WWW.MATHSTIMES.COM

WWW.MATHSTIMES.COM| MODELPRACTICE PAPER-2|Answer key

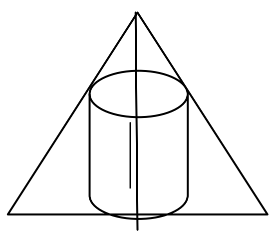
Q.No.	Value points	Marks
1.	X =5/3	1
2.	$(a\otimes b)\otimes c \neq a\otimes(b\otimes c)$	1
3.	A.(adjA) = $\lvert A\rvert$ I=6 I	1
4	$\vec{a}\,.\,\vec{b}$ = $\cos\theta$ can not be $\sqrt{3}$	1
5	$(B^{/}AB)^{/} = (AB)^{/}B$ $= B^{/}A^{/}B$ $= B^{/}(-A)B$ $= -B^{/}AB$, So is skew symmetric.	½ ½ ½ ½
6	$dy/dx = 3x^2 + 1$ Slope of the tangent at (1, 3) = 4 & at (- 1, - 1) = 4 So, they are parallel	½ 1 ½
7.	Put $\sin^2 x = t \Rightarrow I = \frac{1}{2}\int \frac{dt}{\sqrt{a^2 - t^2}}$ $= \frac{1}{2}\sin^{-1}\frac{\sin^2 x}{a} + c$	1 1
8	$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}dx$ $I = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}}dx$ $2I = \int_0^{\pi} dx = \pi \therefore I = \frac{\pi}{2}$	 1 1
9.	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 4$ Points (2, 13/3) and (- 2, - 1)	1 1

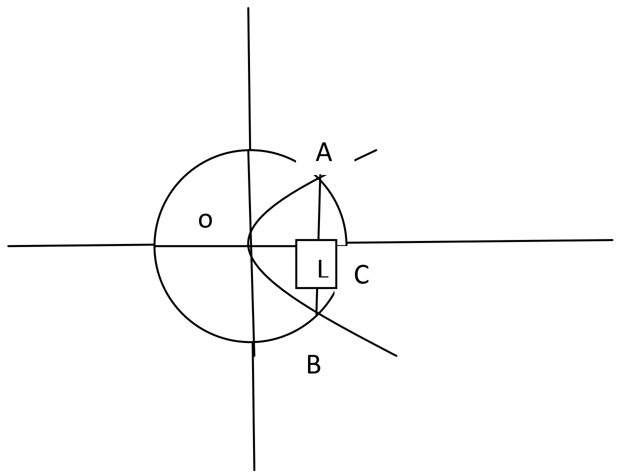
10.	Y = m(x – 3) dy/dx = m and equation is $y = y_1(x - 3)$	1 1
11.	$\frac{dy}{dx}+\frac{1}{x}y=\frac{1}{x}$ $I.F. = e^{\int \frac{1}{x}dx} = x$ Sol^n: yx = x + c	½ ½ 1
12.	Using $\left(\hat{a}+\hat{b}\right)^2 = (\hat{c})^2$, $2\,\hat{a}\cdot\hat{b} = -1$ $\left(\hat{a}-\hat{b}\right)^2 = 3 \Rightarrow \left\lvert\hat{a}-\hat{b}\right\rvert = \sqrt{3}$	1 1
13.	$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$ $\cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\} = \pi - \cos^{-1}z = -z$ Solving $x^2 + y^2 + z^2 + 2xyz = 1$	½ $1\frac{1}{2}$ 2
14.	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $y = (x-2)^2 + 1$ $x = 2 + \sqrt{y-1}$ Range = [1, ∞)	2 ½ 1 ½
15.	$C_2 \to C_2 - p.C_1$ $C_3 \to C_3 - (q.C_1 + p.C_2)$ Simplifying getting 1	1 2 1
16.	L.H.L. = $\lim_{h\to 0} f(0-h)$ $= \lim_{h\to 0}\frac{1-e^{-1/h}}{1+e^{-1/h}}$ = 1 R.H.L. = -1 L.H.L. ≠ R.H.L., hence not continuous	½ ½ ½ 3/2 1

