HIGHER SECONDARYSECOND YEAR CENTUM ASSURED MODEL EXAM 2017

Time : 3.00 hrs. **MATHEMATICS** Max. Marks : 200

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Section – A

Note: 1. ***Answer all the questions.***

2. ***Choose the most suitable answer from the given four alternatives.*** 40 x 1 = 40

1. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

a) $\frac{1}{k^2} I$ b) $\frac{1}{k^3} I$ c) $\frac{1}{k} I$ d) k I

2. If A is a square matrix of order n then $|adjA|$ is

a) $|A|^2$ b) $|A|^n$ c) $|A|^{n-1}$ d) $|A|$

3. In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta=0$ and $\Delta_x = 0$, $\Delta_y \neq 0$, $\Delta_z = 0$ then the system has

a) unique solution b) two solutions c) infinitely many solution d) no solution.

4 . In the system of 3 linear equations with three unknowns, in the non-homogeneous system $\rho(A) = \rho(A,B) = 2$ then the system a) has unique solution b) reduces to 2 equations and has infinitely many solution c) reduces to a single equation and has infinitely many solution d) is inconsistent

5. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ then the angle between $\vec{a}$ and $\vec{b}$ is,

a) $\theta = \frac{\pi}{6}$ b) $\theta = \frac{2\pi}{3}$ c) $\theta = \frac{5\pi}{3}$ d) $\theta = \frac{\pi}{2}$

6. If $\vec{p}$ and $\vec{q}$ and $\vec{p}+\vec{q}$ are vectors of magnitude λ then the magnitude of $|\vec{p}-\vec{q}|$ is

a) 2λ b) $\sqrt{3}\lambda$ c) $\sqrt{2}\lambda$ d) 1

7. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar and $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$ then the value of $[\vec{a},\vec{b},\vec{c}]$ is,

a) 2 b) 3 c) 1 d) 0

8.. The equation of the plane passing through the point (2, 1, -1) and the line of intersection of the planes $\vec{r}\cdot(\vec{i}+3\vec{j}-\vec{k})=0$ and $\vec{r}\cdot(\vec{j}+2\vec{k})=0$ is,

a) x+4y-z=0 b) x+9y+11z=0 c) 2 x+y-z+5=0 d) 2x-y+z=0

9..If $|\vec{u}|=3, |\vec{v}|=4$ and $\vec{a}\cdot\vec{b}=9$ then $|\vec{a}\times\vec{b}|$ is

a) $3\sqrt{7}$ b) 63 c) 69 d) $\sqrt{69}$

10.Chord AB is a diameter of the sphere $\left|\vec{r}-\left(2\vec{i}+\vec{j}-6\vec{k}\right)\right|=\sqrt{18}$ with coordinate of A as (3,2,-2) The coordinates of B Is

a) (1,0,10) b) (-1,0,-10) c) (-1,0,10) d) (1,0,-10)

11. The modulus and amplitude of the complex number $\left[e^{3-i\pi/4}\right]^3$ are respectively

a) $e^9, \frac{\pi}{2}$ b) $e^9, \frac{-\pi}{2}$ c) $e^6, \frac{-3\pi}{4}$ d) $e^9, \frac{-3\pi}{4}$

12. If $a=3+i$ and $z=2-3i$ then the points on the Argand diagram representing $az, 3az$ and $-az$ are

a) vertices of a right angled triangle b) vertices of an equilateral triangle
c) vertices of an isosceles triangle d) collinear

13. The polar form of the complex number $(i^{25})^3$ is

a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ b) $\cos\pi+i\sin\pi$ c) $\cos\pi-i\sin\pi$ d) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

14. Polynomial equation P(x)=0 admits conjugate pairs of roots only if the coefficients are

a) imaginary b) complex c) real d) either real or complex

15. The angle between the two tangents drawn from the point (-4, 4) to $y^2 = 16x$ is.

a) 45° b) 30° c) 60° d) 90°

16. The point of intersection of the tangents $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is.

a) $(6t^2, 8t)$ b) $(8t, 6t^2)$ c) $(t^2, 4t)$ d) $(4t, t^2)$

17. The directrix of the hyperbola $x^2 - 4(y-3)^2 = 16$ is

a) $y = \pm\frac{8}{\sqrt{5}}$ b) $x = \pm\frac{8}{\sqrt{5}}$ c) $y = \pm\frac{\sqrt{5}}{8}$ d) $x = \pm\frac{\sqrt{5}}{8}$

