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CLASS – XII
SUBJECT: - MATHEMATICS
MARKING SCHEME

Q.No.	STEPS	Step Marks	Total marks
1	1x5,5x1	1	1
2	$\frac{dy}{dx} = 1 - \frac{1}{x^2}$	1	1
3	$\pi/4$	1	1
4	$3, -4, -2$	1	1
5	Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix A is skew symmetric $\therefore \quad A = -A'$ $\Rightarrow \quad a_{ij} = -a_{ji} \ \forall\, i, j$ For diagonal elements $i = j$, $\Rightarrow \quad 2a_{ii} = 0$ $\Rightarrow \quad a_{ii} = 0 \Rightarrow$ diagonal elements are zero.	1 1	2
6	$f(x) = 4x^3 - 18x^2 + 27x - 7$ $f'(x) = 12x^2 - 36x + 27$ $= 3(2x-3)^2 \geq 0 \ \forall\, x \in R$ $\Rightarrow$ f(x) is increasing on R	1 1	2
7	$\frac{dv}{dt} = \pi 2r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}$ $= 182\,\pi = 572\ cm^3/sec$	1 1	2
8	$\frac{dy}{dx} = \frac{-2\sin 2t}{3\cos 3t}$ $= \frac{-2\sqrt{2}}{3}$	1 1	2

9	$\int \frac{dx}{x^2+4x+8} = \int \frac{dx}{(x+2)^2+(2)^2}$ $= \frac{1}{2}\tan^{-1}\frac{x+2}{2}+C$	1 1	2
10	Equation of given line is $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1\rangle$ Equation of required line is $\bar{r} = (\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$ 6.		
11	Let x necklaces and y bracelets are manufactured $\therefore$ L.P.P. is Maximize profit, $P = 100x + 300y$ subject to constraints $x + y \leq 24$ $\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ $x, y, \geq 1$	1 1	2
12	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= P(A) + P(B) - P(A)\,P(B)$ as A and B are independent events $\therefore 0.6 = 0.4 + p - (0.4)p$ $\Rightarrow p = \frac{1}{3}$	1 1	2

13	Given, $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}$ $\Rightarrow \tan^{-1}\left(\frac{3x+2x}{1-3x\times 2x}\right)=\frac{\pi}{4}$ $\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$ $\Rightarrow \frac{5x}{1-6x^2}=1$ $\Rightarrow 5x=1-6x^2$ $\Rightarrow 6x^2+5x-1=0$ $\Rightarrow 6x^2+6x-x-1=0$ $\Rightarrow 6x(x+1)-1(x+1)=0$ $\Rightarrow (6x-1)(x+1)=0$ $\therefore x=\frac{1}{6}$ or $x=-1.$	1 1 1 1	4
14	RHL=$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{1-cos4x}{8x^2}$ $=\lim_{x\to 0^+}\frac{2sin^2 2x}{8x^2}$ $=\lim_{x\to 0^+} 2\times\left(\frac{sin2x}{2x}\right)^2\times\frac{1}{2}$ $=1$ At $x=0$, we have $f(0)=k$ Since f(x) is continuous $\therefore$ k=1	2 1 1	4
OR	Since $f(x)$ is continuous at $x=3$ $\Longrightarrow \lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3)$ .... (i) Now, $\lim_{x\to 3^-} f(x)=\lim_{h\to 0} f(3-h)$ $=\lim_{h\to 0} a(3-h)+1$ $=\lim_{h\to 0} 3a-ah+1$ $=3a+1$ .... (ii) $\lim_{x\to 3^+} f(x)=\lim_{h\to 0} f(3+h)$	2	4

