MODEL HIGHER SECONDARY EXAMINATION – 2017

KEY FOR MATHEMATICS

Q.No.	Option	Answer (A)	Q.No.	Option	Answer (B)
1.	(1)	an increasing function in $\left(0,\frac{\pi}{2}\right)$	1.	(1)	$-t_2$
2.	(3)	$3a+b=0$	2.	(1)	$\cos x$
3.	(2)	$\frac{-16}{3} m/\sec$	3.	(3)	two loops between $x=-a$ and $x=a$
4.	(3)	not in the indeterminate form as $x \to 0$	4.	(4)	$e^{\int Pdx}$
5.	(1)	$8\sqrt{5}\pi$	5.	(2)	$\frac{d^2y}{dx^2}=0$
6.	(1)	48	6.	(4)	$\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$
7.	(2)	$\frac{1}{30}$	7.	(1)	1
8.	(3)	$\int_0^{2a} f(x)dx$	8.	(1)	$k^3 \det(A)$
9.	(3)	$x e^{2x}$	9.	(3)	reduces to a single equation and has infinitely many solutions
10.	(1)	$9x-8y-72=0$	10.	(2)	$-\sqrt{3}$
11.	(2)	$\frac{(x+4)^2}{16}+\frac{(y-6)^2}{4}=1$	11.	(3)	ω^{n-k}
12.	(4)	$y=1$	12.	(2)	5
13.	(4)	$\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$	13.	(4)	$\frac{1}{2}$
14.	(1)	1	14.	(2)	16 Var(X)
15.	(1)	$k^3 \det(A)$	15.	(2)	all possible values between certain given limits
16.	(3)	reduces to a single equation and has infinitely many solutions	16.	(2)	$\frac{y'}{y}$
17.	(2)	$-\sqrt{3}$	17.	(4)	$Q-\{0\}$
18.	(3)	$\frac{5\sqrt{3}}{2}$	18.	(3)	(i), (iii), (iv)
19.	(2)	2	19.	(3)	$(Z, .)$
20.	(1)	(0, 0, –4)	20.	(1)	an increasing function in $\left(0,\frac{\pi}{2}\right)$
21.	(4)	28	21.	(3)	$3a+b=0$
22.	(1)	$[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}]=0$	22.	(2)	$\frac{-16}{3} m/\sec$

23.	(4)	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$	23.	(3)	not in the indeterminate form as $x \to 0$
24.	(2)	$z_{1,} + z_3 = z_2 + z_4$	24.	(1)	$8\sqrt{5}\pi$
25.	(3)	-16	25.	(1)	48
26.	(3)	$\arg(z_1 + z_2) = \arg z_1 + \arg z_2$	26.	(1)	$[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}] = 0$
27.	(1)	$-t_2$	27.	(4)	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
28.	(1)	$\cos x$	28.	(2)	$z_{1,} + z_3 = z_2 + z_4$
29.	(3)	two loops between $x = -a$ and $x = a$	29.	(3)	–16
30.	(4)	$e^{\int Pdx}$	30.	(3)	$\arg(z_1 + z_2) = \arg z_1 + \arg z_2$
31.	(2)	$\frac{d^2y}{dx^2} = 0$	31.	(2)	$\frac{1}{30}$
32.	(2)	$\frac{y'}{y}$	32.	(3)	$\int_0^{2a} f(x)dx$
33.	(4)	$Q - \{0\}$	33.	(3)	$x e^{2x}$
34.	(3)	(i), (iii), (iv)	34.	(1)	$9x - 8y - 72 = 0$
35.	(3)	$(Z, .)$	35.	(2)	$\frac{(x+4)^2}{16} + \frac{(y-6)^2}{4} = 1$
36.	(3)	ω^{n-k}	36.	(3)	$\frac{5\sqrt{3}}{2}$
37.	(2)	5	37.	(2)	2
38.	(4)	$\frac{1}{2}$	38.	(1)	(0, 0, –4)
39.	(2)	16 Var(X)	39.	(4)	28
40.	(2)	all possible values between certain given limits	40.	(4)	$y = 1$

