dfrac{n\pi}{4}, n \in N$

47. Show that $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is a strictly increasing function in the interval $\left(0, \dfrac{\pi}{4}\right)$.

48. Newton's law of cooling is given by $\theta = \theta_0° e^{-kt}$, where the excess of temperature at zero time is θ_0°C and at time t seconds is θ° C. Determine the rate of change of temperature after 40 s, given that θ_0= 16°C and k = –0.03. [$e^{1.2}$= 3.3201]

49. (a) Evaluate $\int_0^{\pi/2} e^{2x}\cos x\, dx$

(b) Evaluate $\int_{-\pi/4}^{\pi/4} x^3 \sin^2 x\, dx$

50. If u is a homogenous function of x and y of degree n, prove that $x\dfrac{\partial^2 u}{\partial x \partial y} + y\dfrac{\partial^2 u}{\partial y^2} = (n-1)\dfrac{\partial u}{\partial y}$

51. Solve $(x^2 - y)dx + (y^2 - x)dy = 0$, if it passes through the origin.

52. Construct the truth table for $(p \wedge q) \vee (\sim r)$.

53. Find the order of each element in the group (Z_7- {[0]}, .7)

54. A discrete random variable X has the following probability distributions.

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Find the value of a (ii) Find P(x <3) (iii) Find P(3 <x< 7).

55. (a) Find the equation of the rectangular hyperbola which has its centre at (2, 1), one of its asymptotes $3x - y - 5 = 0$ and which passes through the point (1, –1).

(OR)

(b) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

SECTION – C

Note: (i) Answer any *ten* questions.
(ii) Question No.70 is compulsory and choose any NINE questions from the remaining.
(iii) Each question carries ten marks. 10 x 10 = 100

56. Solve:

$x + y + 2z = 0$
$3x + 2y + z = 0$
$2x + y - z = 0$

57. Show that the lines $\dfrac{x-1}{1} = \dfrac{y+1}{-1} = \dfrac{z}{3}$ and $\dfrac{x-2}{1} = \dfrac{y-1}{2} = \dfrac{-z-1}{1}$ intersect and find their point of intersection.

58. Find the vector and Cartesian equations of the plane passing through the points (–1, 1, 1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.

59. P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$.

60. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

61. Find the eccentricity, centre, foci, vertices of the ellipse and draw the diagram.
$x^2 + 4y^2 - 8x - 16y - 68 = 0$.

62. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, (2, 4) is the centre of the hyperbola and it passes through (2, 0).

63. Find the intervals of concavity and the points of inflection of the following functions $f(\theta) = \sin 2\theta$ in $(0, \pi)$

64. Using Euler's theorem, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$, if $u = \sin^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$

65. Find the area bounded by the curve $y = x^3$ and the line $y = x$.

66. Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between t = 0 and π.

67. Solve: $dx + x\,dy = e^{-y} \sec^2 y\, dy$

68. Show that the set M of complex numbers z with the condition $|z| = 1$ forms a group with respect to the operation of multiplication of complex numbers.

69. A urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn. (i) with replacement (ii) without replacement.

70. (a) Find the equations of those tangents to the circle $x^2 + y^2 = 52$, which are parallel to the straight line $2x+3y = 6$.

(OR)

(b) The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1hour. Show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

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