	$= \lim_{h\to 0} b(3+h)+3$ $= \lim_{h\to 0} 3b+bh+3$ $= 3b+3$ ………… (iii) From equation (i), (ii) and (iii), $3a+1=3b+3$ Or $a-b=\frac{2}{3}$, which is the required relation.	2	
15	Given integral can be written as- $\int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \left(\frac{x}{(x^2+1)} - \frac{x}{(x^2+3)}\right) dx = \int \frac{x}{(x^2+1)}dx - \int \frac{x}{(x^2+3)}dx$ $=\frac{1}{2}\int \frac{2x}{(x^2+1)}dx - \frac{1}{2}\int \frac{2x}{(x^2+3)}dx = \frac{1}{2}\log\|(x^2+1)\| - \frac{1}{2}\log\|(x^2+3)\| + c$, where c is integral constant. $=\frac{1}{2}\log\left\|\frac{x^2+1}{x^2+3}\right\| + c = \log\sqrt{\frac{x^2+1}{x^2+3}} + c.$	2 2	4
16	$I=\int_0^4 \|x-1\|dx$ $=\int_0^1 \|x-1\|dx + \int_1^4 \|x-1\|dx$ $=\int_0^1 (x-1)dx + \int_1^4 (x-1)\,dx$ $=\left[x-\frac{x^2}{2}\right]_0^1 + \left[x-\frac{x^2}{2}\right]_1^4$ $=1-\frac{1}{2}-0+\frac{16}{2}-4-\left(\frac{1}{2}-1\right)$ =4+1=5 Ans.	1 1 1 1	4
OR	let $I=\int x\sin^{-1}x\,dx$ Integrating by parts we have--- $I=\frac{x^2}{2}\sin^{-1}x - \int \frac{x^2}{2\sqrt{1-x^2}}dx$ $I=\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$ $=\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int(\sqrt{1-x^2})\,dx - \frac{1}{2}\sin^{-1}x$ $=\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\sin^{-1}x + \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right] + c$ $=\frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + c$	1 1 1 1	4

	$=\frac{1}{4}\left[(2x^2-1)\sin^{-1}x+x\sqrt{1-x^2}\right]+c$		
17	$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{(x^2+1)^2}$ $\Rightarrow I.F=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$ $\Rightarrow y.(1+x^2)=\int\frac{1}{(1+x^2)^2}\times(1+x^2)dx$ $\Rightarrow\int\frac{1}{1+x^2}dx=\tan^{-1}x+c$ When x=0, y=0 0 = $\tan^{-1}0+c$ => c=0 $\therefore y(1+x^2)=\tan^{-1}x$	1 1 1 1	4
OR	$\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$ Let y=vx$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\Rightarrow v+x\frac{dv}{dx}=\frac{v}{1+v^3}\Rightarrow x\frac{dv}{dx}=\frac{v}{1+v^3}-v$ (1) $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^3}\Rightarrow\int\frac{1+v^3}{v^3}dx=-\int\frac{1}{x}dx\Rightarrow\int\frac{1}{v^3}dv+\int dv=-\log x+c$ $\Rightarrow\frac{v^{-2}}{-2}+v=-\log x+c\Rightarrow-\frac{1}{2v^2}+v=-\log x+c\Rightarrow\frac{-x^2}{2y^2}+\frac{y}{x}=-\log x+c$ (2)	1	4
18	$\vec{a}-\vec{b}=-\hat{i}+\hat{j}+\hat{k};\ \vec{c}-\vec{b}=\hat{i}-5\hat{j}-5\hat{k}$ 1½ m $(\vec{a}-\vec{b})\times(\vec{c}-\vec{b})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&1\\1&-5&-5\end{vmatrix}=-4\hat{j}+4\hat{k}$ 1½ m $\therefore$ Unit vector perpendicular to both of the vectors $=-\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}$ 1 m		4

19	Given, that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar $\therefore\ [\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 0$ i.e. $(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1 $(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1 $\Rightarrow\ \vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$ $\Rightarrow\ 2[\vec{a},\vec{b},\vec{c}] = 0$ or $[\vec{a},\vec{b},\vec{c}]=0$ $\frac{1}{2}$ $\Rightarrow\ \vec{a},\vec{b},\vec{c}$ are coplanar.		
20	One marks for applying one properties		4
21	Let the manufacturer produce x packages of nut and y packages of bolts each day. Then profit function is $Z = 2.5x + y$ Subject to linear constraints $x + 3y \leq 12$ $3x + y \leq 12$ $x \geq 0, y \geq 0$ For correct graphs and common region Z is maximum at x=3,y=3 and maximum value is 10.50 Hence, maximum profit is ? 10.50.	1 2 1	4
22	Let E and F be the events of solving specific problem by A and B respectively then $P(E)=\frac{1}{2}$, $P(F)= \frac{1}{3}$ $P(\bar{E})=1-\frac{1}{2}=\frac{1}{2}$, $P(\bar{F})=1-\frac{1}{3}=\frac{2}{3}$ Probability that the problem is solved= P(EF or $\bar{E}$F or $\bar{F}$E) $= P(E)P(F)+P(\bar{E})P(F)+P(E)P(\bar{F})$ $= \frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{6}+\frac{1}{6}+\frac{2}{6}=\frac{4}{6} =\frac{2}{3}$ b.Probability that exactly one of them solves the problem	2 2	4