SECTION – B

41. $|A| = -1$ **- 1 mark**

Adj A = $\begin{bmatrix} -2 & -1 & 1 \\ 0 & -2 & -1 \\ -5 & -2 & 3 \end{bmatrix}$ **-3 marks**

$A^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ **-2 marks**

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42. $\Delta = 0$ **- 2 marks**

$\Delta x \neq 0$ **- 2 marks**

Since $\Delta = 0$ and $\Delta x \neq 0$ the system is inconsistent.

i.e. It has no solution. **- 2 marks**

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43. $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$ - 1 mark

$\vec{a}=\pm\lambda(\vec{b}\times\vec{c})$ - 2 marks

$\lambda = 2$ - 2 marks

$\vec{a}=\pm\ 2(\vec{b}\times\vec{c})$ - 1 mark

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44. Area of Δ ABC $= \frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|$

$= \frac{1}{2}\left|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right|$ - 2 marks

Area of Δ ABC $= \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$ - 2 marks

Condition for collinear $\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|=0$

(or)

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$ - 2 marks

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45. (a) Given (1 + i) (1 + 2i) (1 + 3i) ……. (1 + ni) = $x + iy$

Taking modulus

$\left|(1+i)(1+2i)\ (1+3i).....(1+ni)\right| = \left|x+iy\right|$ -1 mark

$\sqrt{1+1}\quad\sqrt{1+4}\quad\sqrt{1+9}\qquad\sqrt{1+n^2} = \sqrt{x^2+y^2}$ - 1 mark

Squaring on both sides

$2 . 5 . 10\ldots(1+n^2) = x^2+y^2$ - 1 mark

(b) The given expression $= \dfrac{\left(e^{-i2\theta}\right)^{7}\left(e^{i3\theta}\right)^{-5}}{\left(e^{i4\theta}\right)^{12}\left(e^{-i\theta}\right)^{-6}}$ -1 mark

$= \dfrac{e^{-i14\theta}\ e^{-i15\theta}}{e^{-i52\theta}\ e^{i6\theta}}$ - 1 mark

$= e^{-i107\theta} = cis(-107\theta)$ - 1 mark

Note : For any other method full marks may be given.

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46. $1+i = \sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ - 3 marks

$\therefore (1+i)^n = \left(\sqrt{2}\right)^n\ (\cos n\frac{\pi}{4}+i\sin n\frac{\pi}{4})$ - 1 mark

$(1-i)^n = \left(\sqrt{2}\right)^n\ (\cos n\frac{\pi}{4}-i\sin n\frac{\pi}{4})$ - 1 mark

$(1+i)^n+(1-i)^n = 2^{\frac{n+2}{2}}\ \cos n\frac{\pi}{4}$ - 1 mark

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47. $f(x)=\tan^{-1}(\sin x+\cos x)$

$f^1(x)=\dfrac{\cos x-\sin x}{2+\sin 2x}>0$ -2 marks

Since *cos x-sin x > 0* in the interval

$\left(0, \frac{\pi}{4}\right)$ and 2+ sin 2x > 0 **-2 marks**

$\therefore$ $f(x)$ is strictly increasing function of x in the interval $\left(0, \frac{\pi}{4}\right)$ **-2 marks**

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48. $\frac{d\theta}{dt} = -k\theta_0^{\circ} e^{-kt}$ **-2 marks**

$\left(\frac{d\theta}{dt}\right)_{t=40} = 0.48 \times e^{1.2}$ **-2 marks**

= 1.5936° c/s **-2 marks**

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49. (a) $\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = \left[\frac{e^{2x}}{(2)^2 + (1)^2}(2\cos x + \sin x)\right]^{\frac{\pi}{2}}$ **- 2 marks**

$= \frac{1}{5}(e^{\frac{\pi}{2}} - 2)$ **- 1 mark**

(b) $f(x) = x^3 \sin^2 x$

$f(-x) = -x^3 \sin^2 x$

$f(-x) = -f(x)$ **- 1 mark**

$f(x)$ is an odd function **- 1 mark**

$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^2 x \, dx = 0$ **- 1 mark**

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50. $x\,(Uy)_x + y(Uy)_y = (n-1)Uy$ **- 2 marks**

