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KEY FOR MATHEMATICS

		A			B
Q.No.	Option	Answer	Q.No.	Option	Answer
1.	(3)	$\|A\|^{n-1}$	1	(4)	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
2.	(3)	$\frac{1}{k}I$	2	(3)	-16
3.	(4)	no solutions	3	(1)	Real axis
4.	(1)	1	4	(3)	4
5.	(3)	$\vec{u} = \vec{0}$	5	(2)	x +3 =0
6.	(3)	a =4 , b =4, c = −5	6	(4)	$\frac{\sqrt{5}}{2}$
7.	(2)	8	7	(3)	$\frac{2\pi}{3}$
8.	(3)	2	8	(4)	$\frac{\pi}{2}$
9.	(1)	$3\sqrt{7}$	9	(1)	1.5 cm^2 /sec
10.	(4)	$\vec{r}.\hat{n} = p$	10	(4)	5
11.	(3)	−1	11	(2)	$\frac{24}{169}$
12.	(4)	$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$	12	(1)	16
13.	(3)	−16	13	(1)	1
14.	(1)	Real axis	14	(1)	0, 1
15.	(3)	4	15	(3)	−1
16.	(2)	x + 3 = 0	16	(1)	1
17.	(4)	$\frac{\sqrt{5}}{2}$	17	(3)	cos θ
18.	(3)	$\frac{2\pi}{3}$	18	(4)	x = 0
19.	(4)	$\frac{\pi}{2}$	19	(2)	0
20.	(1)	1.5 cm^2 /sec	20	(1)	$\frac{3}{2}$
21.	(4)	5	21.	(1)	20π
22.	(1)	1	22.	(2)	$2\int_0^a f(x)dx$
23.	(3)	cos θ	23.	(2)	$x = ce^{-my}$
24.	(4)	x = 0	24.	(2)	$\frac{d^2y}{dx^2} - 9y = 0$
25.	(2)	0	25.	(4)	$\frac{xe^{ax}}{g(a)}$
26.	(1)	$\frac{3}{2}$	26.	(4)	(2, 2)

27.	(1)	20π	27.	(4)	8
28.	(2)	$2\int_0^a f(x)dx$	28.	(4)	[3]
29.	(2)	$x = ce^{-my}$	29.	(2)	If every element of a group is its own inverse, then the group is abelian.
30.	(2)	$\frac{d^2y}{dx^2} - 9y = 0$	30.	(3)	4
31.	(4)	$\frac{xe^{ax}}{g(a)}$	31.	(2)	8
32.	(4)	(2, 2)	32.	(3)	2
33.	(4)	8	33.	(1)	$3\sqrt{7}$
34.	(4)	[3]	34.	(4)	$\vec{r}.\vec{n} = p$
35.	(2)	If every element of a group is its own inverse, then the group is abelian.	35.	(3)	$\|A\|^{n-1}$
36.	(3)	4	36.	(3)	$\frac{1}{k} I$
37.	(2)	$\frac{24}{169}$	37.	(4)	no solutions
38.	(1)	16	38.	(1)	1
39.	(1)	1	39.	(3)	$\vec{u} = \vec{0}$
40.	(1)	0, 1	40.	(3)	a =4, b= 4,c= −5

SECTION – B

41. AB $= \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ **- 1 mark**

$A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ **- 1 mark**

$B^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ **- 1 mark**

$(AB)^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ **- 1 mark**

$B^{-1} A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ **- 1 mark**

$(AB)^{-1} = B^{-1} A^{-1}$ **- 1 mark**

42. A ~ $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & -3 & 0 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix} R_1 \leftrightarrow R_3$ **- 1 mark**

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 2 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 2R_1 \\ R_3 \end{matrix}$ **- 2 marks**

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 2 & -1 \\ 0 & 0 & 12 & 4 \end{bmatrix} R_3 \to 5R_3 + R_2$$ **- 2 marks**

$\rho(A) = 3$ **- 1 mark**

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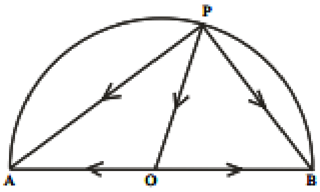
43. Rough Diagram **- 1 mark**



$\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$ **- 1 mark**

$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = \overrightarrow{PO} - \overrightarrow{OA}$ **- 1 mark**

$\overrightarrow{PA}.\overrightarrow{PB} = (\overrightarrow{PO} + \overrightarrow{OA}).(\overrightarrow{PO} - \overrightarrow{OA})$