	$=P(\bar{E}F \text{ or } \bar{F}E)=P(\bar{E})P(F)+P(E)P(\bar{F})=\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{6}+\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$		
23	Let E_1 and E_2 be the events that the ball drawn in the first draw is red & black ball respectively then $P(E_1)=\frac{5}{10}=\frac{1}{2}$, $P(E_2)=\frac{5}{10}=\frac{1}{2}$ Now let A be the event that the ball drawn in second draw is red then $P(A/E_1)=\frac{7}{12}$, $P(A/E_2)=\frac{5}{12}$ Now, according to the law of total probability $P(A)=P(E_1)P(A/E_1)+P(E_2)P(A/E_2)=\frac{1}{2}\times\frac{7}{12}+\frac{1}{2}\times\frac{5}{12}=\frac{7}{24}+\frac{5}{24}=\frac{12}{24}=\frac{1}{2}$	1 1 2	4
OR	Let E_1 and E_2 be the events of drawing ace in first and second draw respectively. Let X be the random variable representing the no. of aces then the value of X may be 0,1 and 2. So $P(X=0)=P(\overline{E1}\overline{E2})=P(\overline{E1})P(\overline{E2}/\overline{E1})=\frac{48}{52}\times\frac{47}{51}=\frac{188}{221}$ $P(X=1)=P(\overline{E1}\,E2 \text{ or } E1\,\overline{E2})=P(\overline{E1})P(E2/\overline{E1})+P(E1)P(\overline{E2}/E1)=$ $=\frac{48}{52}\times\frac{4}{51}+\frac{4}{52}\times\frac{48}{51}=\frac{32}{221}$ $P(X=2)=P(E1\,E2)=P(E1)P(E2/E1)=\frac{4}{52}\times\frac{3}{51}=\frac{1}{221}$ So the probability distribution of random variable X is shown in the following table:	1 1 1 1	4

$X=x_i$	$P(X=x_i)=p_i$	x_i^2	p_ix_i	$p_ix_i^2$
0	$\frac{188}{221}$	0	0	0
1	$\frac{32}{221}$	1	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	4	$\frac{2}{221}$	$\frac{4}{221}$
			$\sum p_ix_i=\frac{2}{13}$	$\sum p_ix_i^2=\frac{36}{221}$

Mean(μ)= $\sum p_ix_i=\frac{2}{13}$

Variance=$\sum p_ix_i^2-(\sum p_ix_i)^2=\frac{36}{221}-\frac{4}{169}$ $=\frac{400}{2873}$

24

Given $A = N \times N$

$*$ is a binary operation on A defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

(*i*) Commutativity: Let $(a, b), (c, d) \in N \times N$

Then $(a, b) * (c, d) = (a + c, b + d) = (c + a, d + b)$ — 2

$(\because a, b, c, d \in N, a + c = c + a$ and $b + d = d + c)$

$= (c, d) * b$

Hence, $(a, b) * (c, d) = (c, d) * (a, b)$

$\therefore$ $*$ is commutative.

(*ii*) Associativity: let $(a, b), (b, c), (c, d)$

Then $[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = ((a + c) + e, (b + d) + f)$

$= [a + (c + e), b + (d + f)]$ $(\because$ set N is associative$)$ — 6

$= (a, b) * (c + e, d + f) = (a, b) * \{(c, d) * (e, f)\}$

Hence, $[(a, b) * (c, d)] * (e, f) = (a, b) * \{(c, d) * (e, f)\}$ — 2

$\therefore$ $*$ is associative.

