A

Register Number

+2 MODEL EXAMINATION

PART III – MATHEMATICS

[English Version]

Time allowed: 3 Hours] **[Maximum Marks: 200**

SECTION – A

Note: **(i) All questions are compulsory.**
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. **40 x 1 = 40**

1. If A is a square matrix of order n then $|adjA|$ is
   (1) $|A|^2$ (2) $|A|^n$ (3) $|A|^{n-1}$ (4) $|A|$

2. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is
   (1) $\frac{1}{k^2}I$ (2) $\frac{1}{k^3}I$ (3) $\frac{1}{k}I$ (4) kI

3. In a system of 3 linear non –homogeneous equation with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has
   (1) unique solution (2) two solutions (3) infinitely many solutions (4) no solutions

4. The rank of the matrix $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ is
   (1) 1 (2) 2 (3) 0 (4) 8

5. If $\vec{u} = \vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b})$, then
   (1) u is a unit vector (2) $\vec{u} = \vec{a}+\vec{b}+\vec{c}$ (3) $\vec{u} = \vec{0}$ (4) $\vec{u} \neq \vec{0}$

6. The vectors $2\vec{i}+3\vec{j}+4\vec{k}$ and $a\vec{i}+b\vec{j}+c\vec{k}$ are perpendicular when
   (1) a=2, b=3, c= –4 (2) a=4 , b=4, c=5 (3) a=4, b=4, c= –5 (4) a= –2, b=3, c=4

7. If $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = 64$ then $[\vec{a}\ \vec{b}\ \vec{c}]$ is
   (1) 32 (2) 8 (3) 128 (4) 0

8. The shortest distance of the point (2, 10, 1) from the plane $\vec{r}.\left(3\vec{i}-\vec{j}+4\vec{k}\right) = 2\sqrt{26}$ is
   (1) $2\sqrt{26}$ (2) $\sqrt{26}$ (3) 2 (4) $\frac{1}{\sqrt{26}}$

9. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $\vec{a}.\vec{b} = 9$ then $|\vec{a}\times\vec{b}|$ is
   (1) $3\sqrt{7}$ (2) 2 (3) 69 (4) $\sqrt{69}$

10. The vector equation of a plane whose distance from the origin is P and perpendicular to a unit vector $\hat{n}$ is

(1) $\vec{r}.\vec{n} = p$ (2) $\vec{r}.\hat{n} = q$ (3) $\vec{r}\times\vec{n} = p$ (4) $\vec{r}.\hat{n} = p$

11. The value of $\left[\frac{-1+i\sqrt{3}}{2}\right]^{100} + \left[\frac{-1-i\sqrt{3}}{2}\right]^{100}$ is

(1) 2 (2) 0 (3) –1 (4) 1

12. The polar form of the complex number $(i^{25})^3$ is

(1) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (2) $\cos\pi + i\sin\pi$ (3) $\cos\pi - i\sin\pi$ (4) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

13. If ω is a cube root of unity then the value of $(1-\omega+\omega^2)^4 + (1+\omega-\omega^2)^4$ is

(1) 0 (2) 32 (3) –16 (4) –32

14. If $Z_1 = a + ib$, $Z_2 = -a + ib$ then $Z_1 - Z_2$ lies on

(1) real axis (2) imaginary axis (3) the line y = x (4) the line y = –x

15. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is

(1) 8 (2) 6 (3) 4 (4) 2

16. The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line.

(1) $x - 3 = 0$ (2) $x + 3 = 0$ (3) $y + 3 = 0$ (4) $y - 3 = 0$

17. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{5}{3}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{5}}{2}$

18. The angle between the asymptotes of the hyperbola $24x^2 - 8y^2 = 27$ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{3} or \frac{2\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) $-\frac{2\pi}{3}$

19. The angle between the curves $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\frac{x^2}{8} - \frac{y^2}{8} = 1$ is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

20. If the length of the diagonal of a square is increasing at the rate of 0.1 cm /sec. What is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}}$ cm?

