Class: **XII**
Sub : **Mathematics** ***Marks : 200***
Time : 2 ½ hrs.

I. Choose the best answer: **40 x 1=40**

1. The rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{bmatrix}$ is
   (a) 1 (b) 2 (c) 3 (d) 4
2. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is
   (a) $\frac{I}{k^2}I$ (b) $\frac{I}{k^3}I$ (c) $\frac{I}{k}I$ (d) kI
3. If A is a square matrix of order *n* then $|adjA|$ is
   (a) $|A|^2$ (b) $|A|^n$ (c) $|A|^{n-1}$ (d) $|A|$
4. If A = $\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$, then A^{12} is
   (a) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
5. If $\vec{a}$ is a non- zero vector and *m* is a non-zero scalar then *m* $\vec{a}$ is a unit vector if
   (a) m = ± 1 (b) a = $|m|$ (c) $a = \frac{1}{|m|}$ (d) a = 1
6. The vectors $2\vec{i}+3\vec{j}+4\vec{k}$ and $a\vec{i}+b\vec{j}+c\vec{k}$ are perpendicular when
   (a) a =2, b =3, c = -4 (b) a =4 , b = 4, c =5 (c) a =4, b =4, c= -5 (d) a =-2 , b =3, c =4
7. The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ is
   (a) $10\sqrt{3}$ (b) $6\sqrt{30}$ (c) $\frac{3}{2}\sqrt{30}$ (d) $3\sqrt{30}$
8. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then
   (a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ is perpendicular to $\vec{b}$ (c) $|\vec{a}| = |\vec{b}|$ (d) $\vec{a}$ and $\vec{b}$ are unit vectors.
9. If the magnitude of moment about the point $\vec{j}+\vec{k}$ of a force $\vec{i}+a\vec{j}-\vec{k}$ acting through the point $\vec{i}+\vec{j}$ is $\sqrt{8}$ then the value of *a* is
   (a) 1 (b) 2 (c) 3 (d) 4
10. If $\vec{a},\vec{b},\vec{c}$ are a right handled triad of mutually perpendicular vectors of magnitude a, b, c then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is (a) $a^2b^2c^2$ (b) 0 (c) $\frac{1}{2}abc$ (d) abc
11. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-2}{3}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is
    (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2\sqrt{6}}$

12. The following two lines are $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z-1}{2}$

(a) parallel (b) intersecting (c) skew (d) perpendicular

13. 18. $\vec{r}=s\vec{i}+t\vec{j}$ is the equation of

(a) a straight line joining the points $\vec{i}$ and $\vec{j}$ (b) xoy plane

(c) yoz plane (d) zox plane

14. The value of $\left[\vec{i}+\vec{j},\vec{j}+\vec{k},\vec{k}+\vec{i}\right]$ is equal to (a) 0 (b) 1 (c) 2 (d) 4

15. If – i +3 is a root of $x^2 – 6x + k = 0$ then the value of *k* is

(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10

16. If $A+ iB = (a_1 + ib_1) (a_2 + ib_2) (a_3+ ib_3)$ then A^2+B^2 is

(a) $a_1^2+ b_1^2 + a_2^2 + b_2^2 + a_3^2 +b_3^2$ (b) $(a_1+ a_2+ a_3)^2 + (b_1+ b_2+ b_3)^2$

(c) $(a_1^2+b_{12}) (a_2^2+b_2^2) (a_3^2+b_3^2)$ (d) $(a_1^2+a_2^2+a_3^2) (b_1^2+b_2^2+b_3^2)$

17. If $-\bar{z}$ lies in the third quadrant then z lies in the

(a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant

18. If $x^2+y^2 = 1$ then the value of $\frac{1+x+iy}{1+x-iy}$ is

(a) x-iy (b) 2x (c) -2 iy (d) x + iy

19. $a = \cos\alpha – i \sin\alpha$, $b = \cos\beta – i \sin\beta$, $c = \cos\gamma – i \sin\gamma$ then $(a^2 c^2 - b^2) / abc$ is

(a) $\cos 2(\alpha-\beta+y) + i \sin 2(\alpha- \beta+\gamma)$ (b) $-2 \cos(\alpha- \beta + \gamma)$