$= PO^2 - OA^2$ **- 1 mark**

$\overrightarrow{PA}.\overrightarrow{PB} = 0$

$\angle APB = \dfrac{\pi}{2}$ **- 1 mark**

∴ AB subtends a right angle at P on the surface. **- 1 mark**

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44.**(a)** Vector equation of sphere

$\left|\vec{r} - \frac{1}{2}\left(-3\vec{i} + \vec{j} - 4\vec{k}\right)\right| = 2$ **- 1 mark**

Centre is $\left(\dfrac{-3}{2}, \dfrac{1}{2}, -2\right)$ **- 1 mark**

And radius is 2. **- 1 mark**

(b) $\vec{n}_1 = 2\vec{i} + \lambda\vec{j} - 3\vec{k}$ and $\vec{n}_2 = \lambda\vec{i} + 3\vec{j} + \vec{k}$ **- 1 mark**

Since the planes are perpendicular

$\vec{n}_1.\vec{n}_2 = 0$ **- 1 mark**

$\lambda = \dfrac{3}{5}$ **- 1 mark**

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45. Since $2 + i\sqrt{3}$ is a root, $2 - i\sqrt{3}$ is also root. **- 1 mark**

The corresponding factor is $x^2 - 4x + 7$ **- 1 mark**

Other factor is $(x^2 + 4x + 5)$ - 2 **marks**

$x^2 + 4x + 5 = 0 \Rightarrow x = -2 \pm i$ - 2 **marks**

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46. $\dfrac{i^4 + i^9 + i^{16}}{3 - 2i^8 - i^{10} - i^{15}} = \dfrac{1 + i + 1}{3 - 2 - i^2 - i^3}$ **- 3 marks**

$= \dfrac{2 + i}{1 - (-1) - (-i)}$ **- 1 mark**

$= \dfrac{2 + i}{2 + i}$ **- 1 mark**

$= 1$ **- 1 mark**

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47. Equation of the tangent is

$y = mx \pm \sqrt{3m^2 - 2}$ **- 2 marks**

It passes (1, 2)

$x = m \pm \sqrt{3m^2 - 2}$ **- 1 mark**

$m = -3, 1$ **- 1 mark**

Equation of the tangents

$3x + y - 5 = 0$ and $x - y + 1 = 0$ **- 2 marks**

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48. $\frac{dI}{dV} = 8\times10^{-4}V$ **- 2 marks**

$\frac{dI}{dV} = +0.6$ **- 1 mark**

V = $\frac{0.6}{8\times10^{-4}}$ **- 1 mark**

V = 750 volts. **- 2 marks**

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49. $\frac{dy}{dx} = 6x - 6$ **- 1 mark**

$\frac{dy}{dx} = 0 \Rightarrow x = 1$ **- 1 mark**

When x = 1, y = −3 **- 1 mark**

If x is slightly less than 1, say 0.9 then $\frac{dy}{dx}$ = 6(0.9) – 6 = − 0.6 < 0 **- 1 mark**

If x is slightly greater than 1, say 1.1 than $\frac{dy}{dx}$ = 6 (1.1) −6 = 0.6 > 0 **- 1 mark**

Since the gradient changes its sign from negative to positive (1, -3) is a minimum point. **- 1 mark**

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50. $\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$ **- 2 marks**

Sin 2x $\frac{\partial u}{\partial x} = \frac{2\tan x}{\tan x + \tan y + \tan z}$ **- 1 mark**

Sin 2y $\frac{\partial u}{\partial y} = \frac{2\tan y}{\tan x + \tan y + \tan z}$ **- 1 mark**

Sin 2z $\frac{\partial u}{\partial z} = \frac{2\tan z}{\tan x + \tan y + \tan z}$ **- 1 mark**

$\sum \sin 2x \frac{\partial u}{\partial x} = 2$ **- 1 mark**

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51. $I = \int_0^3 \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{3-x}}$ **- 1 mark**

$I = \int_0^3 \frac{\sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx$ **- 2 marks**

2I = $\int_0^3 dx = (x)^3{}_0 = 3$ **- 2 marks**

I = $\frac{3}{2}$ **- 1 mark**

52.

p	q	r	p Λq	(p Λ q) V r
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

1st, 2nd, 3rd, 4th columns **- 4 marks**

5th column **- 2 marks**

53.

p	q	~p	~q	(~p) vq	pΛ(~q)	((~p) v q) v (pΛ(~q)
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

The last column contains only T.
The given statement is a tautology.