$x\ U yx + y\ Uyy = (n-1)\ Uy$ **- 2 marks**

$x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$ **-2 marks**

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51. $x^2 dx + y^2 dy = d(xy)$ **-2 marks**

Integrating

$\frac{x^3}{3} + \frac{y^3}{3} = xy + c$ **-1 mark**

It passes (0 ,0) , C = 0 **-1 mark**

The required solution is

$\frac{x^3}{3} + \frac{y^3}{3} = xy\ (or) x^3 + y^3 = 3xy$ **-1 mark**

52.

p	q	r	p Λq	~ r	(p Λ q) V(~ r)
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F

F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

Each columns (1x 6) **- 6 marks**

Note: The order of the truth values of p, q, r need not same as in the table.

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53. G = { [1] , [2], ……………..[6] }

$0[1] = 1$ **- 1 mark**

$0[2] = 3$ **- 1 mark**

$0[3] = 6$ **- 1 mark**

$0[4] = 3$ **- 1 mark**

$0[5] = 6$ **- 1 mark**

$0[6] = 2$ **- 1 mark**

54. (i) $a = \frac{1}{81}$ **- 2marks**

(ii) $P(x<3) = \frac{9}{81}$ (or) $\frac{1}{9}$ **- 2 marks**

(iii) $P(3 < x < 7) = \frac{33}{81}$ or $\frac{11}{27}$ **- 2 marks**

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55. (a) The other asymptote is $x + 3y + k = 0$

It passes centre (2, 1), k = –5, $x + 3y - 5 = 0$ **-1 mark**

Combined equation of the asymptote is

$(3x- y-5)\ (x+3y-5) =0$ **- 1 mark**

The equation of the rectangular hyperbola

$(3x - y - 5)(x + 3y - 5) + C = 0$ **-1 mark**

It passes (1, –1), C = –7 **-1 mark**

$(3x - y - 5)(x + 3y - 5) - 7 = 0$ **-2 marks**

(b) When X=12 , Z=2 **- 1 mark**

$P(X \leq 12) = P(Z \leq 2)$

$= P(-\infty < Z < 2) = P(-\infty < Z < 0) + P(0 < Z < 2)$ **-2 marks**

$P(X \leq 12) = 0.9772$ **- 1 mark**

Pairs of Shoes = 4886 **-2 marks**

SECTION – C

56. $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$ **- 1 mark**

Since $\Delta = 0$, it has infinitely many solutions **- 1 mark**

$Z = k$ **- 1 mark**

$x + y = -2k,\ 2x + y = k$ **- 1 mark**

$\Delta = -1$ **- 1 mark**

$\Delta x = -3k$ **- 1 mark**

$\Delta y = 5k$ **- 1 mark**

By Cramer's Rule

$x=3k,\ y = -5k$ **-2 marks**

Solution is $(x, y, z) = (3k, -5k, k)$ **- 1 mark**

Note: Award full marks for any other correct suitable methods.

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57. Condition for intersecting is

$$\left[(\vec{a}_2 - \vec{a}_1)\,\vec{u}\,\vec{v}\right] = 0 \qquad \text{(OR)} \qquad \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$ **- 1 mark**

$$\left[(\vec{a}_2 - \vec{a}_1)\ \vec{u}\ \vec{v}\right] = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = 0$$ **- 2 marks**

Further $\vec{u}$ and $\vec{v}$ are not parallel $\therefore$ The lines intersect **- 1 mark**

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3} = \lambda$$

Any point on this line is of the form

$(\lambda + 1, -\lambda - 1, 3\lambda)$ **- 1 mark**

$$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{-1} = \mu$$

Any point on this line is of the form

$(\mu + 2, 2\mu + 1, -\mu - 1)$ **- 1 mark**

$(\lambda + 1, -\lambda - 1, 3\lambda) = (\mu + 2, 2\mu + 1, -\mu - 1)$ **- 1 mark**

$\mu = -1,\ \lambda = 0$ **-2 marks**

The point of intersection is (1, -1, 0) **- 1 mark**

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58. $\vec{a} = -\vec{i} + \vec{j} + \vec{k}$ **- 1 mark**

