HIGHER SECONDARYSECOND YEAR - MODEL EXAM (3)

Time : 3.00 hrs. **MATHEMATICS** Max. Marks : 200

Section – A

Note: 1. ***Answer all the questions.***

2. ***Choose the most suitable answer from the given four alternatives.*** **40 x 1 = 40**

1. The gradient of the curve $y=-2x^3+3x+5$ at $x=2$ is

a) -20 b) 27 c) -16 d) -21

2. The parametric equations of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ are.

a) $x=a\sin^3\theta$; $y=a\cos^3\theta$ b) $x=a\cos^3\theta$; $y=a\sin^3\theta$

c) $x=a^3\sin\theta$; $y=a^3\cos\theta$ d) $x=a^3\cos\theta$; $y=a^3\sin\theta$

3. The angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is.

a) $2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\pi/2$ d) $\pi/4$

4. The law of the mean can also be put in the form

a) $f(a+h)=f(a)-hf'(a+\theta h)$ $0<\theta<1$ b) $f(a+h)=f(a)+hf'(a+\theta h)$ $0<\theta<1$

c) $f(a+h)=f(a)+hf'(a-\theta h)$ $0<\theta<1$ d) $f(a+h)=f(a)-hf'(a-\theta h)$ $0<\theta<1$

5. Identify the true statements in the following

i. if a curve is symmetrical about the origin, then it is symmetrical about both axes.

ii. if a curve is symmetrical about the both the axes, then it is symmetrical about the origin.

iii. A curve f(x,y) =0 is symmetrical about the line y = x if f(x,y) = f(y, x)

iv. for the curve f(x,y) =0, if f(x,y) = f(-y,-x), then it is symmetrical about the origin

a) (ii), (iii) b) (i), (iv) c) (i), (iii) d) (ii), (iv)

6. If $u = f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to.

a) 0 b) 1 c) 2 u d) u

7. The area between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and its auxillary circle is

a) $\pi b(a-b)$ b) $2\pi a(a-b)$ c) $\pi a(a-b)$ d) $2\pi b(a-b)$

8. The area bounded by the parabola $y^2 = x$ and its latus rectum is.

a) 4 / 3 b) 1 / 6 c) 2 / 3 d) 8 / 3

9. The area of the region bounded by the graph of y = sinx and y = cos x between x = 0 and x = $\pi/4$ is.

a) $\sqrt{2}+1$ b) $\sqrt{2}-1$ c) $2\sqrt{2}+1$ d) $2\sqrt{2}+2$

10. $\int_a^b f(x)\ dx =$

a) $2\int_0^a f(x)\ dx$ b) $\int_a^b f(a-x)\ dx$ c) $\int_a^b f(b-x)\ dx$ d) $\int_a^b f(a+b-x)\ dx$

11. Integrating factor of $\frac{dy}{dx}+\frac{1}{x\log x}.y=\frac{2}{x^2}$ is

a) e^x. b) log x. c) 1/ x d) e^{-x}.

12. The differential equation of the family of lines y = mx is.

a) $\frac{dy}{dx}=m$ b) y d x – x dy = 0 c) $\frac{d^2y}{dx^2}=0$ d) y d x + x dy = 0

13. The complementary function of $(D^2 + 1)\, y = e^{2x}$ is.

a) $(Ax+B)e^x$. b) A cos x + B sin x. c) $(Ax+B)e^{2x}$. d) $(Ax+B)e^{-x}$.

14. The order and degree of the differential equation are $\frac{d^2y}{dx^2}-y+\left(\frac{dy}{dx}+\frac{d^3y}{dx^3}\right)^{\frac{3}{2}}=0$

a) 2,3 b) 3,3 c) 3,2 d) 2,2

15. Which of the following is a tautology ?

a) p v q. b) p ^ q c) p v ~p. d) p ^ ~p.

