

Time allowed: 3 hours

Maximum marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of **29** questions divided into three sections-A, B C and D.
- (iii) Questions **1-4** in Section A are very short-answer type questions carrying **1** mark each.
- (iv) Questions **5-12** in Section B are short-answer type questions carrying **2** marks each.
- (v) Questions **13-23** in Section C are long-answer I type questions carrying **4** marks each.
- (vi) Questions **24-29** in Section D are long-answer II type questions carrying **6** marks each.
- (vii) There is no overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

1. If $\sin \left\{ \sin^{-1} \frac{3}{5} + \sec^{-1} x \right\} = 1$, find x .
2. Let \otimes be a binary operation defined on N given by $a \otimes b = 2^{ab}$; $a, b \in N$. Is ' \otimes ' associative?
3. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find $A(\text{adj}A)$.
4. If \vec{a} & \vec{b} are unit vectors, then can $\vec{a} \cdot \vec{b} = \sqrt{3}$?

SECTION – B

5. If A is skew symmetric then show that the matrix $B^T A B$ is skew symmetric.
6. Show that the tangents to the curve $y = x^3 + x + 1$ at $(1, 3)$ & $(-1, -1)$ are parallel.
7. Evaluate: $\int \frac{\sin x \cos x dx}{\sqrt{a^2 - (\sin^2 x)^2}}$

8. Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

9. A particle moves on the curve $3y = x^3 + 5$. Find the points on the curve at which the y co-ordinate is changing 4 times as fast as the x co-ordinate.

10. Form the differential equation of the family of lines passing through (3, 0).

11. Solve: $x \frac{dy}{dx} + y = 1$.

12. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

SECTION – C

13. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

14. Show that the function $f: [2, \infty) \rightarrow Y$, defined by $f(x) = x^2 - 4x + 5$ is one-one. for f to be onto find Y .

15. Using properties of the determinant prove that:
$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$

16. Discuss the continuity of the function $f(x) = \begin{cases} \frac{1-e^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ at $x = 0$.

17. If $y = \tan^{-1} \frac{5ax}{a^2 - 6x^2}$, prove that $\frac{dy}{dx} = \frac{3a}{a^2 + 9x^2} + \frac{2a}{a^2 + 4x^2}$

OR

If $y = \cos^{-1} \frac{x^2 - 1}{x^2 + 1}$, find $\frac{dy}{dx}$

18. Let x be the length of one of the equal sides of an isosceles triangle and let θ be the angle between them. If x is increasing at the rate of $(1/2)$ m/hr and θ is increasing at the rate of $\frac{\pi}{180}$ radian/hr, then find the in m^2/hr at which the area of the triangle is increasing when $x = 12m$ and $\theta = \pi/4$.

19. Evaluate: $\int \frac{\log x}{(1 + \log x)^2} dx$

OR

Evaluate: $\int \frac{(3 \sin x - 2) \cos x}{6 - \cos^2 x - 4 \sin x} dx$

20. Solve: $(x^2 - y^2) dx + 2xy dy = 0$

21. Let vector $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{d} which is perpendicular to the plane containing \vec{a} and \vec{b} , and $\vec{c} \cdot \vec{d} = 15$.
22. Show that two lines $\vec{r} = 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$ & $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \mu(-\hat{i} + 2\hat{j} + 2\hat{k})$ intersect also find the point of intersection.
23. A problem in mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively.
- (i) Find the probability that only one of them solves.
- (ii) Find the probability that the problem is solved.

OR

An urn contains 5 red and 2 black balls. Two balls are drawn one by one with replacement. Let X represents the number of black balls drawn. What are the possible values of X? Find mean and variance of X.

SECTION – D

24. Two institutions decided to award their employees for three values of resourcefulness, competence and determination in the form prizes at the rate of Rs. X, Rs. Y and Rs. Z respectively. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of Rs. 37000 and second institution decided to award respectively 5, 3 and 4 employees with a total prize money of Rs. 47000. If all the three prizes per person together amount to Rs.12000, then using matrix method find the values of x, y and z. What values are described in this question?
25. Show that the height of the cylinder of greatest volume which can be inscribed on a right circular cone of height h and semi vertical angle α is $\frac{1}{3}$ that of the cone and greatest volume of cylinder is $\frac{4}{27} \pi h^3 \tan^2 \alpha$

OR

A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into a square and other into a circle. What should be the lengths of two pieces so that the combined area of the circle and the square is minimum.

26. Find the are of the region: $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

OR

Using integration find the area of the region bounded by the triangle whose vertices are (0, 2), (3, -2) and (5, 4)

27. Find the distance of the point (1, 0, 1) from the line $\frac{x}{1} = \frac{y-1}{-2} = \frac{z-1}{1}$ measured parallel to the plane

$$2x + y + z + 3 = 0.$$

OR

A variable plane which remains at a constant distance $3p$ from origin cuts the co-ordinate axes at A, B and C.

Show that locus of centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class than first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an L.P.P. and solve it graphically.
29. In a family there are five children. Two children are selected at random, if they are found to be girls find the probability that the family has three girls and two boys.

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