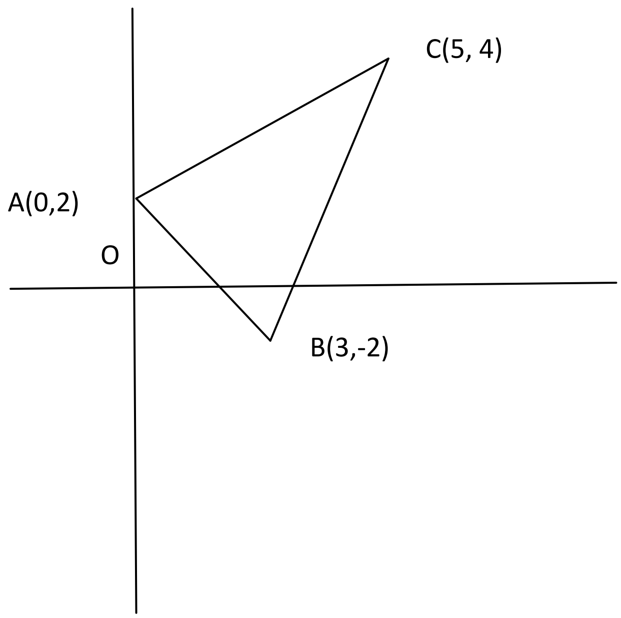
17.	$y = \tan^{-1} \dfrac{5\frac{x}{a}}{1-6\frac{x^2}{a^2}}$ = $\tan^{-1}\dfrac{3x}{a}$ + $\tan^{-1}\dfrac{2x}{a}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{3a}{a^2+9x^2} + \dfrac{2a}{a^2+4x^2}$	1 2 1
17.	**OR** y = $\cos^{-1}\dfrac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}}$ Putting $\dfrac{1}{x}$ = $\tan\theta$, getting y = 2 $\tan^{-1}\dfrac{1}{x}$ $\dfrac{dy}{dx}$ = - $\dfrac{2}{1+x^2}$	1 2 1
18	A = $\dfrac{1}{2}x^2 \sin\theta$ $\dfrac{dA}{dt}$ = $\dfrac{1}{2}\left[2x\sin\theta\dfrac{dx}{dt} + x^2 oos\theta\dfrac{d\theta}{dt}\right]$ = $\dfrac{1}{2}\left[2\cdot 12\cdot\dfrac{1}{12}\cdot\dfrac{1}{\sqrt{2}} + 12^2\cdot\dfrac{1}{\sqrt{2}}\cdot\dfrac{\pi}{180}\right]$ = $\sqrt{2}\left[\dfrac{1}{2}+\dfrac{\pi}{5}\right]$	1 1 1 1
19.	Put logx = t $\therefore$ I = $\int e^t \dfrac{t}{(1+t)^2}dt$ = $\int e^t\left[\dfrac{1}{1+t} + \dfrac{-1}{(1+t)^2}\right]dt$ = $\int e^t\{f(t) + f^{/}(t)\}\,dt$ = e^t f(t) + C = x/(1+logx) + C	 1 1 1 1

19.	**OR** I = $\int \frac{(3\sin x - 2)\cos x}{\sin^2 x - 4\sin x + 5}dx$ Putting sinx = t $\Rightarrow$ cosx dx = dt $\therefore$ I = $\int \frac{(3t-2)dt}{t^2 - 4t + 5}$ = $\int \frac{\frac{3}{2}(2t-4)+4}{t^2 - 4t + 5}dt$ = $\frac{3}{2}$ log(t^2 – 4t + 5) +4$\tan^{-1}$(t – 2) + C = $\frac{3}{2}$ log($\sin^2$x – 4sinx + 5) + 4$\tan^{-1}$(sinx – 2) + C	1 1 3/2 1/2
20	$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ Put y = vx $\therefore$ $\frac{dy}{dx}$ =v + x$\frac{dv}{dx}$ $\int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$ Log(1+v^2) = - logx + C X + y^2/x = C	1 2 1
21.	$\vec{d}$ is parallel to $\vec{a}\times\vec{b} \Rightarrow \lambda(\vec{a}\times\vec{b})$ $\therefore$ $\vec{d}$ = 32 $\lambda\hat{i}$ - $\lambda\hat{j} - 14\lambda\hat{k}$ Using $\vec{c}\cdot\vec{d} = 15, \lambda = 5/3$ $\therefore$ $\vec{d} = \frac{5}{3}$ (32 $\hat{i}$ - $\hat{j} - 14\hat{k}$	1 1 1 1