(c) $-2 i \sin(\alpha- \beta+ \gamma)$ (d) $2 \cos(\alpha- \beta + \gamma)$

20. If the amplitude of a complex number is $\pi/2$ then the number is

(a) purely imaginary (b) purely real

(c) 0 (d) neither real nor imaginary

21. The line $4x +2y = c$ is a tangent to the parabola $y^2 =16x$ then c is

(a) -1 (b) -2 (c) 4 (d) -4

22. The point of intersection of the tangents at $t^1 = t$ and $t^2 = 3t$ to the parabola $y^2 = 8x$ is

(a) $(6t^2 ,8t)$ (b) $(8t , 6t^2)$ (c) $(t^2, 4t)$ (d) $(4t, t^2)$

23 The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is

(a) 8 (b) 6 (c) 4 (d) 2

24.The length of the latus rectum of the parabola whose vertex is (2,-3) and the directrix x =4 is

(a) 2 (b) 4 (c) 6 (d) 8

25.The focus of the parabola $x^2 =16 y$ is

(a) (4, 0) (b) (0, 4) (c) (-4, 0) (d) (0,-4)

26. The distance between the foci of the ellipse $9x^2 + 5y^2 =180$ is

(a) 4 (b) 6 (c)8 (d) 2

27.The gradient of the curve $y = -2x^3 + 3x + 5$ at x =2 is

(a) -20 (b) 27 (c) -16 (d) -21

28.The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at x = 0 is

(a) 3 (b) 2 (c) 1 (d) -1

29.What is the surface area of a sphere when the volume is increasing at the same rate as its radius?

(a) 1 (b) $\frac{1}{2\pi}$ (c) 4π (d) $\frac{4\pi}{3}$

30.If a normal makes an angle θ with positive x- axis then the slope of the curve at the point where the normal is drawn is

(a) – cot θ (b) tan θ (c) – tan θ (d) cot θ

31.The value of *c* in Rolle's Theorem for the function $f(x)=\cos\frac{x}{2} on[\pi,3\pi]$ is

(a) 0 (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

32.The curve $ay^2 = x^2 (3a-x)$ cuts the y axis at.

a) x = - 3a , x = 0 b) x =0, x = 3a. c) x = 0, x = a. d) x = 0

33.If $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to

a) yx^{y-1}. b) u log x c) u log y d) xy^{x-1}.

34. In the homogeneous system $\rho(A)$ < the number of unknowns then the system has

a) only trivial solution b) trivial solution and infinitely many non-trivial solutions

c) only non-trivial solutions d) no solution

35. The work done in moving a particle from the point A, with position vector $2\vec{i} - 6\vec{j} + 7\vec{k}$, to the point B , with position vector $3\vec{i} - \vec{j} - 5\vec{k}$, by a force $\vec{F} = \vec{i} + 3\vec{j} - \vec{k}$ is

a) 25 b) 26 c) 27 d) 28

36. The vector equation of a sphere whose centre is origin and radius 'a' is

a) $r = \vec{a}$ b) $\vec{r} - \vec{c} = \vec{a}$ c) $\left|\vec{r}\right| = \left|\vec{a}\right|$ d) $\vec{r} = a$