16. Which of the following is not a binary operation on R ?

a) a * b = ab. b) a * b = a – b c) a * b = $\sqrt{ab}$ d) a * b = $\sqrt{a^2+b^2}$

17. The order of –i in the multiplicative group of 4^{th} roots of unity

a)4 b) 3 c) 2 d) 1

18. If p is true and q is false then which of the following statements is not true ?

a) $p\rightarrow q$ is false b) $p\vee q$ is true c) $p\wedge q$ is falsed) $p\leftrightarrow q$ is true

19. A random variable X has the following p.d.f.

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

The value of k is

a) 1/ 8 b) 1/ 10 c) 0 d) -1 or 1/10

20.The mean of binomial distribution is 5 and its standard deviation is 2. Then the value of n and p are.

a) $\left(\frac{4}{5},25\right)$ b) $\left(25,\frac{4}{5}\right)$ c) $\left(\frac{1}{5},25\right)$ d) $\left(25,\frac{1}{5}\right)$

21. If a random variable X follows Poisson distribution such that $E(X^2) = 30$ then the variance of the distribution is.

a) 6 b) 5 c) 30 d) 25

22. The p.d.f of the standard normal variate Z is $\varphi(z)=$

a) $\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{1}{2}z^2}$ b) $\frac{1}{\sqrt{2\pi}}e^{-z^2}$ c) $\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}z^2}$ d) $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$

23. If $ae^x + be^y = c$; $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is

a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ b) $\left(\log\frac{\Delta_2}{\Delta_1}, \log\frac{\Delta_3}{\Delta_1}\right)$ c) $\left(\log\frac{\Delta_1}{\Delta_3}, \log\frac{\Delta_1}{\Delta_2}\right)$ d) $\left(\log\frac{\Delta_1}{\Delta_2}, \log\frac{\Delta_1}{\Delta_3}\right)$

24. In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta=0$ and $\Delta_x = 0$, $\Delta_y \neq 0$, $\Delta_z = 0$ then the system has

a) unique solution b) two solutions
c) infinitely many solution d) no solution.

25. If A and B are any two matrices such that AB = 0 and A is non-singular, then

a) B = 0 b) B is singular c) B is non-singular d) B

26. In the homogeneous system $\rho(A) <$ the number of unknowns then the system has

a) only trivial solution b) trivial solution and infinitely many non-trivial solutions
c) only non-trivial solutions d) no solution

27. The centre and radius of the sphere given by $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

a) (-3, 4, -5) , 49 b) (-6, 8, -10) , 1 c) (3, -4, 5) , 7 d) (6, -8, 10) ,

28. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{+4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

a) (0, 0, -4) b) (1, 0, 0) c) (0, 2, 0) d) (1, 2, 0)

29. The equation of the plane passing through the point (2, 1, -1) and the line of intersection of the

planes $\vec{r} \cdot (\vec{i} + 3\vec{j} - \vec{k}) = 0$ and $\vec{r} \cdot (\vec{j} + 2\vec{k}) = 0$ is,

a) x+4y-z=0 b) x+9y+11z=0 c) 2 x+y-z+5=0 d) 2x-y+z=0

30. $\vec{r} = s\vec{i} + t\vec{j}$ is the equation of

a) a straight line joining the points $\vec{i}$ and $\vec{j}$ b) xoy plane
c) yoz plane d) zox plane

31. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then $|\vec{a} + \vec{b} + \vec{c}| =$

a) 3 b) 9 c) $3\sqrt{3}$ d) $\sqrt{3}$

32. The projection of $3\vec{i} + \vec{j} - \vec{k}$ on $4\vec{i} - \vec{j} + 2\vec{k}$ is

a) $\frac{9}{\sqrt{21}}$ b) $\frac{-9}{\sqrt{21}}$ c) $\frac{81}{\sqrt{21}}$ d) $\frac{-81}{\sqrt{21}}$

33.The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

a) i b) -i c) 1 d) -1

34. If $\frac{1-i}{1+i}$ is a root of $ax^2 + bx + 1 = 0$, where a, b are real then (a,b) is

a) (1,1) b) (1,-1) c) (0,1) d) (1,0)

35. If ω is the nth root of unity then

a) $1 + \omega^2 + \omega^4 + ...= \omega + \omega^3 + \omega^5 + ...$ b) $\omega^n = 0$
c) $\omega^n = 1$ d) $\omega = \omega^{n-1}$

36. The conjugate of $(2+i)(3-2i)$ is

a) $8-i$ b) $-8-i$ c) $-8+i$ d) $8+i$

37. The angle between the two tangents drawn from the point (-4, 4) to $y^2=16x$ is.

a) 45° b) 30° c) 60° d) 90°

38. The radius of the director circle of the conic $9x^2+16y^2=144$ is

a) $\sqrt{7}$ b) 4 c) 3 d) 5.