22.	$\vec{r} = 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$ $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \mu(-\hat{i} + 2\hat{j} + 2\hat{k})$ $\therefore 2 + \lambda = 1 - \mu \quad (i)$ $\lambda = -1 + \mu \quad (ii)$ $-1 - 2\lambda = 1 + 2\mu \quad (iii)$ $Solving\ (i)\ and\ (ii)\ \lambda = -1 \& \mu = 0$ which satisfy $(iii) \therefore$ they intersect point $(1, -1, 1)$	½ ½ ½ 3/2 1
23.	$P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C}) + P(\bar{A}\bar{B}C)$ $= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} + \frac{1}{2}\times\frac{1}{3}\times\frac{3}{4} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}$ $= \frac{11}{24}$ $P(problem\ is\ solved) = 1 - P(\bar{A}\bar{B}\bar{C})$ $= 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$ $= \frac{3}{4}$	1 ½ ½ 1 ½ 1/2
23.	**OR** Possible values of X are 0, 1 and 2 <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>25/49</td><td>20/49</td><td>4/49</td></tr></table>E(X)= $\sum x_i p_i$ =28/49, E(X^2) = $\sum x_i^2 p_i$ = 36/49 Var(X) = E(X^2) – $\{E(X)\}^2$ =20/49	3/2 1 3/2
24.	4x + 3y + 2z = 37000 5x + 3y + 4z =47000 X + y + z = 12000 A = $\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \& B = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$ AX=B $\Rightarrow$ X = A^{-1}B	1 1

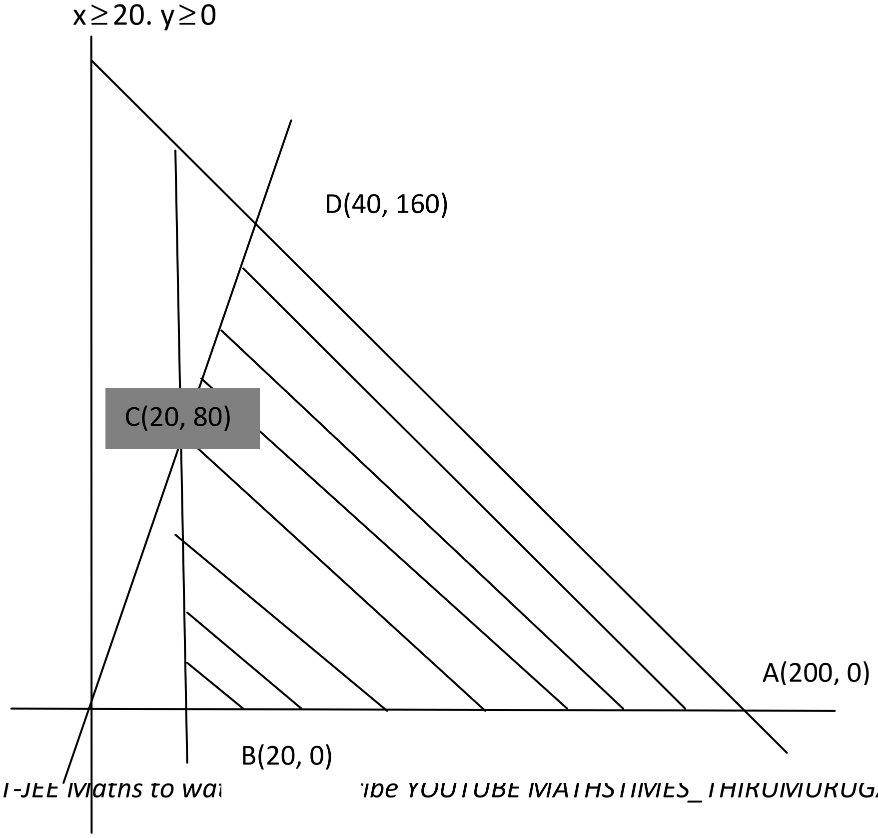
	$\|A\|$ = - 3, $A^{-1} = \frac{1}{-3}\begin{bmatrix}-1 & -1 & 6\\ -1 & 2 & -6\\ 2 & -1 & -3\end{bmatrix}$ $X = A^{-1}B = \begin{bmatrix}4000\\ 5000\\ 3000\end{bmatrix}$ Value	2 1 1
25.	 Height of the cone is h , x be the radius of the cylinder $V = \pi x^2(h - x\cot\alpha)$ $\frac{dv}{dx} = 2\pi hx - 3\pi x^2\cot\alpha = 0$ $\Rightarrow x = \frac{2h}{3}\tan\alpha$ Prove that second derivative is negative V= $\frac{4}{27}\pi h^3\tan^2\alpha$, correct proof	1 2 1 1 1
25.	OR Let length of the wire for circle = x cm and for square =28 – x cm Therefore combined area A = $\frac{x^2}{4\pi} + \frac{1}{16}(28 - x)^2$ For A to be maximum $\frac{dA}{dx} = \frac{x}{2\pi} - \frac{2}{16}(28 - x) = 0$ $\Rightarrow$ x = $\frac{28\pi}{4+\pi}$	1 1 1 3/2