18. The locus of the foot of perpendicular from the focus on any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

19. If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero is.

a) $\frac{32}{3}$ m / sec b) $\frac{-16}{3}$ m / sec c) $\frac{16}{3}$ m / sec d) $\frac{-32}{3}$ m / sec

20. Which of the following curves is concave down ?

a) $y = -x^2$ b) $y = x^2$ c) $y = e^x$. d) $y = x^2 + 2x - 3$.

21. If $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to

a) yx^{y-1}. b) u log x c) u log y d) xy^{x-1}.

22. The least possible perimeter of a rectangle of area 100 m^2 is.

a) 10 b) 20 c) 40 d) 60

23. $\lim_{x \to 0} \frac{x}{\tan x}$ is a)1 b) - 1 c) 0 d) ∞

24. The curve $y^2(x-2) = x^2(1+x)$ has

a) an asymptote parallel to x-axis b) an asymptote parallel to y-axis

c) asymptotes parallel to both axes d) no asymptotes

25. The value of $\int_0^{\pi/2} \frac{\cos^{5/3} x}{\cos^{5/3} x + \sin^{5/3} x} dx$ a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c)0 d) π

26. The area of the region bounded by the graph of y = sin x and y = cos x between x = 0 and x = $\pi/4$ is a) $\sqrt{2}+1$ b) $\sqrt{2}-1$ c) $2\sqrt{2}+1$ d) $2\sqrt{2}+2$

27. $\int_0^\infty x^6 e^{-\frac{x}{2}} dx$ = a). $\frac{\angle 6}{2^7}$ b). $\frac{\angle 6}{2^6}$ c) $2^6 \angle 6$ d). $2^7 \angle 6$

28. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is.

a) 48 b) 24 c) 12 d) 96

29. In finding the differential equation corresponding to $y = e^{mx}$ where m is the arbitrary constant, then m is

a) $\frac{y}{y'}$ b) $\frac{y'}{y}$ c) y' d) y

30. The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ is

a) $\int Pdx$ b) $\int Qdx$ c) $e^{\int Qdx}$ d) $e^{\int Pdx}$

31. The complementary function of $(D^2 + 1)y = e^{2x}$ is.

a) $(Ax+B)e^x$. b) A cos x + B sin x . c) $(Ax+B)e^{2x}$. d) $(Ax+B)e^{-x}$.

32. .The differential equation satisfied by all the straight lines in xy plane is

a) $\frac{dy}{dx} = \text{a constant}$ b) $\frac{d^2y}{dx^2} = 0$ c) $y + \frac{dy}{dx} = 0$ d) $\frac{d^2y}{dx^2} + y = 0$

33. $p \leftrightarrow q$ is equivalent to

a) $p \to q$ b) $q \to p$ c) $(p \to q) \vee (q \to p)$ d) $(p \to q) \wedge (q \to p)$

34. The order of [7] in (Z_9, $+_9$) is… a) 9 b) 6 c) 3 d) 1.

35. In the group (G, .), $G = \{1, -1, i, -i\}$, the order of –i is

a) 2 b) 0 c) 4 d) 3

36. Given E(X +c) = 8 and E (X –c) = 12 then the value of c is.

a) -2 b) 4 c) -4 d) 2.

37. If in a Poisson distribution P(X=0) = k then the variance is.

a) log 1/ k b) log k. c) e^{λ} d) 1 / k

38. A monoid becomes a group if it also satisfies the

a) closure axiom. b) associative axiom c) identity axiom d) inverse axiom.