(*iii*) Let (x, y) be identity element for $*$ on A,

Then $(a, b) * (x, y) = (a, b)$

$\Rightarrow$ $(a + x, b + y) = (a, b)$

$\Rightarrow$ $a + x = a,$ $b + y = b$

$\Rightarrow$ $x = 0,$ $y = 0$

But $(0, 0) \notin A$

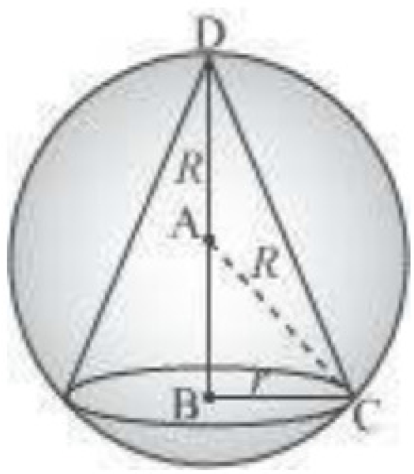
$\therefore$ For $*$, there is no identity element. — 2

25

Corresponding Linear equation

x+y+z=6000 — 1

x+3z=11000 — 1

	x-2y+z= 0 Matrix equation AX =B, where A= $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{bmatrix}$ B= $\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}$ And X= $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ A^{-1} = $1/6.\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}$ So x=500,y=2000,z=3500	2 2	
26	Let r and h be the radius and height of the cone respectively inscribed in a sphere of radius R.  Let V be the volume of the cone. Then, $V=\frac{1}{3}\pi r^2 h$ Height of the cone is given by, h = R + AB $=R+\sqrt{R^2-r^2}$ [ABC is a right triangle]	1	6

$$\therefore V = \frac{1}{3}\pi r^2\left(R + \sqrt{R^2 - r^2}\right)$$

$$= \frac{1}{3}\pi r^2 R + \frac{1}{3}\pi r^2\sqrt{R^2 - r^2}$$

$$\therefore \frac{dV}{dr} = \frac{2}{3}\pi r R + \frac{2}{3}\pi r\sqrt{R^2 - r^2} + \frac{1}{3}\pi r^2 \cdot \frac{(-2r)}{2\sqrt{R^2 - r^2}}$$

$$= \frac{2}{3}\pi r R + \frac{2}{3}\pi r\sqrt{R^2 - r^2} - \frac{1}{3}\pi\frac{r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{2}{3}\pi r R + \frac{2\pi r\left(R^2 - r^2\right) - \pi r^3}{3\sqrt{R^2 - r^2}}$$

$$= \frac{2}{3}\pi r R + \frac{2\pi r R^2 - 3\pi r^3}{3\sqrt{R^2 - r^2}}$$

$$\frac{d^2V}{dr^2} = \frac{2\pi R}{3} + \frac{3\sqrt{R^2 - r^2}\left(2\pi R^2 - 9\pi r^2\right) - \left(2\pi r R^2 - 3\pi r^3\right)\cdot\frac{(-2r)}{6\sqrt{R^2 - r^2}}}{9\left(R^2 - r^2\right)}$$

$$= \frac{2}{3}\pi R + \frac{9\left(R^2 - r^2\right)\left(2\pi R^2 - 9\pi r^2\right) + 2\pi r^2 R^2 + 3\pi r^4}{27\left(R^2 - r^2\right)^{\frac{3}{2}}}$$

$$\text{Now, } \frac{dV}{dr} = 0 \Rightarrow \frac{2}{3}\pi\ rR = \frac{3\pi r^3 - 2\pi r R^2}{3\sqrt{R^2 - r^2}}$$

$$\Rightarrow 2R = \frac{3r^2 - 2R^2}{\sqrt{R^2 - r^2}} \Rightarrow 2R\sqrt{R^2 - r^2} = 3r^2 - 2R^2$$

$$\Rightarrow 4R^2\left(R^2 - r^2\right) = \left(3r^2 - 2R^2\right)^2$$

$$\Rightarrow 4R^4 - 4R^2r^2 = 9r^4 + 4R^4 - 12r^2R^2$$

$$\Rightarrow 9r^4 = 8R^2r^2$$

$$\Rightarrow r^2 = \frac{8}{9}R^2$$

When $r^2 = \frac{8}{9}R^2$, then $\frac{d^2V}{dr^2} < 0$.

$\therefore$ By second derivative test, the volume of the cone is the maximum when $r^2 = \frac{8}{9}R^2$.