$\vec{b} = \vec{i} - \vec{j} + \vec{k}$ **- 1 mark**

$\vec{r} = \vec{i} + 2\vec{j} + 2\vec{k}$ **- 1 mark**

Vector Form

$\vec{r} = (1-s)\left(-\vec{i} + \vec{j} + \vec{k}\right) + s\left(\vec{i} - \vec{j} + \vec{k}\right) + t\left(\vec{i} + 2\vec{j} + 2\vec{k}\right)$ **- 2 marks**

Cartesian Form

$$\begin{vmatrix} x+1 & y-1 & z-1 \\ 2 & -2 & 0 \\ 1 & 2 & 2 \end{vmatrix} = 0$$ **- 3 marks**

$2x + 2y - 3z + 3 = 0$ **- 2 marks**

Note: Other vector form equation (or) Cartesian equation may be adopted.

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59. $\arg(z-1) - \arg(z+3) = \dfrac{\pi}{2}$ **- 2 marks**

$\tan^{-1}\left(\dfrac{y}{x-1}\right) - \tan^{-1}\left(\dfrac{y}{x+3}\right) = \dfrac{\pi}{2}$ **- 2 marks**

$$\tan^{-1}\left(\frac{\dfrac{y}{x-1} - \dfrac{y}{x+3}}{1 + \dfrac{y}{x-1}\cdot\dfrac{y}{x+3}}\right) = \frac{\pi}{2}$$ **- 2 marks**

$\Rightarrow 1 + \dfrac{y}{x-1}\cdot\dfrac{y}{x+3} = 0$ **- 2 marks**

$\therefore$ Locus of P is $x^2 + y^2 + 2x - 3 = 0$

Note: Award full marks for any other correct suitable method. **-2 marks**

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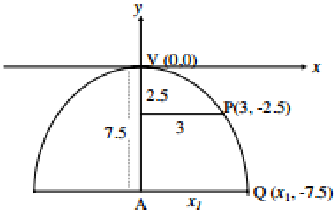
60. Rough Diagram - **2 marks**

$x^2 = -4ay$ - **1 mark**

$a = \frac{9}{10}$ - **2 marks**



$x^2 = -4x\frac{9}{10}y$ -**1 mark**

The point (x_1, −7.5) lies on the parabola - **2 marks**

$x_1 = 3\sqrt{3}$ - **2 marks**

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61. **Rough diagram** - **2 marks**

$$\frac{(x-4)^2}{100} + \frac{(y-2)^2}{25} = 1$$ - **2 marks**

$e = \frac{\sqrt{3}}{2}$ - **1 mark**

Centre (4, 2) - **1 mark**

$F_1(4+5\sqrt{3}, 2)$ - **1 mark**

$F_2(4-5\sqrt{3}, 2)$ - **1 mark**

A (14 ,2) - **1 mark**

A' (− 6, 2) - **1 mark**

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62. Equations of the two asymptotes are of the form

$x + 2y + l = 0$ and $x - 2y + m = 0$ - **2 marks**

$l = -10$, m = 6 - **2 marks**

Equations of the asymptotes are

$x+2y-10=0$ and $x-2y+6=0$ - **1 mark**

Combined equation of the asymptotes is

$(x+2y-10)(x-2y+6)=0$ - **1 mark**

The equation of the hyperbola is of the form

$(x+2y-10)(x-2y+6)+k=0$ - **1 mark**

k = 64 - **2 marks**

Equation of the hyperbola is of the form

$(x+2y-10)(x-2y+6)+64=0$ - **1 mark**

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63. $f'(\theta) = 2\cos 2\theta$ - **1 mark**

$f''(\theta) = -4\sin 2\theta$ - **1 mark**

$f''(\theta) = 0 \Rightarrow \theta = \frac{\pi}{2}$ - **2 marks**

$(\pi/2, \pi)$ → concave upward (or) convex downward - **2 marks**

$(0, \pi/2)$ →concave downward (or) convex upward - **2 marks**

Point of inflection $(\frac{\pi}{2}, 0)$ - **2 marks**

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64. f = sin u - **2 marks**

Degree = 2 - **2 marks**

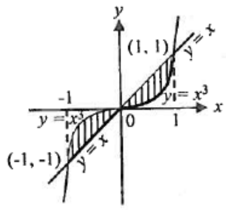
$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f$ -**3 marks**