(1) 1.5 cm^2 /sec (2) 3 cm^2 /sec (3) $3\sqrt{2}$ cm^2 /sec (4) 0.15cm^2/ sec

21. If $f(x) = x^2 - 4x - 5$ on [0,3] then the absolute maximum value is

(1) 2 (2) 3 (3) 4 (4) 5

22. $\lim_{x\to 0} x\cot x$ is

(1) 1 (2) –1 (3) 0 (4) ∞

23. If x = r cos θ, y = r sin θ, then $\frac{\partial r}{\partial x}$ is equal to

(1) sec θ (2) sin θ (3) cos θ (4) cosec θ

24. The curve $ay^2 = x^2 (3a - x)$ cuts the y-axis at

(1) $x = -3a, x = 0$ (2) $x = 0, x = 3a$ (3) $x = 0, x = a$ (4) $x = 0$

25. The value of $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x} dx$ is

(1) $\frac{\pi}{2}$ (2) 0 (3) $\frac{\pi}{4}$ (4) π

26. The area bounded by the line y = x , the x-axis , the ordinates x = 1, x = 2 is

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7}{2}$

27. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

(1) 20π (2) 40π (3) 10π (4) 30π

28. If $f(x)$ is even then $\int_{-a}^{a} f(x)dx$ is

(1) 0 (2) $2\int_0^a f(x)dx$ (3) $\int_0^a f(x)dx$ (4) $-2\int_0^a f(x)dx$

29. Solution of $\frac{dx}{dy} + mx = 0$, where m<0 is

(1) $x = ce^{my}$ (2) $x = ce^{-my}$ (3) $x = my + c$ (4) $x = c$

30. The differential equation obtained by eliminating *a* and *b* from $y = ae^{3x} + be^{-3x}$ is

(1) $\frac{d^2y}{dx^2} + ay = 0$ (2) $\frac{d^2y}{dx^2} - 9y = 0$ (3) $\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0$ (4) $\frac{d^2y}{dx^2} + 9x = 0$

31. The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D - a)g(D), g(a) \neq 0$ is

(1) me^{ax} (2) $\frac{e^{ax}}{g(a)}$ (3) $g(a)e^{ax}$ (4) $\frac{xe^{ax}}{g(a)}$

32. The order and degree of the differential equation are $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$

(1) (2, 1) (2) (1, 2) (3) $(2, \frac{1}{2})$ (4) (2, 2)

33. The number of rows in the truth table of ~ [p ∧ (~ q)] is

(1) 2 (2) 4 (3) 6 (4) 8

34. The value of $[3] +_{11} ([5] +_{11} [6])$ is

(1) [0] (2) [1] (3) [2] (4) [3]

35. Which of the following is correct?

(1) An element of a group can have more than one inverse.

(2) If every element of a group is its own inverse, then the group is Abelian.

(3) The set of all 2×2 real matrices forms a group under matrix multiplication.

(4) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$

36. In the group $(z_5 - \{[0]\}, *_5)$, $O([3])$ is

(1) 5 (2) 3 (3) 4 (4) 2

37. X is a discrete random variable which takes the values 0, 1, 2 and $P(X = 0) = \frac{144}{169}$, $P(X = 1) = \frac{1}{169}$ then the value of $P(X = 2)$ is

(1) $\frac{145}{169}$ (2) $\frac{24}{169}$ (3) $\frac{2}{169}$ (4) $\frac{143}{169}$

38. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then the mean of the random variable X is

(1) 16 (2) 5 (3) 2 (4) 1

39. If $f(x)$ is a p.d.f of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)\,dx$ is

(1) 1 (2) 0.5 (3) 3 (4) 0.25

40. For a standard normal distribution the mean and variance are

(1) 0, 1 (2) 1, 1 (3) μ, σ (4) μ, σ^2

SECTION - B

Note: (i) Answer any *ten* questions.

(ii) Question No.55 is compulsory and choose any nine questions from the remaining.

(iii) Each question carries six marks. 10 x 6 = 60

41. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$.

42. Find the rank of the matrices: $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$

43. Angle in a semi-circle is a right angle. Prove by vector method.

44. (a) Find the centre and radius : $\left|2\vec{r} + (3\vec{i} - \vec{j} + 4\vec{k})\right| = 4$

(b) The planes $\vec{r}.(2\vec{i} + \lambda\vec{j} - 3\vec{k}) = 10$ and $\vec{r}.(\lambda\vec{i} + 3\vec{j} + \vec{k}) = 5$ are perpendicular. Find λ.

45. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, if one of its roots is $2 + \sqrt{3}\,i$.