39. The distribution function F(x) of a random variable X is.

a) a decreasing function b) a non decreasing
c) a constant function d) increasing first and then decreasing

40. For a standard normal distribution the mean and variance are

a) μ , σ^2 b) μ , σ c) 0,1 d) 1,1

Section – B

Note: 1. ***Answer any 10 questions*** 2. ***Question No. 55 is compulsory and choose any nine questions from the remaining.*** 10××6=60

41. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result $A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,.\,I$

42. Solve the following non-homogeneous equations of three unknown

$x + y + 2z = 4$; $2x + 2y + 4z = 8$; $3x + 3y + 6z = 10$

43 . Angle in a semi-circle is a right angle. Prove by vector method.

44. (a) Find the coordinates of the centre and the radius of the sphere whose vector equation is $(\vec{r})^2 - \vec{r} \circ \left(4\vec{i} + 2\vec{j} - 6\vec{k}\right) - 11 = 0$

(b)Find the angle between the following lines. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-4}{6}$ and $x+1 = \frac{y+2}{2} = \frac{z-4}{2}$

45. If n is a positive integer , prove that $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos n\left(\frac{\pi}{2} - \theta\right) + i\sin n\left(\frac{\pi}{2} - \theta\right)$

46. Find the square root of $(8 + 6i)$

47. At what angle θ do the curves $y = a^x$ and $y = b^x$ intersect $(a \neq b)$?

48. Determine for which values of x, the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing and for which it is decreasing. Also determine the points where the tangents to the graph of the function are parallel to the x axis.

49. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ if $w = \log\left(x^2 + y^2\right)$ where $x = r\cos\theta$, $y = r\sin\theta$

50.Evaluate : $\int_0^1 x(1-x)^{10}\, dx$

51. Solve: $3e^x \tan y\, dx + \left(1 + e^x\right)\sec^2 y\, dy = 0$

52. Show that $\sim(p \wedge q) \equiv ((\sim p) \vee (\sim q))$

53. (a) For the probability density function $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$,findF(2)

(b)Prove that the total probability is one.

54.Suppose that the amount of cosmic radiation to which a person is exposed when flying by jet across the United States is a random variable having a normal distribution with a mean of 4.35 m rem and a standard deviation of 0.59 m rem. What is the probability that a person will be exposed to more than 5.20 m rem of cosmic radiation of such a flight.

55.(a) A standard rectangular hyperbola has its vertices at(5,7) and (-3,-1). Find its equation and asymptotes.

(b)Show that the cube roots of unity forms a finite abelian group under multiplication.

Section - C

Note: 1. ***Answer any 10 questions.*** 2. ***Question No. 70 is compulsory and choose any nine questions from the remaining*** $10\times10=100$

56. Solve the following non-homogeneous system of linear equations determinant method:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=4\ ;\ \frac{1}{x}-\frac{1}{y}+\frac{1}{z}=2\ ;\ \frac{2}{x}+\frac{1}{y}-\frac{1}{z}=1$$

57. Show that the lines $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x-2}{1}=\frac{y-1}{2}=\frac{-z-1}{1}$ intersect and find their point of intersection.

58. Derive the equation of the plane in the intercept form.

59. If P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right)=\frac{\pi}{2}$

60. A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet nearer to the sun?(Take the orbit as open rightward).

61.Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram: $x^2+4y^2-8x-16y-68=0$

62. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x+2y-5=0$ and passes through the points (6,0) and (-3,0).

63. The distance x metres traveled by a vehicle in time t seconds after the brakes are applied is given by : $x=20t-5/3t^2$.Determine

64. Using Euler's theorem prove that ,If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u.$

65. Find the length of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{a}\right)^{2/3}=1$

66. Solve: $dx+x\,dy=e^{-y}\sec^2 y\,dy$

67. A drug is excreted in a patients urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of drug at time t = 0 , which is excreted at a Rate of $-3t^{1/2}$ mg/h.(i) What is the general equation for the amount of drug in the patient at time t > 0 ? (ii) When will the patient be drug free?

68. Show that the set $G=\{a+b\sqrt{2}\ /\ a,\ b\in Q\}$ is an infinite abelian group with respect to addition.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year . $\left[e^{-3}=0.0498\right]$.

70.(a) A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What ar the dimensions of the field that has the largest area?(

(b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$