39. The angle between the asymptotes to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is.

a) $\pi-2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\pi-2\tan^{-1}\left(\frac{4}{3}\right)$ c) $2\tan^{-1}\frac{3}{4}$ d) $2\tan^{-1}\left(\frac{4}{3}\right)$

40. The directrix of the parabola $x^2=-4y$ is

a) x = 1 b) x = 0 c) y = 1 d) y = 0

Section – B

Note: 1. ***Answer any 10 questions.*** **10×6=60**

2. ***Question No. 55 is compulsory and choose any nine questions from the remaining***

41. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

42. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $\left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]=0$

43. Angle in a semi-circle is a right angle.Prove by vector method.

44. (i) Express the following complex number in polar form. $-1-i$

(ii) Prove that if $\omega^3=1$, then $\left(\frac{-1+i\sqrt{3}}{2}\right)^5+\left(\frac{-1-i\sqrt{3}}{2}\right)^5=-1$

45. Find the square root of $(-8-6i)$

46. Prove that the product of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2+b^2}$

47. (i) Obtain the Maclaurin's Series for e^x

(ii) Evaluate: $\lim_{x\to\infty}\frac{x^2}{e^x}$

48. Determine for which values of x, the function $f(x)=2x^3-15x^2+36x+1$ is increasing and for which it is decreasing. Also determine the points where the tangents to the graph of the function are parallel to the x axis.

49. If $U=(x-y)(y-z)(z-x)$ then show that $U_x+U_y+U_z=0$

50. Evaluate $\int_0^{\pi/2}\log(\tan x)\,dx$

Find the equation of the curve passing through (1,0) and which has slope $1+\frac{y}{x}$ at (x, y).

51. Show that $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$

52. State and prove cancellation laws on groups.

53. Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x} & , 0 < x < \infty \\ 0 & , elsewhere \end{cases}$

54. The overall percentage of passes in a certain examination is 80. If 6 candidates appear in the examination what is the probability that at least 5 pass the examination.

55. (a) Examine the consistency of the following system of equation. If it is consistent then solve the same. $x-4y+7z=14$; $3x+8y-2z=13$; $7x-8y+26z=5$ **(OR)**

(b) Find the equation of the curve passing through (1,0) and which has slope $1+\frac{y}{x}$ at (x, y).

Section - c

Note: 1. ***Answer any 10 questions.*** **10×10=100**

2. ***Question No. 70 is compulsory and choose any nine questions from the remaining***

56. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of red chair is Rs. 240, cost of blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chair is Rs. 25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased

57. Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B.$

58. Find the vector and Cartesian equations of the plane passing through the points (1,2,.3) and (2,3,1) and perpendicular to the plane $3x-2y+4z-5=0$

59. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$

60. Find the axis , vertex, focus, equation of directrix, latus rectum, length of the latus rectum for the following parabola and hence sketch their graph. $y^2 + 8x - 6y + 1 = 0$

61. A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet nearer to the sun?(Take the orbit as open rightward).

62. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radiu r.

63. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $k = \pm\frac{1}{2\sqrt{2}}$

64. Verify Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$

65. Show that the surface area of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x-axis is $2\pi\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$

66. The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal ? $(\log_e 2 = 0.6931)$

67. Solve : $dx + xdy = e^{-y} \sec^2 y dy$

68. Show that the set M of complex numbers z with the condition $|z| = 1$ forms a group with respect to the operation of multiplication of complex numbers.

69. Find c , μ and σ^2 of the normal distribution whose probability function is given by

$f(x) = ce^{-x^2+3x} \quad -\infty < X < \infty.$

70. (a) Find the equation of the hyperbola if its asymptotes are parallel to $x+2y-12=0$ and $x-2y+8=0$, (2,4) is the centre of the hyperbola and it passes through (2,0).

(OR)

(b) Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $2x - y = 4$.