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u$ **- 3 marks**

65. Rough Diagram **- 2 marks**

The points of intersection at

$x = \{0, \pm 1\}$ **- 2 marks**



- 1 mark

Area = $\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx$ **- 2 marks**

$= \left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}-\frac{x^4}{4}\right]_{0}^{1}$ **- 2 marks**

= ½ sq units **- 2 marks**

66. $\frac{dx}{dt}$ = a(1 – cos t) **- 2 marks**

$\frac{dy}{dt}$ = a sin t **- 2 marks**

$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 = 4a^2\sin^2\left(\frac{t}{2}\right)$

(or)

$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = 2a\sin\frac{t}{2}$ - 3 **marks**

Length = $\int_{0}^{\pi}2a\sin\frac{t}{2}dt$ - 1 **mark**

Correct integration **- 1 mark**

Length = 4a **- 1 mark**

67. $\frac{dx}{dy}+x = e^{-y}\sec^2 y$ **- 2 marks**

I.F = e^y **- 3 marks**

Required solution is

$xe^{y}=\int e^{-y}\sec^2 y\, e^{y}\, dy$ **- 3 marks**

$xe^{y} = \tan y + c$ **- 2 marks**

68. M= $\{z\in c \,|\, |z|=1\}$ **- 1 mark**

(i) **Closure axiom:** **- 1 mark**

$Z_1\ Z_2 \in M$ **- 1 mark**

(ii) **Associative axiom:** **- 1 mark**

The multiplication of complex number is always associative **- 1 mark**

(iii) **Identity axiom:** **- 1 mark**

1 is the identity axiom **- 1 mark**

(iv) **Inverse axiom:** **- 1 mark**

$\frac{1}{z}$ is the inverse of Z **- 1 mark**

∴ M is a group with respect to the multiplication of complex numbers **- 1 mark**

69. **With replacement**

$P(R) = \frac{3}{7}$ **- 1 mark**

$P(W) = \frac{4}{7}$ **- 1 mark**

$P(X=0) = \frac{64}{343}$ **- 1 mark**

$P(X=1) = \frac{144}{343}$ **- 1 mark**

$P(X=2) = \frac{108}{343}$ **- 1 mark**

$P(X=3) = \frac{27}{343}$ **- 1 mark**

<u>Without replacement</u>

$P(X=0) = \frac{4}{35}$ **- 1 mark**

$P(X=1) = \frac{18}{35}$ **- 1 mark**

$P(X=2) = \frac{12}{35}$ **- 1 mark**

$P(X=3) = \frac{1}{35}$ **- 1 mark**

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70. (a) We have

$x^2+y^2 = 52$

$\frac{dy}{dx} = \frac{-x}{y}$ **- 2 marks**

$\left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-x_1}{y_1}$

Given line is $2x+3y=6$

$\left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-2}{3}$ **- 1 mark**

We have $\frac{-x_1}{y_1} = \frac{-2}{3}$ (or) $x_1 = \frac{2}{3}y_1$ **- 1 mark**

$y_1 = \pm 6$ **- 1 mark**

$y_1 = 6 \Rightarrow x_1 = 4$ **- 1 mark**

$y_1 = -6 \Rightarrow x_1 = -4$ **- 1 mark**

Points are (4, 6) and (−4 , −6) **- 2 marks**

Equation of tangents $2x+3y \pm 26 = 0$ **- 2 marks**

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70. (b) $\frac{dA}{dt} = kA$ **- 2 marks**

$A = C e^{kt}$ **- 1 mark**

When t = 0, assume that A = Ao

Ao = C **-2 Marks**

When $t = 1$, $A = 3Ao$ **-1 mark**

$e^k=3$ **-2 marks**

When $t = 5$, $A = 3^5$ Ao **-2 marks**

Note: Instead of Ao one may take any different symbol.

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