When $r^2 = \frac{8}{9}R^2$, $h = R + \sqrt{R^2 - \frac{8}{9}R^2} = R + \sqrt{\frac{1}{9}R^2} = R + \frac{R}{3} = \frac{4}{3}R.$

Therefore,

$$= \frac{1}{3}\pi\left(\frac{8}{9}R^2\right)\left(\frac{4}{3}R\right)$$

$$= \frac{8}{27}\left(\frac{4}{3}\pi R^3\right)$$

$$= \frac{8}{27}\times(\text{Volume of the sphere})$$

Hence, the volume of the largest cone that can be inscribed in the sphere is $\frac{8}{27}$ the volume of the sphere.

OR	Step 1: Let the length, breadth and height of the box be $l, x\ and\ y$ respectively. Area = c^2 sq. units. $\therefore x^2+4xy=c^2$ $y=\frac{c^2-x^2}{4x}$ Let v be the volume of the box, then $v=x^2y$ $\Rightarrow v=x^2\left(\frac{c^2-x^2}{4x}\right)$ $v=\frac{c^2}{4}x-\frac{x^3}{4}$ Step 2: Differentiate w.r.t x we get, $\frac{dv}{dx}=\frac{c^2}{4}-\frac{3x^2}{4}$ Again differentiate w.r.t x we get, $\frac{d^2v}{dx^2}=-\frac{3x}{2}$ For maximum or minimum , we must have, $\frac{dv}{dx}=0 \Rightarrow \frac{c^2}{4}-\frac{3x^2}{4}=0$ $\Rightarrow \frac{3x^2}{4}=\frac{c^2}{4}$ $\Rightarrow x=\frac{c}{\sqrt{3}}$ $\left(\frac{d^2v}{dx^2}\right)_{x=\frac{c}{\sqrt{3}}}=\frac{-3c}{2\sqrt{3}}<0$ Thus, v is maximum when $x=\frac{c}{\sqrt{3}}$ Step 4: Put $x=\frac{c}{\sqrt{3}}$, we get $y=\frac{c}{2\sqrt{3}}$ $\therefore$ The maximum volume of the box is given by $v=x^2y$ $=\frac{c^2}{3}\times\frac{c}{2\sqrt{3}}$ $=\frac{c^3}{6\sqrt{3}}$ cubic units		6
27	For correct figure $Y^2=4ax$......(i) $X^2=4ay$......(ii Putting $x=\frac{y^2}{4a}$ from (i) in (ii) we get $y^4-64a^3y=0$ or, $y(y^3-64a^3)=0$ or,y=0	1 1	6

	or,y=4a Thus, points of intersection of two parabolas are O(0,0) and A (4a,4a). Draw AD⊥OX .Then ,point D is (4a,0) Required area = area OCABO = (area OBADO)-(area OCADO) $=\int_0^{4a} y\,dx$ for (y^2=4ax)-$\int_0^{4a} y dx$ (for x^2=4ay) $=\int_0^{4a} 2\sqrt{ax}$ dx-$\int_0^{4a}\frac{x^2}{4a}$dx . $=\left[\frac{32a^2}{3}-\frac{16a^2}{3}\right]$ $=\left[\frac{16a^2}{3}\right]$sq. units **Hence the required area is $\frac{16a^2}{3}$ sq. units.**	2	
OR	For correct figure Both ellipse and the straight line passes through (a,0) and (0.b).We have to find the area between them. Required area =(Area between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1 and the x-axis from x=0 to x=a)- (Area between the straight line $\frac{x}{a}+\frac{y}{b}$=1 and the x-axis from x=0 to x=a) $=[\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\ dx$ -$\int_0^a \frac{b(a-x)}{a}\ dx$ $=[\frac{\pi ab}{4}-\frac{ab}{2}]$ sq.units	1 1 3 1	
28	The given differential equation can be rewritten as cot y cosec y $\frac{dy}{dx}+\frac{1}{x}\cos ecy=\frac{1}{x^2}$ On putting $v=-\cos ecy and \frac{dv}{dx}=\cos ecy\cot y\frac{dy}{dx}$	1 1	6

	$\frac{dv}{dx}-\frac{1}{x}v=\frac{1}{x^2}$ Here P = $-\frac{1}{x}$ and Q = $\frac{1}{x^2}$ I.F = $e^{\int pdx}=e^{\int -\frac{1}{x}dx}=\frac{1}{x}$ $\therefore$ Required solution is $v\frac{1}{x}=\int\frac{1}{x}.\frac{1}{x^2}\quad dx+c=-\frac{1}{2x^2}+c$ $2x+(1-2cx^2)\sin y=0$	2 2	
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