3^{rd}, 4^{th}, 5^{th}, 6^{th} columns - **- 4 marks**
7^{th} column - **- 2 marks**

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54. $f(x) = ke^{-2(x^2-2x+1)}$ s **- 1 mark**

$= ke^{-2(x-1)^2}$ **- 1 mark**

$= ke^{-\frac{1}{2}\left(\frac{x-1}{1/2}\right)^2}$ **- 1 mark**

$\sigma = \frac{1}{2}$, μ = 1 **- 1 mark**

k= $\sqrt{\frac{2}{\pi}}$ **- 2 marks**

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55. (a) Put x + y = z **- 1 mark**

$\frac{dy}{dx} = \frac{dz}{dx} - 1$ **- 1 mark**

$\frac{dz}{dx} = 1 + \sin z$ **- 1 mark**

$\frac{dz}{1+\sin z} = dx$ **- 1 mark**

$\int \frac{1-\sin z}{\cos^2 z}\, dz = \int dx + c$ **- 1 mark**

tan (x + y) – sec (x + y) = x + c **- 1 mark**

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55. (b) $p = \frac{4}{5}$ **- 1 mark**

$q = \frac{1}{5}$ **- 1 mark**

p (Atleast 5 pass) = p (x ≥ 5) **- 1mark**

= p (x = 5) + p (x = 6) **- 1 mark**

P (X ≥ 5) = $\frac{2048}{5^5}$ **- 2 marks**

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SECTION – C

56. [A, B] = $\begin{bmatrix} k & 1 & 1 & 1 \\ 1 & k & 1 & 1 \\ 1 & 1 & k & 1 \end{bmatrix}$ **- 1 mark**

$\sim \begin{bmatrix} 1 & 1 & k & 1 \\ 0 & k-1 & 1-k & 0 \\ 0 & 0 & 2-k-k^2 & 1-k \end{bmatrix}$ **- 1 mark**

$$\sim \begin{bmatrix} 1 & 1 & k & 1 \\ 0 & k-1 & 1-k & 0 \\ 0 & 0 & (2+k)(1-k) & 1-k \end{bmatrix}$$ **- 2 marks**

Case (i): k = 1

The system is consistent and has infinitely many solutions. **- 2 marks**

Case (ii): k = -2

The system is inconsistent or has no solution. **- 2 marks**

Case (iii): k ≠ -2 and k≠ 1

The system is consistent and has unique solutions. **- 2 marks**

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57. $\vec{b}\times\vec{c} = -5\vec{i} - 6\vec{j} - 2\vec{k}$ **- 1 mark**

$\vec{a}\times(\vec{b}\times\vec{c}) = -12\vec{i} + 9\vec{j} + 3\vec{k}$ **- 2 marks**

$\vec{a}.\vec{c} = 6$ **- 1 mark**

$(\vec{a}.\vec{c})\vec{b} = -12\vec{i} + 30\vec{k}$ **- 1 mark**

$\vec{a}.\vec{b} = -9$ **- 1 mark**

$(\vec{a}.\vec{b})\vec{c} = -9\vec{j} + 27\vec{k}$ **- 1 mark**

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = -12\vec{i} + 9\vec{j} + 3\vec{k}$ **- 2 marks**

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ **- 1 mark**

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58. $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$

$\vec{b} = -\vec{i} + 2\vec{j} - \vec{k}$

$\vec{v} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ **- 3 marks**

$\vec{v} = (1-s)(\vec{i} - 2\vec{j} + 3\vec{k}) + s(-\vec{i} + 2\vec{j} - \vec{k}) + t(2\vec{i} + 3\vec{j} + 4\vec{k})$

(OR)

$\vec{v} = (\vec{i} - 2\vec{j} + 3\vec{k}) + s(-2\vec{i} + 4\vec{j} - 4\vec{k}) + t(2\vec{i} + 3\vec{j} + 4\vec{k})$ **- 2 marks**

Cartesian form

$$\begin{vmatrix} x+1 & y+2 & z-3 \\ -2 & 4 & -4 \\ 2 & 3 & 4 \end{vmatrix} = 0$$ **- 3 marks**

$2x - z + 1 = 0$ **- 2 marks**

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59. (i) $\sqrt{abc} = \cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)$ **- 2 marks**

$\frac{1}{\sqrt{abc}} = \cos(\alpha+\beta+\gamma) - i\sin(\alpha+\beta+\gamma)$ **- 1 mark**

$\sqrt{abc} + \frac{1}{\sqrt{abc}} = 2\cos(\alpha+\beta+\gamma)$ **- 2 marks**

(ii) $\frac{ab}{c} = \cos(2\alpha+2\beta-2\gamma) + i\sin(2\alpha+2\beta-2\gamma)$ **- 2 marks**

$\frac{c}{ab} = \cos(2\alpha+2\beta-2\gamma) + i\sin(2\alpha+2\beta-2\gamma)$ **- 1 mark**