	For square 28 - $\frac{28\pi}{4+\pi}$ $\frac{d^2A}{dx^2} < 0$	½ 1
26.	 Solving $y^2 = 4x$ & $4x^2 + 4y^2 = 9$, points of intersection are $(1/2, \sqrt{2})$ & $(1/2, -\sqrt{2})$ Required area = 2 area OAC = 2 {area OAL + area ALC} $=2\{ \int_0^{1/2} 2\sqrt{x}dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2 dx} \}$ $=2\{ 2\left[\frac{2}{3}x^{3/2}\right]_0^{1/2} + \left[\frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\right]_{1/2}^{3/2}$ $= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$	1 1 1 2 1

26.	OR  	1
	Equation of AB is x = $\frac{-3}{4}$ (y – 2), AC is x = $\frac{5}{2}$ (y – 2) & BC is x = $\frac{1}{3}$ (y +11)	3/2
	Required area = $\int_{-2}^{2}\frac{-3}{4}(y-2)dx+\int_{2}^{4}\frac{5}{2}(y-2)dx-\int_{-2}^{4}\frac{1}{3}(y+11)dx$	3/2
	= $\frac{-3}{4}\left[\frac{(y-2)^2}{2}\right]_{-2}^{2}+\frac{5}{2}\left[\frac{(y-2)^2}{2}\right]_{2}^{4}-\frac{1}{3}\left[\frac{(y-2)^2}{2}\right]_{-2}^{4}$	
	= 13 sq.unit	2
27.	Let A(1, 0, 1) be the point.	
	AB is drawn parallel to the plane to meet the given line at B	1
	B is on the given line, So coordinates of B are (λ, -2 λ +1, λ +1)	1
	Direction ratios of AB λ -1, λ +1, λ	1
	Normal is perpendicular to AB	1
	$\Rightarrow$ 2(λ -1) + (λ +1)+ λ =0	1
	Find B and hence AB	1

<table>
<tr><td>27.OR</td><td>Let A(a, 0, 0), B(0, b, 0) &C(0, 0, c)
Eq^n of the plane is x/a + y/b + z/c = 1
Let centroid be (u, v, w) $\Rightarrow$ a = 3u, b = 3v & c = 3w
Distance of the plane from the origin is $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = 3p$
$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{9p^2}$
Putting the values of a, b & c, we get
$\Rightarrow \frac{1}{u^2}+\frac{1}{v^2}+\frac{1}{w^2} = \frac{1}{p^2}$
Replacing u, v & w by x, y &z we get the locus.</td><td>1
1
1

1

1
1</td></tr>
<tr><td>28.</td><td>Let no. of first class ticket = x
and no. of second class ticket = y
Maximize Z = 400x + 300y
Constraints: x + y$\leq$ 200
4x - y$\leq$0
x$\geq$20. y$\geq$0


</td><td>

1

3/2

3/2</td></tr>
</table>

Corner point	Value of z =400x + 300y
A(200, 0)	Rs. 80000
B(20, 0)	Rs 8000
C(20, 80)	Rs 32000
D(40, 160)	Rs. 64000 (Maximum profit

2

29. E: The family has 2 girls & 3 boys.

F: The family has 3 girls & 2 boys.

G: The family has 4 girls & 1 boy. 1

H: The family has 5 girls

A: Two selected children are girls 1

P(E) = P(F) = P(G) =P(H) = 1/4 1

P(A/E) = $\frac{C(2,2)}{C(5,2)}$, P(A/F) = $\frac{C(3,2)}{C(5,2)}$, P(A/G) = $\frac{C(4,2)}{C(5,2)}$ & P(A/H) = $\frac{C(5,2)}{C(5,2)}$ 2

P(E/A)= $\frac{P(E).P(A/E}{\sum P(E).P(A/E)}$ 1

= $\frac{1}{10}$

*****

Email: Thirumurugan.kirithish@gmail.com
K.THIRUMURUGAN|PGT|GHSS|VAZHUTHAVUR|VILLUPURAM DT|TAMILNADU|9787062570