37. If Z = 0 then the $\arg(Z)$ is

a) 0 b) π c) $\frac{\pi}{2}$ d) indeterminate

38. The axis of the parabola $y^2 = 4x$ is

a) x = 0 b) y = 0 c) x = 1 d) y = 1

39. The foot of a perpendicular from a focus of the hyperbola on an asymptote lies on the ---------

a) Centre b) corresponding directrix c) vertex d) Latusrectum

40. Let " h " be the height of the tank. Then the rate of change of pressure " p " of the tank with respect to height is

a) $\frac{dh}{dt}$ b) $\frac{dp}{dt}$ c) $\frac{dh}{dp}$ d) $\frac{dp}{dh}$

SECTION- B

II. ***Note:*** ***(i) Answer any TEN questions.***
(ii) Question No.55 is compulsory ***10 x 6=60***

22. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$
23. Solve by matrix inversion method: 2x – y = 7 3x - 2y = 11.
24. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and verify the result $(A^T)^{-1} = (A^{-1})^{-T}$
25. Angle in a semi-circle is a right angle. Prove by vector method.
26. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$
27. Show that the two lines $\vec{r} = (\vec{i} - \vec{j}) + t(2\vec{i} + \vec{k})$ and $\vec{r} = (2\vec{i} - \vec{j}) + s(\vec{i} + \vec{j} - \vec{k})$ and skew lines and find the distance between them.
28. Find the vector and Cartesian equation of the sphere on the join of the points A and B having position vectors $2\vec{i} + 6\vec{j} - 7\vec{k}$ and $-2\vec{i} + 4\vec{j} - 3\vec{k}$ respectively as a diameter. Find also the centre and radius of the sphere.
29. Show that the points representing the complex numbers 7 + 9i, -3 + 7i and 3 +3i in the Argand diagram respectively.
30. If $x = \cos\alpha + i\sin\alpha; y = \cos\beta + i\sin\beta$ prove that $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$
31. The head light of a motor vehicle is a parabolic reflector of diameter 12 cm and depth 4cm. Find the position of bulb on the axis of the reflector for effective functioning of the headlight.
32. Prove that the product of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is constant and the values is $\frac{a^2b^2}{a^2 + b^2}$
33. Prove that the curve $2x^2 + 4y^2 = 1$ and $6x^2 - 12y^2 = 1$ cut each other at right angles.
34. Obtain the Maclaurin's Series is : arc tan x or $\tan^{-1} x$.
35. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ if $w = \log\left(x^2 + y^2\right)$ where $x = r\cos\theta$, $y = r\sin\theta$
36. (a) Solve the following non-homogeneous system of linear equations :
    2x + 2y + z =5 x –y + z =1 3x + y + 2z = 4
    (OR)
    (b) Find the square root of (– 8 – 6i)

SECTION-C

Note: ***(i) Answer any Ten questions.*** ***10X10=100***
(ii) Question no. 70 is compulsory.

37. Discuss the solutions of the system of equations for all values of λ.
    x + y + z =2, 2x +y - 2z = 2, λx + y + 4z = 2
38. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of a blue chair is Rs. 260 and the cost of a green chair is Rs.300. The

total cost of chair is Rs. 25, 000. Find at least 3 different solution of the number of chairs in each colour to be purchased.

39. Prove that sin (A + B) = sin A cos B + cos A sin B.
40. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect and hence find the point of intersection.
41. Find the vector and Cartesian equations of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$
42. If $\vec{a}=2\vec{i}+3\vec{j}-5\vec{k}$, $\vec{b}=-\vec{i}+5\vec{k}$, $\vec{c}=2\vec{j}-3\vec{k}$ verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$.
43. P represents the variable complex number z, find the locus of P if $\arg\left(\frac{z-1}{z+3}\right)=\frac{\pi}{2}$
44. Find all the values of $\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{3/4}$ and hence prove that the product of the values is 1.
45. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12mts away from the starting point. Find the angle of projection.
46. A ladder of length 15 m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the focus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.
47. A kho-kho player in a practice session while running realizes that the sum of the distances from the two kho-kho poles from him is always 8m. Find the equations of the path traced by him if he distance between the poles is 6m.
48. Find the equation of the hyperbola if its asymptotes are parallel to x + 2y − 12 = 0 and x − 2y + 8 = 0, (2, 4) is the centre of the hyperbola and it passes through (2, 0).
49. A man is at a point P on a bank of a straight river, 3km wide, and wants to reach point Q, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point R and then run to Q, or he could row directly to Q, or he could row to some point S between Q and R then run to Q. If he can row at 6 km/h and run at 8 km/h where should he land to reach Q as soon as possible?
50. Trace the curve $y^2 = 2x^3$
51. (a) Let P be a point on the curve $y = x^3$ and suppose that the tangent line at P intersects the curve again at Q. Prove that the slope at Q is four times the slope at P.

(OR)

(b) Find the eccentricity, centre, foci and vertices of the following hyperbolas and draw their diagrams: $x^2 - 4y^2+6x+16y-11 =0$.

*****wish you all the best******

K.Thirumurugan