$\frac{ab}{c} + \frac{c}{ab} = 2\cos(2\alpha+2\beta-2\gamma)$ **- 1 mark**

$\frac{a^2b^2+c^2}{abc} = 2\cos\ 2(\alpha+\beta-\gamma)$ **- 1 mark**

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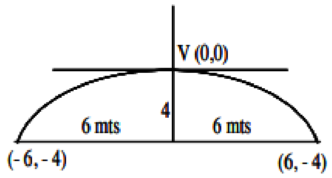
60. Rough Diagram - 2 marks



$x^2 = -4ay$ - 1 mark

$a = \frac{9}{4}$ (or) $4a = 9$ - 2 marks

$x^2 = -9y$ - 1 mark

$\frac{dy}{dx} = \frac{-2}{9}x$ - 2 marks

$\theta = \tan^{-1}\frac{4}{3}$ - 2 marks

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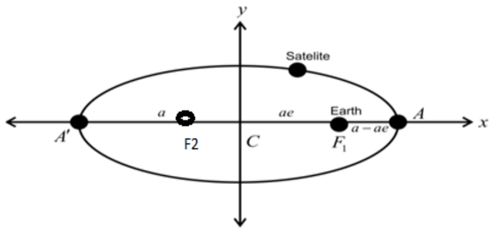
61. Rough diagram - 2 marks



$F_1 A = 400$ km, $e = \frac{1}{2}$

$a - ae = 400$ - 2 marks

$a = 800$ km - 2 marks

$F_1A^1 = F_1C + CA^1$

$= a + ae$ - 2 marks

$= 1200$ km - 2 marks

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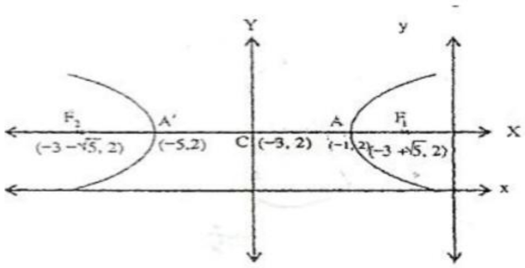
62. $\frac{(x+3)^2}{4} - \frac{(y-2)^2}{1} = 1$ - 2 marks



$e = \frac{\sqrt{5}}{2}$ - 1 mark

C (-3, 2) - 1 mark

F_1 $(-3 + \sqrt{5}, 2)$ - 1 mark

F2 $(-3 - \sqrt{5}, 2)$ - 1 mark

A (-1, 2) - 1 mark

A^1 (-5, 2) - 1 mark

Rough diagram - 2 marks

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63. $x_1^2 = \frac{b_1 - b}{ab_1 - a_1b}$ - 1 mark

$y_1^2 = \frac{a - a_1}{ab_1 - a_1b}$ - 1 mark

$m_1 = \frac{-ax_1}{by_1}$

$m_2 = \frac{-a_1x_1}{b_1y_1}$ - 2 marks

- 2 marks

For orthogonal, $m_1m_2 = -1$

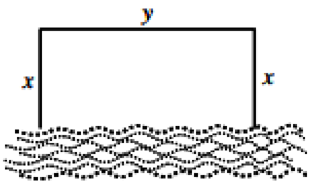
$\frac{aa_1x_1^2}{bb_1y_1^2} = -1$

$a a_1x_1^2 + bb_1y_1^2 = 0$ - 2 marks

$\frac{b_1 - b}{bb_1} + \frac{a - a_1}{aa_1} = 0$ - 1 mark

$\frac{1}{b} - \frac{1}{b_1} + \frac{1}{a_1} - \frac{1}{a} = 0$ (OR) $\frac{1}{a} - \frac{1}{a_1} = \frac{1}{b} - \frac{1}{b_1}$ - 1 mark

64. Rough Diagram - **2 marks**



A = xy - **1 mark**

$2x + y = 2400$ - **1 mark**

A(x) = $2400\,x - 2x^2$ - **2 marks**

$A^1\,(x) = 2400 - 4x$

2400 – 4x = 0 which gives x = 600

A(0) = 0, A (600) = 7,20,000

and A (1200) = 0 - **2 marks**

The maximum value is

A(600) = 7 ,20,000

x = 600, y = 1200 - **2 marks**

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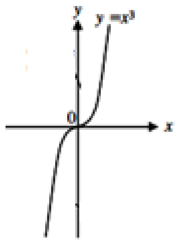
65. (i) Domain , extent, intercepts and origin - **1 mark**

The domain i.e. $(-\propto, \propto)$. The horizontal extent is $-\propto < x < \propto$ and the vertical extent is $-\propto < x < \propto$.