46. Express the following in the standard form a + ib : $\dfrac{i^4+i^9+i^{16}}{3-2i^8-i^{10}-i^{15}}$

47. Find the equation of the two tangents that can be drawn from the point (1, –1) to the hyperbola $2x^2 - 3y^2 = 6$.

48. The luminous intensity I candelas of a lamp at varying voltage V is given by: $I = 4 \times 10^{-4} V^2$. Determine the voltage at which the light is increasing at a rate of 0.6 candelas per volt.

49. Locate the extreme point on the curve $y = 3x^2 - 6x$ and determine its nature by examining the sign of the gradient on either side.

50. If $u = \log(\tan x + \tan y + \tan z)$ prove that $\sum \sin 2x \dfrac{\partial u}{\partial x} = 2$

51. Evaluate : $\displaystyle\int_0^3 \frac{\sqrt{x}dx}{\sqrt{x}+\sqrt{3-x}}$

52. Construct the truth table for (p $\wedge$ q) v r.

53. Using the truth table to determine whether is statement ((~p) v q) v (p $\wedge$ (~q)) is a tautology.

54. Find k, μ and σ of the normal distribution whose probability function is given by $f(x) = ke^{-2x^2+4x-2}$

55. (a) Solve : $\dfrac{dy}{dx} = \sin(x+y)$

(OR)

(b) The overall percentage of passes in a certain examination is 80. If 6 candidates appear in the examination, what is the probability that atleast 5 pass the examination

SECTION - C

Note: (i) Answer any *ten* questions.
(ii) Question No.70 is compulsory and choose any Nine questions from the remaining.
(iii) Each question carries ten marks. 10 x 10 = 100

56. For what values of *k*, the system of equations $kx+y+z=1$, $x+ky+z=1$, $x+y+kz=1$ have
(i) unique solution (ii) more than one solution (iii) no solution

57. If $\vec{a} = 2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b} = -2\vec{i}+5\vec{k}$, $\vec{c} = \vec{j}-3\vec{k}$ .Verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

58. Find the vector and cartesian equation of the plane passing through the points A (1, –2, 3) and B (–1, 2, –1) and is parallel to the line $\dfrac{x-2}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$.

59. If a = cos 2α + i sin 2α , b = cos 2β + i sin 2β and c = cos 2γ + i sin 2γ prove that

(i) $\sqrt{abc}+\frac{1}{\sqrt{abc}}=2\cos(\alpha+\beta+\gamma)$ (ii) $\frac{a^2b^2+c^2}{abc}=2\cos2(\alpha+\beta-\gamma)$

60. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6mts away from the point of projection. Finally it reaches the ground 12mts away from the starting point. Find the angle of projection.

61. A satellite is travelling around the earth in elliptical orbit having the earth at a focus and of eccentricity ½. The Shortest distance that the satellite gets to the earth is 400kms. Find the longest distance that the satellite gets from the earth.

62. Find the eccentricity, centre, foci, vertices of the hyperbola $x^2 - 4y^2 + 6x + 16y - 11 = 0$ and draw the diagram

63. Find the condition for the curves $ax^2 + by^2 = 1$, $a_1x^2 + b_1y^2 = 1$ to intersect orthogonally.

64. A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

65. Trace the curve $y = x^3$.

66. Find the area of the region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

67. A drug is excreted in a patient's urine. The urine is monitored continuously using a catheter. A patient is administered 10mg of drug at time t = 0, which is excreted at a rate of $-3t^{1/2}$ mg/h.(i)What is the general equation for the amount of drug in the patient at time t >0?(ii)When will the patient be drug free?

68. Show that the set G = {a + b$\sqrt{2}$ / a, b ∈ Q} is an infinite abelian group with respect to addition.

69. If the number of incoming buses per minute at a bus terminus is a random variable having a Poisson distribution with λ = 0.9 , find the probability that there will be

(i) Exactly 9 incoming buses during a period of 5 minutes.

(ii) Fewer than 10 incoming buses during a period of 8 minutes.

(iii) At least 14 incoming buses during a period of 11 minutes.

70. (a) Solve: $dx + xdy = e^{-y}\sec^2 y\, dy$

(OR)

(b) Derive the formula for the volume of a right circular cone with radius 'r' and height 'h'.