It passes through the origin. - **1 mark**



(ii) Symmetry

Its symmetrical about the origin. - **1 mark**

(iii) Asymptotes

The curve does not admit any asymptote. - **1 mark**

(iv) Monotoricity

The curve is increasing (or strictly increasing) in $(-\propto, \propto)$ - **1 mark**

(v) Special points: - **2 marks**

(0, 0) is a point of inflection, The curve is concave upward in (0, $\propto$) and convex upward in $(-\propto, 0)$

Rough Diagram - **3 marks**

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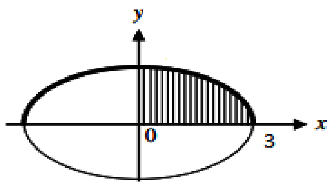
66. Rough diagram - **2 marks**



A $= 4\int_o^3 y\,dx$ - **2 marks**

$= 4\int_o^3 \frac{2}{3}\sqrt{9-x^2}\,dx$ - **1 mark**

$= \frac{8}{3}\int_o^3 \sqrt{9-x^2}\,dx$

$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]^3$ - **2 marks**

$= \frac{8}{3}\left(\frac{9}{2}\sin^{-1}(1)\right)$

$= \frac{8}{3}\left(\frac{9}{2}\left(\frac{\pi}{2}\right)\right)$ - **1 mark**

A = 6 π sq units. - **2 marks**

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67. (i) $\frac{dA}{dt} = -3t^{\frac{1}{2}}$ - **1 mark**

A = $-2t^{\frac{3}{2}} + c$ - **2 marks**

When t = 0 , A = 10 , C =10 - **1 mark**

A= 10 $-2t^{\frac{3}{2}}$ - **2 marks**

(ii) A = 0 , t^3 = 25 **- 1 mark**

$t = 2.9$ hours **- 2 marks**

Hence the patient will be drug free in 2.9 hours or 2 hours 54 min. **- 1 mark**

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68. (i) Closure axiom : **- 1 mark**

$x = a + b\sqrt{2}$, $y = c + d\sqrt{2}$

$x + y = (a + c) + (b + d)\sqrt{2} \in G$

∴ G is closed with respect to addition **- 1 mark**

(ii) Associative axiom: **- 1 mark**

Since the elements of G are all real numbers, addition is associative **- 1 mark**

(iii) Identity axiom: **- 1 mark**

0 is the identity element of G **- 1 mark**

(iv) Inverse axiom: **- 1 mark**

$(-a) + (-b)\sqrt{2}$ is the inverse of $a + b\sqrt{2}$ **- 1 mark**

(v) Commutative axiom:

$x + y = y + x$ **- 1 mark**

(G, +) is an abelian group **- 1 mark**

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69. (i) $\lambda = 4.5$ **- 1 mark**

$$P(x = 9) = \frac{e^{-4.5} \times (4.5)^9}{\angle 9}$$ **- 2 marks**

(ii) $\lambda = 7.2$ **- 1 mark**

$$P(x < 10) = \sum_{x=0}^{9} \frac{e^{-7.2} \times (7.2)^x}{\angle x}$$ **- 2 marks**

(iii) $\lambda = 9.9$ **- 1 mark**

$P(x \geq 14) = 1 - p(x < 14)$ **- 1 mark**

$$P(x \geq 14) = 1 - \sum_{x=0}^{13} \frac{e^{-9.9} \times (9.9)^x}{\angle x}$$ **- 2 marks**

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70. (a) $\dfrac{dx}{dy} + x = e^{-y}\sec^2 y$ **- 2 marks**

I. F = e^y **- 3 marks**

$xe^y = \int e^{-y}\sec^2 y\, e^y dy$ **- 3 marks**

$xe^y = \tan y + c$ **- 2 marks**

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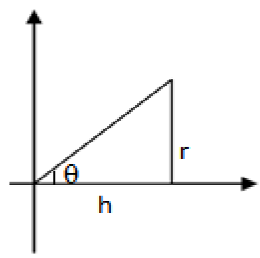
70. (b) Rough diagram **- 2 marks**



$\tan\theta = \dfrac{r}{h}$ **- 1 mark**

$y = \dfrac{r}{h}x$ **- 1 mark**

$$V = \pi\int_o^h \frac{r^2}{h^2}x^2 dx$$ **- 2 marks**

$$V = \frac{\pi r^2}{h^2}\left(\frac{x^3}{3}\right)_o^h$$ **- 2 marks**

$$V = \frac{1}{3}\pi r^2 h$$ **- 2marks**